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CONFIDENCE INTERVALS FOR PROCESS CAPABILITY INDICES WHEN DATA EXHIBITS AUTOCORRELATION

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Abstract. The paper discusses the effect of autocorrelations on process capability indices (PCIs).Confidence intervals are constructed for PCIs when data are autocorrelated. Approximate lower confidence limits for various Cpk are computed for AR(1) model. Industrial examples are considered to illustrate the results.

1. Introduction

Persistent pressure passed on to manufacturers from escalating consumer expectations and the ever growing global competitiveness have produced a rapidly rising interest in the development of various manufacturing strategy models. Academic and industrial circles are taking keen interest in the field of manufacturing strategy. Many manufacturing strategies are currently centered on the traditional concepts of focused manufacturing capabilities such as quality, cost, dependability and innovation.

Process capability analysis is conducted assuming that the process under study is in statistical control and independent observations are generated over time. However, in practice it is very common to come across processes which, due to their inherent natures, generate autocorrelated observations. The degree of autocorrelation affects the behaviour of patterns on control charts. Even, small levels of autocorrelation between successive observations can have considerable effects on the statistical properties of conventional control charts. When observations are autocorrelated the classical control charts exhibit non random patterns and lack of control. Many authors have considered the effect of autocorrelation on the performance of Statistical Process Control (SPC) charts.

Shore [22] investigated the effect of autocorrelations on PCIs and models the autocorrelation structure of a set of data, using an autoregressive model of order three (AR(3)). Zhang [25] discusses the use of the process capability indices Cp and Cpk when the process data are autocorrelated. Interval estimation procedures for Cp and Cpk are proposed and their properties are also studied. Process capability analysis when observations are autocorrelated is addressed using time series modelling and regression analysis by Noorosana [18]. Guevara and Vargas [6] deals with the comparison of process capability indices Cp , Cpk , Cpm and Cpmk when data are autocorrelated. Variances for their estimators are derived and coverage probabilities of some confidence intervals are calculated.

Key words and phrases. Auto correlation, confidence intervals, process capability indices, specification limits, stationary Gaussian processes .

With the development of measurement technology and data acquisition technology, sampling frequency is getting higher and the existence of autocorrelation cannot be ignored. Lovelace et al.[16] developed lower confidence limits for Cpk when data are uncorrelated as well as autocorrelated. Sun et al. [23] analyzes five estimation schemes of process capability for autocorrelated data and their comparisons are discussed for small sample and large sample data. Properties of Cp and Cpk for autocorrelated data in the presence of random measurement errors are explained in detail by Scagliarini [20,21]. Anis and Kuntal [1] explains some statistical properties of the estimator of Cp when sample observations are autocorrelated and affected by measurement errors. Also they discussed the first-order stationary autoregressive process where measurement error follows a Gaussian distribution.

Jose and Luke [8, 9] developed confidence intervals for process capability index Cpk for the balanced and unbalanced one-way random effect model following Bissells approximation method. Also Jose and Luke [10] introduced a method for comparing two PCIs under one-way random effect ANOVA model using generalized confidence intervals. Their method was found useful in selecting a superior supplier in a manufacturing firm with limited statistical knowledge.

Ke and Zhang [26] used a Monte Carlo simulation study and a real data example to compare asymptotic methods with the various moving average resampling techniques.In this study, they evaluated the finite sample performance of the six tests of autocorrelations for both normal and nonnormal series. Toor and Tanweer[24] study explores the power and size properties of selected autocorrelation tests. They compared autocorrelation tests in terms of their power under the given conditions i.e. specific sample sizes, autocorrelation coefficients and levels of significance.

In this article, we discuss the effects of PCIs when data are autocorrelated. The article is structured as follows. In Section 2, we have briefly reviewed process capability index C_p and some of the available approximate lower confidence limits for C_{pk}. In Section 3, stationary Gaussian processes is explained. Effect of autocorrelation on PCIs is described in Section 4. Confidence intervals for C_p and C_{pk} are computed and given in Section 5. Approximate lower confidence limits for C_{pk} are computed and presented in Section 6. Concluding remarks are given in Section 7.

2. Confidence Intervals for Process Capability Indices

Process Capability Indices are statistical devices used to measure the extent to which the process characteristic X under consideration meets specifications. For a normally distributed process with mean and variance σ^2, and lower and upper specification limits L and U, respectively, the index C_p by Juran et al. [11] is given by

$$C_p = \frac{U - L}{6\sigma}$$

2.1. Approximate lower confidence limits for C_{pk}.

Given the functional form C_{pk}, as a function of μ and σ, it is unlikely that exact lower confidence limits can be found for C_{pk}. Over the years, a number of researchers have derived various approximate lower confidence limits. Several of them are briefly described below, assuming an underlying normal distribution. We first note that if $\bar{X}$ and S denote the sample mean and sample standard deviation based on a random sample of n measurements, a natural estimator of C_{pk} is given by

$$\widehat{C}_{pk} = \frac{d - |\bar{X} - M|}{3S}$$

2.1.1. *Bissell's approximation.*

Bissell [2] derived an approximate expression for the variance of of C_{pk}, given by

$$Var(\widehat{C}_{pk}) = \frac{1}{9n} + \frac{C_{pk}^2}{2n-2}$$

Based on this expression, Bissell [2] proposed a $100(1-\alpha)\%$ lower confidence limit for C_{pk} as

$$B_{pk} = \widehat{C}_{pk} - Z_{1-\alpha}\sqrt{\frac{1}{9n} + \frac{\widehat{C}_{pk}^2}{2n-2}}$$

where $Z_{1-\alpha}$ is the $(1-\alpha)$ percentile value of the standard normal distribution.

2.1.2. *Heavlin's approximation.* Using the probability distribution of $\widehat{C}_{pk}$, Heavlin [7] obtained a $100(1-\alpha)\%$ lower confidence bound for C_{pk} as

$$H_{pk} = \widehat{C}_{pk} - Z_{1-\alpha}\sqrt{\frac{n-1}{9n(n-3)} + \widehat{C}_{pk}^2 \frac{1}{2(n-3)}\left(1 + \frac{6}{n-1}\right)}$$

2.1.3. *Nagata and Nagahata's approximation.* Nagata and Nagahata [17] obtained a $100(1-\alpha)\%$ lower confidence bound for C_{pk} as

$$N_{pk} = \sqrt{1 - \frac{2}{5(n-1)}\widehat{C}_{pk}} - Z_{1-\alpha}\sqrt{\frac{1}{9n} + \frac{\widehat{C}_{pk}^2}{2n-2}}$$

2.1.4. *Kushler and Hurley's approximation.* Kushler and Hurley [15] made an in-depth study of the various approximate confidence intervals. Though the above mentioned confidence intervals for C_{pk} are easy to calculate, they are not a multiple of $\widehat{C}_{pk}$. They examined the effect of various terms in the expressions of the above bounds and finally arrived at a simple multiple of $\widehat{C}_{pk}$ as a lower bound for C_{pk}, given by

$$KH_{pk} = \left[1 - \frac{Z_{1-\alpha}}{\sqrt{2(n-1)}}\right]\widehat{C}_{pk}$$

3. Stationary Gaussian processes

The correlation in a process can be captured using time series models. An important class of time series models are the stationary processes, which assume that the process remains in equilibrium around a constant mean. This type of models can provide a framework for seeking statistical control when monitoring auto correlated processes. A process is said to be stationary Gaussian if it is stationary and Gaussian simultaneously. For more details, refer to Brockwell & Davis [4]. The first-order autoregressive process AR(1) process is given by Box et al. [3] has the structure

$$X_t = \mu + \phi(X_{t-1} - \mu) + \xi_t \tag{3.1}$$

where X_t is the value of observation at time t, μ is the process mean and $\{\xi_t\}$ is a white noise process with zero mean and variance σ_ξ^2 where $\xi_t \sim N(0, \sigma_\xi^2)$. It is also assumed that $-1 < \phi < 1$. For an AR(1) model, the autocorrelation coefficient between X_t and X_{t-k} is given by

$$\rho_k = \phi^k,\ k = 1, 2, ...$$

4. The effect of process capability indices in discrete stationary Gaussian processes under autocorrelated data

Suppose for an industrial production process, a certain characteristic, for example, the concentration of a certain chemical component is observed. Assume that a quality characteristic is normally distributed with mean $\mu = 50$ and standard deviation $\sigma = 7$. The upper and lower specification limits are $U = 80$ and $L = 20$. Consider a process with independent observations and a process with observations following an AR(1) model in equation (3.1) where $\{X_t\}$ is a stationary Gaussian process with $\xi_t \sim N(0, \sigma_\xi^2)$ and $\sigma_\xi^2 = 7$. Also assume that $-1 < \phi < 1$. For different parameter combinations of n, ϕ and σ, we generated 10,000 random samples from a normal distribution with independent observations and a process with observations following an AR(1) model using MATLAB software.

The values of PCIs are computed and compared with and without autocorrelated processes. For each process, the mean, standard deviation and the capability indices C_p, C_{pk}, C_{pmk} and C_{pm} are computed. Through the results of a simulation study given in Table 1, we can ascertain the effect of autocorrelation on the expected value of the sample mean, sample standard deviation and different PCIs. It can be seen that the autocorrelation does not affect the values of sample means but affects the values of sample standard deviations. Higher the autocorrelation level, lower the capability index value.

Table 2 shows the expected values and standard errors of the sample mean and sample standard deviation for processes without and with autocorrelation following an AR(1) model. It reveals that the autocorrelation does not affect the expected value of the sample mean for different parameter combinations. As n increases, the estimated expected value of the standard error increases slightly for autocorrelated data.

TABLE 1. Effect of autocorrelation on mean, standard deviation (SD), C_p, C_{pk}, C_{pmk} and C_{pm} of processes following AR(1) model via simulation

ϕ	n	without AR(1) mean	without AR(1) SD	with AR(1) mean	with AR(1) SD	$C_p = C_{pk}$ without AR(1)	$C_p = C_{pk}$ with AR(1)	$C_{pm} = C_{pmk}$ without AR(1)	$C_{pm} = C_{pmk}$ with AR(1)
-0.75	10	50.7589	3.4664	50.5524	10.3790	2.8849	0.9635	0.2702	0.0753
	50	50.5081	6.8597	50.0931	8.1806	1.4578	1.2224	0.1388	0.1088
	100	50.0615	6.4432	50.1264	6.6910	1.5520	1.0330	0.1503	0.0842
-0.25	10	49.0355	5.3088	49.9378	6.9795	1.8837	1.4328	0.1880	0.1357
	50	49.1524	7.4569	50.0751	7.9541	1.4810	1.3406	0.1441	0.1216
	100	49.4380	6.6170	50.0908	7.0403	1.5113	1.4204	0.1454	0.1341
0.25	10	49.0154	8.2524	49.7602	8.8764	1.2918	1.2696	0.1174	0.1049
	50	49.1886	6.0512	49.6400	7.7150	1.6526	1.2962	0.1623	0.1183
	100	50.0452	7.1520	49.7116	10.8421	1.3982	0.9223	0.1313	0.0701
0.75	10	51.1824	7.1835	48.3467	7.2423	1.3921	1.3808	0.1305	0.1291
	50	50.2729	7.4208	51.4180	12.9319	1.3476	0.7733	0.1249	0.0520
	100	50.8279	6.9558	50.7433	9.0339	1.4377	1.1069	0.1363	0.0938

4.1. The Effect of Variances of PCIs under autocorrelated Data.

Let $\{X_t\}$ be a stationary Gaussian process. Let $\{X_1, X_2, ..., X_n\}$ be a sample of size n from the process $\{X_t\}$. Let $\overline{X} = \sum_{i=1}^{n} \frac{X_i}{n}$ and $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2$ be the sample mean and the sample variance respectively. Under the assumption that $\{X_t\}$ is a discrete stationary Gaussian process, Zhang [25]) derived the statistical properties of $\widehat{C}_p$ and the expected values and variances of $\overline{X}, S^2$ and S as follows.

$$\begin{aligned} E(\overline{X}) &= \mu_x,\ Var(\overline{X}) = \frac{\sigma_X^2}{n} g(n, \rho_i) \\ E(S^2) &= \sigma_X^2 f(n,\rho_i),\ Var(S^2) = \frac{2\sigma_X^4}{(n-1)^2} F(n,\rho_i) \\ E(S) &= [E(S^2)]^{1/2} = \sigma_X [f(n,\rho_i)]^{1/2} \end{aligned} \tag{4.1}$$

and

$$Var(S) = \frac{Var(S^2)}{4E(S^2)} = \left[\frac{\frac{2\sigma_X^4}{(n-1)^2}F(n,\rho_i)}{4\sigma_X^2 f(n,\rho_i)}\right] = \sigma_X^2 \frac{F(n,\rho_i)}{2(n-1)^2 f(n,\rho_i)} \tag{4.2}$$

where $\rho_i = \rho_X(i)$, for $i = 1, 2, ..., n$ is the autocorrelation of X at lag i,

$$f(n,\rho_i) = 1 - \frac{2}{n(n-1)} \sum_{i=1}^{n-1} (n-i)\rho_i$$

$$F(n,\rho_i) = n + 2\sum_{i=1}^{n-1}(n-i)\rho_i^2 + \frac{1}{n^2}\left[n + 2\sum_{i=1}^{n-1}(n-i)\rho_i\right]^2 - \frac{2}{n}\sum_{i=0}^{n-1}\sum_{j=0}^{n-i}(n-i-j)\rho_i\rho_j$$

TABLE 2. Expected values and standard errors of the sample mean and sample standard deviation for processes following an AR(1) model

			Mean		SD	
			without	with	without	with
n	ϕ		AR(1)	AR(1)	AR(1)	AR(1)
10	-0.75	Average	50.0048	49.9997	0.4857	0.6983
		S.E	0.0509	0.0322	0.0369	0.0819
	-0.25	Average	49.9990	50.0040	0.4866	0.5070
		S.E	0.0492	0.0499	0.0371	0.0402
	0.25	Average	49.9983	50.0008	0.4875	0.4920
		S.E	0.0500	0.0647	0.0366	0.0374
	0.75	Average	50.0040	49.9976	0.4867	0.5239
		S.E	0.0505	0.1519	0.0367	0.0532
50	-0.75	Average	49.999	50.0003	0.4979	0.7400
		S.E	0.0099	0.0059	0.0072	0.0183
	-0.25	Average	49.9987	49.9980	0.4971	0.5129
		S.E	0.0099	0.0082	0.0071	0.0080
	0.25	Average	50.0001	50.0033	0.4961	0.5086
		S.E	0.0101	0.0130	0.0071	0.0076
	0.75	Average	49.9985	50.0010	0.4978	0.6899
		S.E	0.0102	0.0375	0.0069	0.0177
100	-0.75	Average	49.9994	49.9991	0.4976	0.7480
		S.E	0.0049	0.0029	0.0035	0.0099
	-0.25	Average	49.9985	49.9988	0.4987	0.5149
		S.E	0.0050	0.0060	0.0036	0.0039
	0.25	Average	49.9998	49.9987	0.4995	0.5126
		S.E	0.0050	0.0067	0.0035	0.0039
	0.75	Average	49.9988	49.9967	0.4998	0.7205
		S.E	0.0049	0.0194	0.0036	0.0096

and

$$g(n,\rho_i) = 1 + \frac{2}{n}\sum_{i=1}^{n-1}(n-i)\rho_i$$

Zhang [25] derived the approximate value of variance of C_p and C_{pk} and are given by

$$Var(\widehat{C}_p) = C_p^2 \frac{F(n,\rho_i)}{2(n-1)^2 f^3(n,\rho_i)}$$

and

$$Var(\widehat{C}_{pk}) = \frac{C_{pk}^2}{f(n,\rho_i)}\left[\frac{g(n,\rho_i)}{9n\hat{C}_{pk}^2} \;+\; \frac{F(n,\rho_i)}{2(n-1)^2 f^2(n,\rho_i)}\right]$$

Guevara and Vargas [6] developed approximate value of variance of C_{pm} and C_{pmk} as follows.

$$Var(\widehat{C}_{pm}) = C_p^2\left[\frac{\frac{2F(n,\rho_i)}{(n-1)^2} \;+\; \frac{4g(n,\rho_i)\xi^2}{n}}{4[f(n,\rho_i)+\xi^2]^3}\right]$$

$$Var(\widehat{C}_{pmk}) = C_{pk}^2\left[\frac{1}{f(n,\rho_i)+\xi^2}\right]\left\{\frac{F(n,\rho_i)}{2(n-1)^2[f(n,\rho_i)+\xi^2]^2} + \frac{g(n,\rho_i)}{9n}\left[\frac{1}{C_{pk}} + \frac{6\xi}{2[f(n,\rho_i)+\xi^2]}\right]^2\right\}$$

where $\xi = \frac{\mu-T}{\sigma}$. If $\mu = T$, then $Var(\widehat{C}_{pm}) \simeq Var(\widehat{C}_p)$ and $Var(\widehat{C}_{pmk}) = Var(\widehat{C}_{pk})$.

Simulation study is done to compare the variances of these estimators for first order stationary autoregressive process with parameter ϕ. Variance of C_p and C_{pk} are plotted on Figures 1 and 2 respectively for different parameter combinations of n and ϕ and fixed value of PCI. As n increases, Var(C_p)and Var(C_{pk}) decreases.

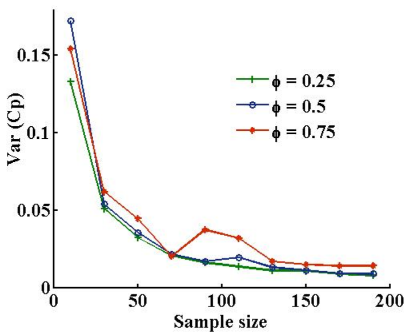

FIGURE 1. Variance of C_p with C_p=1.5

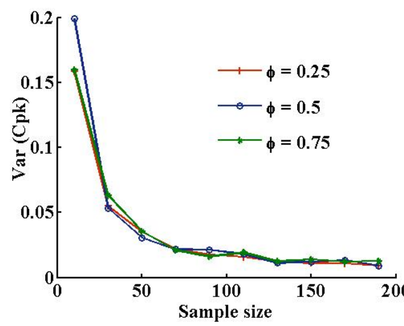

FIGURE 2. Variance of C_{pk} with C_{pk}=1.5

4.2. Numerical Example. To demonstrate how the above procedure may be illustrated through a data set, we consider the example given in Shore [22]. The data consists of 50 camshaft bearing diameters, given below (arranged in batches of N = 5; read each column from top to bottom; each batch mean is given in parentheses).

The autocorrelations and their respective standard errors for lags $k = 1, 2, ..., 10$ are obtained as follows.

$\rho_k = 0.74$	0.54	0.34	0.24	0.13	-0.021	-0.086	-0.21	-0.21	-0.31
S.E = 0.14	0.20	0.23	0.24	0.25	0.25	0.25	0.25	0.25	0.26

The PCIs for processes following an AR(1) model for the data in Table 3 are given in Table 4. It is clear that autocorrelation lowers the value of process capability indices. That is, autocorrelation affects the process capability indices of the production processes.

Table 3. Measurements of 50 camshaft bearing diameters

50	51	50.5	49	50	(50.1)
43	42	45	47	49	(45.2)
46	50	52	52.5	51	(50.3)
52	50	49	54	51	(51.2)
52	46	42	43	45	(45.6)
46	42	44	43	46	(44.2)
42	43	42	45	49	(44.2)
50	51	52	54	51	(51.6)
49	50	49.5	51	50	(49.9)
52	50	48	49.5	49	(49.7)

Table 4. Various values of PCIs for the example given in Table 6.3

	without AR(1)	with AR(1)
C_p	0.5655	0.5612
C_{pk}	0.3741	0.3720
C_{pmk}	0.0620	0.0614
C_{pm}	0.0930	0.0922

5. Confidence intervals for the capability indices under stationary Gaussian processes

Zhang [25] derived the approximate expected value and variance of S^2, variances of $\widehat{C}_p$ and $\widehat{C}_{pk}$ in terms of the process autocorrelation function. Interval estimates of C_p and C_{pk} for autocorrelated processes are computed using the symmetrical construction method. Given the specification limits and a sample of process data $\{X_1, X_2, ..., X_n\}$, interval estimators of C_p and C_{pk} can be constructed as

$$\widehat{C}_p \pm k\widehat{\sigma}_{C_p},\ \ \widehat{C}_{pk} \pm k\widehat{\sigma}_{C_{pk}}$$

where k is a constant chosen by the user, and $\widehat{\sigma}_{C_p}$ and $\widehat{\sigma}_{C_{pk}}$ are the sample standard deviations of $\widehat{C}_p$ and $\widehat{C}_{pk}$, respectively. For selected values of $\widehat{C}_p, \widehat{C}_{pk}, n$ and ϕ for AR(1) processes Zhang [25] investigated coverage probabilities for both indices.

Interval estimation for C_p following AR(1) model for various parameter combinations using $\widehat{C}_p \pm k\widehat{\sigma}_{C_p}$ are reported in Table 5. Also, Table 6 gives confidence intervals for C_{pk} following AR(1) model for various parameter combinations using $\widehat{C}_{pk} \pm k\widehat{\sigma}_{C_{pk}}$. For fixed values of ϕ, the lengths of the intervals decrease when n increases. For a fixed value of n, when $|\phi|$ increases, the length of the intervals increases. Tables 5 and 6 reveals that based on a small sample and large $|\phi|$, the uncertainty in C_p and C_{pk} are relatively large.

TABLE 5. Interval estimation for C_p following AR(1) model for various parameter combinations

k	$\widehat{C}_p$	n	ϕ=0.25	ϕ=0.50	ϕ=0.75
2	1.33	10	(0.6127, 2.0473)	(0.6103, 2.0497)	(0.5793, 2.0807)
		50	(1.0237, 1.6363)	(1.0022, 1.6578)	(0.9553, 1.7047)
		100	(1.0990, 1.5610)	(1.1074, 1.5526)	(1.0218, 1.6382)
3	1.33	10	(0.3490, 2.3110)	(0.3019, 2.3581)	(0.2308, 2.4292)
		50	(0.8649, 1.7951)	(0.7695, 1.8905)	(0.7216, 1.9384)
		100	(1.0091, 1.6509)	(0.9918, 1.6682)	(0.9008, 1.7592)
2	2	10	(0.8061, 3.1939)	(1.0406, 2.9594)	(0.8931, 3.1069)
		50	(1.5360, 2.4640)	(1.5082, 2.4918)	(1.4931, 2.5069)
		100	(1.6711, 2.3289)	(1.6533, 2.3467)	(1.6040, 2.3960)
3	2	10	(0.4698, 3.5302)	(0.4821, 3.5179)	(0.4692, 3.5308)
		50	(1.2984, 2.7016)	(1.2565, 2.7435)	(1.1654, 2.8346)
		100	(1.5136, 2.4864)	(1.5121, 2.4879)	(1.3776, 2.6224)

TABLE 6. Interval estimation for C_{pk} following AR(1) model for various parameter combinations

k	$\widehat{C}_{pk}$	n	$\phi = 0.25$	$\phi = 0.50$	$\phi = 0.75$
2	1.33	10	(0.5902, 2.0698)	(0.5896, 2.0704)	(0.5565, 2.1035)
		50	(1.0188, 1.6412)	(0.9928, 1.6672)	(0.9585, 1.7015)
		100	(1.0945, 1.5655)	(1.1013, 1.5587)	(1.0292, 1.6308)
3	1.33	10	(0.3452, 2.3148)	(0.3126, 2.3474)	(0.1990, 2.4610)
		50	(0.8570, 1.8030)	(0.7639, 1.8961)	(0.7148, 1.9452)
		100	(1.0223, 1.6377)	(0.9913, 1.6687)	(0.9704, 1.6896)
2	2	10	(0.7935, 3.2065)	(1.0383, 2.9617)	(0.8888, 3.1112)
		50	(1.5447, 2.4553)	(1.5026, 2.4974)	(1.4887, 2.5113)
		100	(1.6766, 2.3234)	(1.6580, 2.3420)	(1.6124, 2.3876)
3	2	10	(0.4668, 3.5332)	(0.4757, 3.5243)	(0.4673, 3.5327)
		50	(1.3052, 2.6948)	(1.2622, 2.7378)	(1.2037, 2.7963)
		100	(1.5051, 2.4949)	(1.5067, 2.4933)	(1.3758, 2.6242)

6. Comparative study of effect of PCIs for Autocorrelated Data

For many industrial processes, like oil refinery, paper production etc, it is well known that the level of individual quality characteristics often varies with a wave-like pattern. Observations on such a characteristic, made at equal time intervals, are then supposed to be dependent and the outcome of such a process can be modeled in many ways. Let us consider the lower and upper specifications are $L = 5$ and $U = 17$. For different values of n and ϕ, we calculated the approximate lower confidence limits for $\widehat{C}_{pk}$. To study the distributional characteristics of the capability indices, samples were generated for each of the sample sizes $n = 15, 50, 100, 150$ and 200. We compute the approximate lower confidence limits for C_{pk}. Table 7 gives the comparative study of effect of autocorrelation on PCIs for

TABLE 7. lower confidence limits for C_{pk} when observations are independent and autocorrelated (AR(1)) for various parameter combinations

ϕ	n	Bissell without AR(1)	Bissell with AR(1)	Nagata & Nagahata without AR(1)	Nagata & Nagahata with AR(1)	Heavlin without AR(1)	Heavlin with AR(1)	Kushler & Hurley's without AR(1)	Kushler & Hurley's with AR(1)
0.25	10	4.4896	3.5921	4.3246	3.4600	3.1699	2.5358	4.4949	3.5986
	50	4.6342	4.1206	4.6114	4.1004	4.5587	4.0536	4.6374	4.1243
	100	4.4802	3.5478	4.4699	3.5397	4.4562	3.5289	4.4827	3.5510
	150	4.8924	4.6625	4.8851	4.6556	4.8786	4.6494	4.8944	4.6646
	200	5.1872	4.2130	5.1815	4.2084	5.1778	4.2054	5.1888	4.2150
0.5	10	4.8727	2.9021	4.6937	2.7953	3.4406	2.0482	4.8776	2.9102
	50	4.7348	3.0883	4.7115	3.0731	4.6577	3.0380	4.7379	3.0931
	100	4.4465	3.6557	4.4363	3.6474	4.4228	3.6362	4.4491	3.6588
	150	5.0280	2.2891	5.0205	2.2857	5.0138	2.2826	5.0299	2.2932
	200	4.3588	4.1472	4.3541	4.1427	4.3510	4.1397	4.3608	4.1492
0.75	10	2.3747	1.5732	2.2872	1.5149	1.6754	1.1085	2.3846	1.5881
	50	4.5568	2.8895	4.5345	2.8753	4.4827	2.8426	4.5601	2.8947
	100	4.6945	4.0806	4.6837	4.0712	4.6694	4.0588	4.6969	4.0834
	150	4.5969	3.3076	4.5901	3.3027	4.5839	3.2983	4.5990	3.3105
	200	4.3886	2.9144	4.3838	2.9112	4.3807	2.9092	4.3905	2.9173

autocorrelated data. It shows that the effect of autocorrelation changes the values of PCIs for various parameter combinations.

7. Conclusion

Autocorrelation is prevalent in continuous production processes, such as the processes in the chemical and pharmaceutical industries. The paper explores the properties of PCIs when observations are autocorrelated. PCIs are widely used in manufacturing industries to measure the performance of a process in meeting preset specifications limits. Autocorrelation has bearing on the variances of PCI estimators. It demonstrates that higher the sample size lower the variances. As sample size and autocorrelation increase, the uncertainty in PCIs tends to diminish. We computed approximate lower confidence limits for C_{pk} and established that autocorrelation affects the values of PCIs. Higher the autocorrelation level, lower the capability index.

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