ISSN: 0974-8571 Vol.13 No. 2 December, 2021

International Journal of Computational Intelligence in Control

Representation of Different Properties and Graphs using Nonagonal Neutrosophic Number

Sana Akram[1], Javeria Yasin[2], Nosheen Ilyas[2], Muhammad Farhan Tabassum[1*]
[1]Department of Mathematics, University of Management and Technology, Lahore, Pakistan. [2]Department of Mathematics, Lahore Garrison University, Lahore, Pakistan.
*Corresponding Author: Muhammad Farhan Tabassum, E-mail: farhanuet12@gmail.com

Abstract: The generalize form of fuzzy set is soft set theory, further it is moved to fuzzy soft set theory, followed by to deal with fluctuations Neutrosophic numbers are used. The concept of Neutrosophic set based on membership values of truth, indeterminacy and falsity, which are independent and which play vital role in situations like uncertainty, incomplete and inconsistence. The membership values of truth, indeterminacy and falsity with nine edges, Nonagonal Neutrosophic Numbers have a wide range of applications while handling further variances in the decision-making condition. In this paper Nonagonal Neutrsophic Numbers and their $\propto -$ cuts and graphs are found out. Further it is applied on real life problem which is solved by Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) and VIekriterijumsko KOmpromisno Rangiranje (VIKOR) method, which are Multi Criteria Decision Making (MCDM). After that we compare both methods to check which method will give more accurate result in case of indeterminacy.

Keywords: Neutrosophic Numbers, Nonagonal Numbers, MCDM, TOPSIS Method, VIKOR Method

INTRODUCTION

The hypothesis of fuzzy set, purposed by Lotfi Aliasker Zadeh [12] has picked up affective applications in different areas. He presents the concept of fuzzy set. He isn't as it were the originator of fuzzy set hypothesis but he moreover been one of the foremost imperatives contributes to related with the hypothesis and preserved of numerous of its application.

Soft set theory is a generality of fuzzy set, which was introduced in a parametric way by Molodstov in 1999 to deal with uncertainty. We are familiar with Pawlak's soft sets, which are a different idea that can be used to solve a wide range of isssues.

Smarandache suggested the conception of neutrosophic set [2, 10, 15] in 1995, and idea was issued in 1998. They have three divergent logic components: i. truthfulness, ii. Indeterminacy, and iii. Falsity. This concept also includes a concept of uncertainty and the study has a significant influence across various research domain. In neutrosophic truth membership is stated as $\tilde{T},$ indeterminacy membership is stated by $\tilde{I}$, falsity membership is stated by $\tilde{F}$. These are all self-determining, and their summation is among $0 \leq \tilde{T} + \tilde{I} + \tilde{F} \leq 3$.

Scientists from various fields looked into the properties and fluctuations of neutrosophic numbers, as well as the properties of correlation between them. In terms of decision-making, the neutrosophic fuzzy number is absolutely important, and triangular neutrosophic numbers was just getting started, after that Trapezoidal neutrosophic numbers, then Pentagonal neutrosophic numbers, then Hexagonal neutrosophic numbers, then Heptagonal neutrosophic numbers, then Octagonal neutrosophic numbers and then Nonagonal neutrosophic numbers. Moreover, scholars continue to work to introduce new possibilities.

We have nine edges in expressions of truthness, falsity, and indeteminacy membership value in the case of nonagonal neutrosophic amounts. As a result, give us more flexibility to deal with more volatility. We needed a system that could be used in these difficult situations because decision making is so diverse and has so many possibilities.

Decision-making applications in various fields such as phone collection, game prediction, supplier selection, medical, and staff selection.

There might not always be a finite number of options when making a decision, or there might be several alternatives to the original decision. There is also some probability of the criteria not having an acceptable option. MCDM is an approach designed to estimate concerns with a finite or infinite number of choices.

Hwang and Yoon (1981) [1] were the first to use the TOPSIS system (Also see Chen and Hwang, 1992). In a broad context, it is a human being's need to make "calculated" decisions in a situation with many choices. In scientific terms, the aim is to establish theoretical and computational approaches that consider several options through numerous parameters.

TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) is a MCDM phase of numbrics. This is a tool that can be used in a variety of situations and has a basic mathematical model. Furthermore, it is a very realistic approach that relies on computer assistance. The process has been used for the past three decades, and there are several publications on it. Some recent wrok based on graph theory is reported in [25-28].

The three main stages in the classical TOPSIS technique are as follows.

(1) Predict one negative ideal solution and one positive ideal result for the initial decision-making challenge.

(2) To figure out how far each alternate is from the positive ideal result/negative ideal result, or how close each alternate is to the positive ideal result/negative ideal result.

(3) To mandate the options rendering to their distance or similarity closeness coefficients.

In recent years, VIKOR (VIseKriterijumska Optimizacija I KOmpromisno Resenje) [6] technique, which has valuable facts of understanding the balance among group utility maximization and human regrettably minimization, has been regarded as a useful technique to use in a variety of decision-making fields. As compared to other models, the VIKOR method has the benefit of considering a balance amongst group utility maximisation and individual regret minimization.

Motivation: A lot of research articles regards to neutrosophic arena available, which they apply and exaggerated the concept of MCDM. They presented the notion, properties along with application from triangular to nonagonal neutrosophic numbers.

structure of paper: The article is structured as follow as shown in the figure 1:

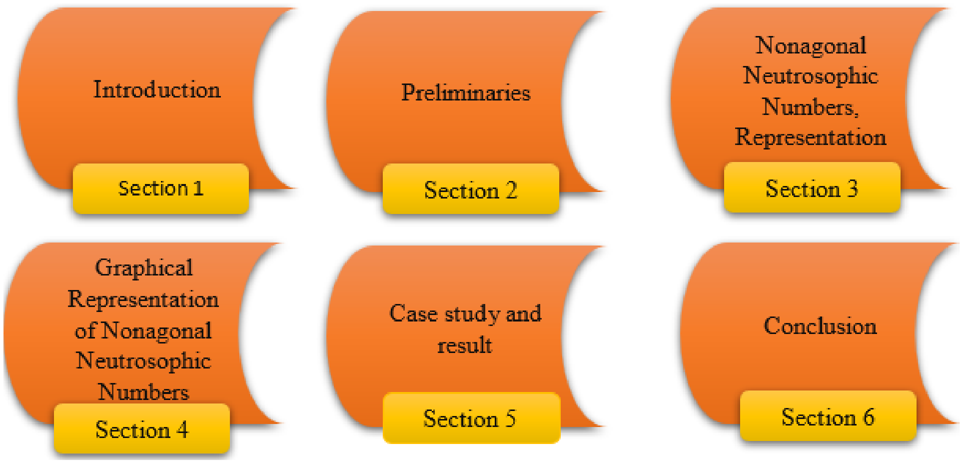


Figure 1. Structure of article

PRELIMINARIES

Fuzzy sets: A collection is said to be fuzzy set[12] $\check{\mathcal{A}}: \mho \longrightarrow [0, 1]$

Neutrosophic sets: Let $\bar{\bar{\mathcal{E}}}$ is a neutrosophic number [4, 10] if

$$\bar{\bar{\mathcal{E}}} = \{< \mathcal{X}: T_{\bar{\bar{\mathcal{E}}}}(x), \quad I_{\bar{\bar{\mathcal{E}}}}(x), \quad F_{\bar{\bar{\mathcal{E}}}}(x) >; \mathfrak{X} \in U\}$$

where $T_{\bar{\bar{\mathcal{E}}}}(x)$: Truth membership function

$I_{\bar{\bar{\mathcal{E}}}}(x)$: Indeterminacy membership function

$F_{\bar{\bar{\mathcal{E}}}}(x)$: Falsity membership function

With condition: $0 \leq T_{\bar{\bar{\mathcal{E}}}}(x) + I_{\bar{\bar{\mathcal{E}}}}(x) + F_{\bar{\bar{\mathcal{E}}}}(x) \leq 3$

Triangular Neutrosophic Numbers: Triangular Neutrosophic Numbers [16] can be defined as $Tr = (c_1, c_2, c_3; g_1, g_2, g_3; h_1, h_2, h_3)$ as well as truth, indeterminacy and falsity are given as:

$$\tilde{T}_{Tr} = \begin{cases} \dfrac{x - c_1}{c_2 - c_1} & for\ c_1 \leq x < c_2 \\ 1 & when\ x = c_2 \\ \dfrac{c_3 - x}{c_3 - c_2} & for\ c_2 < x \leq c_3 \\ 0 & otherwise \end{cases}$$

$$\tilde{I}_{Tr} = \begin{cases} \dfrac{g_2 - x}{g_2 - g_1} & for\ g_1 \leq x < g_2 \\ 0 & when\ x = g_2 \\ \dfrac{x - g_2}{g_3 - g_2} & for\ g_2 < x \leq g_3 \\ 1 & otherwise \end{cases}$$

$$\tilde{F}_{Tr} = \begin{cases} \dfrac{x - h_1}{h_1 - h_2} & for\ h_1 \leq x < h_2 \\ 1 & when\ x = h_2 \\ \dfrac{h_3 - x}{h_3 - h_2} & for\ h_2 < x \leq h_3 \\ 0 & otherwise \end{cases}$$

Where $0 \leq \tilde{T}_{Tr} + \tilde{I}_{Tr} + \tilde{F}_{Tr} \leq 3$

$x \in Tr$

The parametric form of this kind is

$$Tr_{(\mu,\vartheta,\varphi)} = [\tilde{T}_{Tr1}(\tilde{\mu}), \tilde{T}_{Tr2}(\tilde{\mu}); \tilde{I}_{Tr1}(\tilde{\vartheta}), \tilde{I}_{Tr2}(\tilde{\vartheta}); \tilde{F}_{Tr1}(\tilde{\varphi}), \tilde{F}_{Tr2}(\tilde{\varphi})]$$

Where

$$\tilde{T}_{Tr1}(\tilde{\mu}) = c_1 + \tilde{\mu}(c_2 - c_1)\ ;\ \tilde{T}_{Tr2}(\tilde{\mu}) = c_3 - \tilde{\mu}(c_3 - c_2)$$
$$\tilde{I}_{Tr1}(\tilde{\vartheta}) = g_2 - \tilde{\vartheta}(g_2 - g_1)\ ;\ \tilde{I}_{Tr2}(\tilde{\vartheta}) = g_3 + \tilde{\vartheta}(g_3 - g_2)$$
$$\tilde{F}_{Tr1}(\tilde{\varphi}) = h_1 - \tilde{\varphi}(h_2 - h_1)\ ;\ \tilde{F}_{Tr2}(\tilde{\varphi}) = h_3 - \tilde{\varphi}(h_3 - h_2)$$

Here

$$0 < \tilde{\mu}^{\cdot} \leq 1, \qquad 0 < \tilde{\vartheta}^{\cdot} \leq 1, \qquad 0 < \tilde{\varphi}^{\cdot} \leq 1$$

And $0 < \tilde{\mu}^{\cdot} + \tilde{\vartheta}^{\cdot} + \tilde{\varphi}^{\cdot} < 3$

Trapezoidal Neutrosophic Number: Trapezoidal Neutrosophic Number [17] is defined as

$$TrP = [e^{\cdot}, f^{\cdot}, g^{\cdot}, h^{\cdot} : \theta^{\cdot}]\ ;\ [l^{\cdot}, m^{\cdot}, n^{\cdot}, o^{\cdot} : \varepsilon^{\cdot}]\ ;\ [q^{\cdot}, r^{\cdot}, s^{\cdot}, t^{\cdot} : \varphi^{\cdot}]$$

Where $\theta^{\cdot}, \varepsilon^{\cdot}, \varphi^{\cdot} \epsilon\ [0,1]$

With condition: $0 \leq \theta^{\cdot} + \varepsilon^{\cdot} + \varphi^{\cdot} \leq 3$

Pentagonal Neutrosophic Number

Pentagonal Neutrosophic Number [18] is given as

$$Pen = [\infty, \sim, \approx, \nabla, \partial : \rho]\ , [\infty_1, \sim_1, \approx_1, \nabla_1, \partial_1 ; \sigma]\ , [\infty_2, \sim_2, \approx_2, \nabla_2, \partial_2\ ; \tau]$$

Where

$\rho, \sigma, \tau\ \epsilon\ [0, 1]$

Truth membership $T_{Pen}: \mathbb{R} \rightarrow [0, 1]$

Indeterminacy membership $I_{Pen}: \mathbb{R} \rightarrow [0, 1]$

Falsity membership $F_{Pen}: \mathbb{R} \rightarrow [0,1]$

Octagonal Neutrosophic Number

Octagonal Neutrosophic number [5] well-defined as

$$Oct = \left[(m^{\cdot\cdot}, n^{\cdot\cdot}, o^{\cdot\cdot}, p^{\cdot\cdot}, q^{\cdot\cdot}, r^{\cdot\cdot}, s^{\cdot\cdot}, t^{\cdot\cdot}) :\ \ddot{\mathbb{A}}\right] \left[(m^{\cdot\cdot 1}, n^{\cdot\cdot 1}, o^{\cdot\cdot 1}, p^{\cdot\cdot 1}, q^{\cdot\cdot 1}, r^{\cdot\cdot 1}, s^{\cdot\cdot 1}, t^{\cdot\cdot 1}) :\ \ddot{\mathbb{B}}\right] \left[(m^{\cdot\cdot 2}, n^{\cdot\cdot 2}, o^{\cdot\cdot 2}, p^{\cdot\cdot 2}, q^{\cdot\cdot 2}, r^{\cdot\cdot 2}, s^{\cdot\cdot 2}, t^{\cdot\cdot 2}) :\ \ddot{\mathbb{C}}\right]$$

Where $\ddot{\mathbb{A}}, \ddot{\mathbb{B}}, \ddot{\mathbb{C}}\ \in [0,1]$ as well as truth, falsity and indeterminacy well defined as

$$\ddot{\mathbb{A}}_{Oct}(x) = \begin{cases} \ddot{\mathbb{A}}_{Oct0}(x) & m^{\cdot\cdot} \leq x^{\cdot} < n^{\cdot\cdot} \\ \ddot{\mathbb{A}}_{Oct1}(x) & n^{\cdot\cdot} \leq x^{\cdot} < 0^{\cdot\cdot} \\ \ddot{\mathbb{A}}_{Oct2}(x) & o^{\cdot\cdot} \leq x^{\cdot} < p^{\cdot\cdot} \\ \ddot{\mathbb{A}}_{Oct3}(x) & p^{\cdot\cdot} \leq x^{\cdot} < q^{\cdot\cdot} \\ \ddot{\mathbb{A}} & x^{\cdot} = q^{\cdot\cdot} \\ \ddot{\mathbb{A}}_{Oct3}(x) & q^{\cdot\cdot} \leq x^{\cdot} < r^{\cdot\cdot} \\ \ddot{\mathbb{A}}_{Oct2}(x) & r^{\cdot\cdot} \leq x^{\cdot} < s^{\cdot\cdot} \\ \ddot{\mathbb{A}}_{Oct1}(x) & s^{\cdot\cdot} \leq x^{\cdot} < t^{\cdot\cdot} \\ 0 & otherwise \end{cases}$$

$$\ddot{\mathbb{B}}_{Oct}(x) = \begin{cases} \ddot{\mathbb{B}}_{Oct0}(x) & m^{\cdot\cdot 1} \leq x^{\cdot} < n^{\cdot\cdot 1} \\ \ddot{\mathbb{B}}_{Oct1}(x) & n^{\cdot\cdot 1} = x^{\cdot} < o^{\cdot\cdot 1} \\ \ddot{\mathbb{B}}_{Oct2}(x) & o^{\cdot\cdot 1} \leq x^{\cdot} < p^{\cdot\cdot 1} \\ \ddot{\mathbb{B}}_{Oct3}(x) & p^{\cdot\cdot 1} \leq x^{\cdot} < q^{\cdot\cdot 1} \\ \ddot{\mathbb{B}} & x^{\cdot} = q^{\cdot\cdot 1} \\ \ddot{\mathbb{B}}_{Oct3}(x) & q^{\cdot\cdot 1} \leq x^{\cdot} < r^{\cdot\cdot 1} \\ \ddot{\mathbb{B}}_{Oct2}(x) & r^{\cdot\cdot 1} \leq x^{\cdot} < s^{\cdot\cdot 1} \\ \ddot{\mathbb{B}}_{Oct1}(x) & s^{\cdot\cdot 1} \leq x^{\cdot} < t^{\cdot\cdot 1} \\ 1 & otherwise \end{cases}$$

$$\ddot{\mathbb{C}}_{Oct}(x) = \begin{cases} \ddot{\mathbb{C}}_{Oct0}(x) & m^{\cdot\cdot 2} \leq x^{\cdot} < n^{\cdot\cdot 2} \\ \ddot{\mathbb{C}}_{Oct1}(x) & n^{\cdot\cdot 2} \leq x^{\cdot} < o^{\cdot\cdot 2} \\ \ddot{\mathbb{C}}_{Oct2}(x) & o^{\cdot\cdot 2} \leq x^{\cdot} < p^{\cdot\cdot 2} \\ \ddot{\mathbb{C}}_{Oct3}(x) & p^{\cdot\cdot 2} \leq x^{\cdot} < q^{\cdot\cdot 2} \\ \ddot{\mathbb{C}} & x^{\cdot} = q^{\cdot\cdot 2} \\ \ddot{\mathbb{C}}_{Oct3}(x) & q^{\cdot\cdot 2} \leq x^{\cdot} < r^{\cdot\cdot 2} \\ \ddot{\mathbb{C}}_{Oct2}(x) & r^{\cdot\cdot 2} \leq x^{\cdot} < s^{\cdot\cdot 2} \\ \ddot{\mathbb{C}}_{Oct1}(x) & s^{\cdot\cdot 2} \leq x^{\cdot} < t^{\cdot\cdot 2} \\ 1 & otherwise \end{cases}$$

where

$Oct = \left[(m^{\cdot\cdot} < n^{\cdot\cdot} < o^{\cdot\cdot} < p^{\cdot\cdot} < q^{\cdot\cdot} < r^{\cdot\cdot} < s^{\cdot\cdot} < t^{\cdot\cdot}) : \ddot{\mathbb{A}}\right] \left[(m^{\cdot\cdot 1} < n^{\cdot\cdot 1} < o^{\cdot\cdot 1} < p^{\cdot\cdot 1} < q^{\cdot\cdot 1} < r^{\cdot\cdot 1} < s^{\cdot\cdot 1} < t^{\cdot\cdot 1}) : \ddot{\mathbb{B}}\right] \left[(m^{\cdot\cdot 2} < n^{\cdot\cdot 2} < o^{\cdot\cdot 2} < p^{\cdot\cdot 2} < q^{\cdot\cdot 2} < r^{\cdot\cdot 2} < s^{\cdot\cdot 2} < t^{\cdot\cdot 2}) : \ddot{\mathbb{C}}\right]$

Nonagonal Neutrosophic Number

Nonagonal Nutrosophic Numbers (NNN) [4, 10] are defined as

$$\mathcal{G}: [(q^{;} r^{;}, s^{;}, t^{;}, u^{;}, v^{;}, w^{;}, y^{;}, z^{;}): \delta] \, [(q^{;1}, r^{;1}, s^{;1}, t^{;1}, u^{;1}, v^{;1}, w^{;1}, y^{;1}, z^{;1}): \varphi]$$
$$[(q^{;2}, r^{;;2}, s^{;2}, t^{;2}, u^{;2}, v^{;2}, w^{;2}, y^{;2}, z^{;2}): \omega]$$

Where $\delta, \varphi, \omega \in [0,1]$

Membership function truth, indeterminacy anf falsity is define as

$$\delta_{\mathcal{G}}(x) = \begin{cases} \delta_{\mathcal{G}}(x) & q^{;} \le x^{\cdot} < r^{;} \\ \delta_{\mathcal{G}}(x) & r^{;} \le x^{\cdot} < s^{;} \\ \delta_{\mathcal{G}}(x) & s^{;} \le x^{\cdot} < t^{;} \\ \delta_{\mathcal{G}}(x) & t^{;} \le x^{\cdot} < u^{;} \\ \delta & x^{\cdot} = u^{;} \\ \delta_{\mathcal{G}}(x) & u^{;} \le x^{\cdot} < v^{;} \\ \delta_{\mathcal{G}}(x) & v^{;} \le x^{\cdot} < w^{;} \\ \delta_{\mathcal{G}}(x) & w^{;} \le x^{\cdot} < y^{;} \\ \delta_{\mathcal{G}}(x) & y^{;} \le x^{\cdot} < z^{;} \\ 0 & otherwise \end{cases}$$

$$\varphi_{\mathcal{G}}(x) = \begin{cases} \varphi_{\mathcal{G}}(x) & q^{;1} \le x^{\cdot} < r^{;1} \\ \varphi_{\mathcal{G}}(x) & r^{;1} \le x^{\cdot} < s^{;1} \\ \varphi_{\mathcal{G}}(x) & s^{;1} \le x^{\cdot} < t^{;1} \\ \varphi_{\mathcal{G}}(x) & t^{;1} \le x^{\cdot} < u^{;1} \\ \varphi & x^{\cdot} = u^{;1} \\ \varphi_{\mathcal{G}}(x) & u^{;1} \le x^{\cdot} < v^{;1} \\ \varphi_{\mathcal{G}}(x) & v^{;1} \le x^{\cdot} < w^{;1} \\ \varphi_{\mathcal{G}}(x) & w^{;1} \le x^{\cdot} < y^{;1} \\ \varphi_{\mathcal{G}}(x) & y^{;1} \le x^{\cdot} < z^{;1} \\ 1 & otherwise \end{cases}$$

$$\omega_{\mathcal{G}}(x) = \begin{cases} \omega_{\mathcal{G}}(x) & q^{;2} \le x^{\cdot} < r^{;2} \\ \omega_{\mathcal{G}}(x) & r^{;2} \le x^{\cdot} < s^{;2} \\ \omega_{\mathcal{G}}(x) & s^{;2} \le x^{\cdot} < t^{;2} \\ \omega_{\mathcal{G}}(x) & t^{;2} \le x^{\cdot} < u^{;2} \\ \omega & x^{\cdot} = u^{;2} \\ \omega_{\mathcal{G}}(x) & u^{;2} \le x^{\cdot} < v^{;2} \\ \omega_{\mathcal{G}}(x) & v^{;2} \le x^{\cdot} < w^{;2} \\ \omega_{\mathcal{G}}(x) & w^{;2} \le x^{\cdot} < y^{;2} \\ \omega_{\mathcal{G}}(x) & y^{;2} \le x^{\cdot} < z^{;2} \\ 1 & otherwie \end{cases}$$

Representation and properties of Nonagonal Neutrosophic Number: We represent the Nonagonal Neutrosophic Number by Linear and Non-Linear symmetry [4].

Linear NNN with symmetry: Let $\nabla'_l = (a', b', c', d', e', f', g', h', i')$ as linear Nonagonal Neutrosophic Number with membership function is define as:

As, $0 < k < 1$

$A_\alpha = \{x \in X^{\cdot} \backslash T'(X^{\cdot}), F'(X^{\cdot}), I'(X^{\cdot}) \ge \alpha\}$

$$T'_L(x)=\begin{cases}0 & x<a^* \\ k'\left(\frac{x-a^*}{b^*-a^*}\right) & a^*<x<b^* \\ k'\left(\frac{(x-b^*)}{c^*-b^8}\right) & b^*<x<c^* \\ k & c^*<x<d^* \\ k'+(1-k')\left(\frac{x-d^*}{e^*-d^*}\right) & d^*<x<e^* \\ 1 & e^*<x<f^* \\ k'+(1-k')\left(\frac{g^*-x}{g^*-f^*}\right) & f^*<x<g^* \\ k' & g^*<x<h^* \\ k'\left(\frac{(i^*-x)}{i^*-h^*}\right) & h^*<x<i^* \\ 0 & x>i^*\end{cases}$$

$$F'_L(x)=\begin{cases}0 & x<a^{*1} \\ k'\left(\frac{x-a^{*1}}{b^{*1}-a^{*1}}\right) & a^{*1}<x<b^{*1} \\ k'\left(\frac{x-b^{*1}}{c^{*1}-b^{*1}}\right) & b^{*1}<x<c^{*1} \\ k' & c^{*1}<x<d^{*1} \\ k'+(1-k')\left(\frac{x-d^{*1}}{e^{*1}-d^{*1}}\right) & d^{*1}<x<e^{*1} \\ 1 & e^{*1}<x<f^{*1} \\ k'+(1-k')\left(\frac{g^{*1}-x}{g^{*1}-f^{*1}}\right) & f^{*1}<x<g^{*1} \\ k' & g^{*1}<x<h^{*1} \\ k'\left(\frac{i^{*1}-x}{i^{*1}-h^{*1}}\right) & h^{*1}<x<i^{*1} \\ 1 & x>i^{*1}\end{cases}$$

$$I'_L(x)=\begin{cases}0 & x<a^{*2} \\ k\cdot\left(\frac{x-a^{*2}}{b^{*2}-a^{*2}}\right) & a^{*2}<x<b^{*2} \\ k'\left(\frac{x-b^{*2}}{c^{*2}-b^{*2}}\right) & b^{*2}<x<c^{*2} \\ k' & c^{*2}<x<d^{*2} \\ k'+(1-k')\left(\frac{x-d^{*2}}{e^{*2}-d^{*2}}\right) & d^{*2}<x<e^{*2} \\ 1 & e^{*2}<x<f^{*2} \\ k'+(1-k')\left(\frac{g^{*2}-x}{g^{*2}-f^{*2}}\right) & f^{*2}<x<g^{*2} \\ k' & g^{*2}<x<h^{*2} \\ k'\left(\frac{i^{*2}-x}{i^{*2}-h^{*2}}\right) & h^{*2}<x<i^{*2} \\ 1 & x>i^{*2}\end{cases}$$

$\alpha - cut$ of Linear ONN with symmetry: $\alpha - cut$ of the membership function truth, indeterminacy and falsity are define as[4]

$$T'_L(x) = \begin{cases} \mu'_{1L}(\alpha') = a' + \frac{\alpha'}{b'_1}(b' - a')\ for\ \alpha' \in [0, b_1] \\ \mu'_{2L}(\alpha') = b' + \left(\frac{1-\alpha}{1-b'_2}\right)(c' - b')\ for\ \alpha' \in [b'_2, 1] \\ \mu'_{3L}(\alpha') = c' + \left(\frac{1-\alpha}{1-b'_3}\right)(d' - c')\ for\ \alpha' \in [b'_3, 1] \\ \mu'_{4L}(\alpha') = d' + \left(\frac{1-\alpha}{1-b'_4}\right)(e' - d')\ for\ \alpha' \in [b'_4, 1] \\ \mu'_{4R}(\alpha') = e' - \frac{\alpha'}{b'_4}(f' - e')\ for\ \alpha' \in [0, b'_4] \\ \mu'_{3R}(\alpha') = f' - \frac{\alpha'}{b'_3}(g' - f')\ for\ \alpha' \in [0, b'_3] \\ \mu'_{2R}(\alpha') = g' - \frac{\alpha'}{b'_2}(h' - g')\ for\ \alpha' \in [0, b'_2] \\ \mu'_{1R}(\alpha') = h' - \frac{\alpha'}{b'_1}(i' - h')\ for\ \alpha' \in [0, b'_1] \end{cases}$$

$$F'_L(x) = \begin{cases} \mu'_{1L}(\alpha') = a'^1 + \frac{\alpha'}{b'_1}\left(b'^1 - a'^1\right)\ for\ \alpha' \in [0, b_1] \\ \mu'_{2L}(\alpha') = b'^1 + \left(\frac{1-\alpha}{1-b'_2}\right)\left(c'^1 - b'^1\right)\ for\ \alpha' \in [b'_2, 1] \\ \mu'_{3L}(\alpha') = c'^1 + \left(\frac{1-\alpha}{1-b'_3}\right)\left(d'^1 - c'^1\right)\ for\ \alpha' \in [b'_3, 1] \\ \mu'_{4L}(\alpha') = d'^1 + \left(\frac{1-\alpha}{1-b'_4}\right)\left(e'^1 - d'^1\right)\ for\ \alpha' \in [b'_4, 1] \\ \mu'_{4R}(\alpha') = e'^1 - \frac{\alpha'}{b'_4}\left(f'^1 - e'^1\right)\ for\ \alpha' \in [0, b'_4] \\ \mu'_{3R}(\alpha') = f'^1 - \frac{\alpha'}{b'_3}\left(g'^1 - f'^1\right)\ for\ \alpha' \in [0, b'_3] \\ \mu'_{2R}(\alpha') = g'^1 - \frac{\alpha'}{b'_2}\left(h'^1 - g'^1\right)\ for\ \alpha' \in [0, b'_2] \\ \mu'_{1R}(\alpha') = h'^1 - \frac{\alpha'}{b'_1}\left(i'^1 - h'^1\right)\ for\ \alpha' \in [0, b'_1] \end{cases}$$

$$I'_L(x) = \begin{cases} \mu'_{1L}(\alpha') = a'^2 + \frac{\alpha'}{b'_1}\left(b'^2 - a'^2\right)\ for\ \alpha' \in [0, b_1] \\ \mu'_{2L}(\alpha') = b'^2 + \left(\frac{1-\alpha}{1-b'_2}\right)\left(c'^2 - b'^2\right)\ for\ \alpha' \in [b'_2, 1] \\ \mu'_{3L}(\alpha') = c'^2 + \left(\frac{1-\alpha}{1-b'_3}\right)\left(d'^2 - c'^2\right)\ for\ \alpha' \in [b'_3, 1] \\ \mu'_{4L}(\alpha') = d'^2 + \left(\frac{1-\alpha}{1-b'_4}\right)\left(e'^2 - d'^2\right)\ for\ \alpha' \in [b'_4, 1] \\ \mu'_{4R}(\alpha') = e'^2 - \frac{\alpha'}{b'_4}\left(f'^2 - e'^2\right)\ for\ \alpha' \in [0, b'_4] \\ \mu'_{3R}(\alpha') = f'^2 - \frac{\alpha'}{b'_3}\left(g'^2 - f'^2\right)\ for\ \alpha' \in [0, b'_3] \\ \mu'_{2R}(\alpha') = g'^2 - \frac{\alpha'}{b'_2}\left(h'^2 - g'^2\right)\ for\ \alpha' \in [0, b'_2] \\ \mu'_{1R}(\alpha') = h'^2 - \frac{\alpha'}{b'_1}\left(i'^2 - h'^2\right)\ for\ \alpha' \in [0, b'_1] \end{cases}$$

Non-Linear Nonagonal Neutrosophic Numbers (NONNN) with symmetry: Let $\Delta'_{nl}= (j'', k'', l'', m'', n'', o'', p'', q'', r'')_{(x'',y'',c'',d'')}$ as non-linear Nonagonal Neutrosophic Number [4] with membership function is define as:

As, $0 < k < 1$

$$A_\alpha = \{x \in X'' \backslash T'(X''), F'(X''), I'(X'') \geq \alpha\}$$

$$T'_N(x) = \begin{cases} 0 & x < j'' \\ k''\left(\dfrac{x-j''}{k''-j''}\right)^{x''} & j'' < x < k'' \\ k''\left(\dfrac{x-k''}{l''-k''}\right)^{y''} & k'' < x < l'' \\ k'' & l'' < x < m'' \\ k'' + (1-k'')\left(\dfrac{x-m''}{n''-m''}\right)^{c''} & m'' < x < n'' \\ 1 & n'' < x < o'' \\ k'' + (1-k'')\left(\dfrac{o''-x}{o''-p''}\right)^{d''} & o'' < x < p'' \\ k'' & p'' < x < q'' \\ k''\left(\dfrac{q''-x}{q''-r''}\right)^{e''} & q'' < x < r'' \\ 0 & x > r'' \end{cases}$$

$$F'_N(x) = \begin{cases} 0 & x < j''^1 \\ k''\left(\dfrac{x-j''^1}{k''-j''^1}\right)^{x''} & j''^1 < x < k''^1 \\ k''\left(\dfrac{x-k''^1}{l''^1-k''^1}\right)^{y''} & k''^1 < x < l''^1 \\ k'' & l''^1 < x < m''^1 \\ k'' + (1-k'')\left(\dfrac{x-m''^1}{n''^1-m''^1}\right)^{c'} & m''^1 < x < n''^1 \\ 1 & n''^1 < x < o''^1 \\ k'' + (1-k'')\left(\dfrac{o''^1-x}{o''^1-p''^1}\right)^{d'} & o''^1 < x < p''^1 \\ k'' & p''^1 < x < q''^1 \\ k''\left(\dfrac{q''^1-x}{q''^1-r''^1}\right)^{e'} & q''^1 < x < r''^1 \\ 1 & x > r''^1 \end{cases}$$

$$I'_N(x)=\begin{cases} 0 & x<j''^2 \\ k''\left(\frac{x-j''^2}{k''-j''^2}\right)^{x''} & j''^2<x<k''^2 \\ k''\left(\frac{x-k''^2}{l''^2-k''^2}\right)^{y''} & k''^2<x<l''^2 \\ k'' & l''^2<x<m''^2 \\ k''+(1-k'')\left(\frac{x-m''^2}{n''^2-m''^2}\right)^{c''} & m''^2<x<n''^2 \\ 1 & n''^2<x<o''^2 \\ k''+(1-k'')\left(\frac{o''^2-x}{o''^2-p''^2}\right)^{d''} & o''^2<x<p''^2 \\ k'' & p''^2<x<q''^2 \\ k''\left(\frac{q''^2-x}{q''^2-r''^2}\right)^{e''} & q''^2<x<r''^2 \\ 1 & x>r''^2 \end{cases}$$

$\alpha-cut$ of Non- Linear NNN with symmetry: $\alpha-cut$ of nON-LNNN [4] with membership function truth, falsity and indeterminacy are define as

$$T'_N(x)=\begin{cases} \mu'_{1B}(\alpha')=j'+\left(\frac{\alpha}{b'_1}\right)^{x'_1}(k'-j')\, for \alpha'\in[0,b'_1] \\ \mu'_{2B}(\alpha')=k'+\left(\frac{1-\alpha}{1-b'_2}\right)^{x'_2}(l'-k')\, for\ \alpha'\in[b'_2,1] \\ \mu'_{3B}(\alpha')=l'+\left(\frac{1-\alpha}{1-b'_3}\right)^{x'_3}(m'-l')\, for\ \alpha'\in[b'_3,1] \\ \mu'_{4B}(\alpha')=m'+\left(\frac{1-\alpha}{1-b'_4}\right)^{x'_4}(n'-m')\, for\ \alpha'\in[b'_4,1] \\ \mu'_{4D}(\alpha')=n'-\left(\frac{\alpha}{b'_4}\right)^{y'_1}(o'-n')\ for\ \alpha'\in[0,b'_4] \\ \mu'_{3D}(\alpha')=o'-\left(\frac{\alpha}{b'_3}\right)^{y'_2}(p'-o')\ for\ \alpha'\in[0,b'_3] \\ \mu'_{2D}(\alpha')=p'-\left(\frac{\alpha}{b'_2}\right)^{y'_3}(q'-p')\ for\ \alpha'\in[0,b'_2] \\ \mu'_{1D}(\alpha')=q'-\left(\frac{\alpha}{b'_1}\right)^{y'_4}(r'-q')\ for\ \alpha'\in[0,b'_1] \end{cases}$$

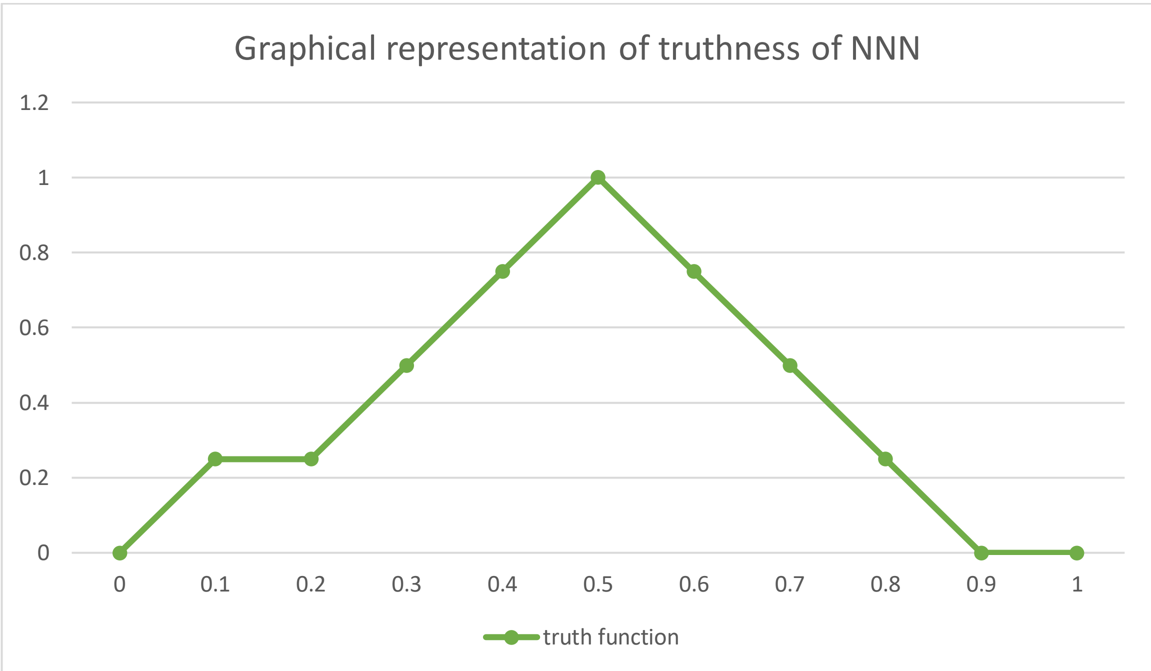
$$F'_N(x) = \begin{cases} \mu'_{1B}(\alpha') = j'^1 + \left(\dfrac{\alpha}{b'_1}\right)^{x'_1} (k'^1 - j'^1)\; for \alpha' \in [0, b'_1] \\ \mu'_{2B}(\alpha') = k'^1 + \left(\dfrac{1-\alpha}{1-b'_2}\right)^{x'_2} (l'^1 - k'^1)\; for\ \alpha' \in [b'_2, 1] \\ \mu'_{3B}(\alpha') = l'^1 + \left(\dfrac{1-\alpha}{1-b'_3}\right)^{x'_3} (m'^1 - l'^1)\; for\ \alpha' \in [b'_3, 1] \\ \mu'_{4B}(\alpha') = m'^1 + \left(\dfrac{1-\alpha}{1-b'_4}\right)^{x'_4} (n'^1 - m'^1)\; for\ \alpha' \in [b'_4, 1] \\ \mu'_{4D}(\alpha') = n'^1 - \left(\dfrac{\alpha}{b'_4}\right)^{y'_1} (o'^1 - n'^1)\; for\ \alpha' \in [0, b'_4] \\ \mu'_{3D}(\alpha') = o'^1 - \left(\dfrac{\alpha}{b'_3}\right)^{y'_2} (p'^1 - o'^1)\; for\ \alpha' \in [0, b'_3] \\ \mu'_{2D}(\alpha') = p'^1 - \left(\dfrac{\alpha}{b'_2}\right)^{y'_3} (q'^1 - p'^1)\; for\ \alpha' \in [0, b'_2] \\ \mu'_{1D}(\alpha') = q'^1 - \left(\dfrac{\alpha}{b'_1}\right)^{y'_4} (r'^1 - q'^1)\; for\ \alpha' \in [0, b'_1] \end{cases}$$

$$I'_N(x) = \begin{cases} \mu'_{1B}(\alpha') = j'^2 + \left(\dfrac{\alpha}{b'_1}\right)^{x'_1} (k'^2 - j'^2)\; for \alpha' \in [0, b'_1] \\ \mu'_{2B}(\alpha') = k'^2 + \left(\dfrac{1-\alpha}{1-b'_2}\right)^{x'_2} (l'^2 - k'^2)\; for\ \alpha' \in [b'_2, 1] \\ \mu'_{3B}(\alpha') = l'^2 + \left(\dfrac{1-\alpha}{1-b'_3}\right)^{x'_3} (m'^2 - l'^2)\; for\ \alpha' \in [b'_3, 1] \\ \mu'_{4B}(\alpha') = m'^2 + \left(\dfrac{1-\alpha}{1-b'_4}\right)^{x'_4} (n'^2 - m'^2)\; for\ \alpha' \in [b'_4, 1] \\ \mu'_{4D}(\alpha') = n'^2 - \left(\dfrac{\alpha}{b'_4}\right)^{y'_1} (o'^2 - n'^2)\; for\ \alpha' \in [0, b'_4] \\ \mu'_{3D}(\alpha') = o'^2 - \left(\dfrac{\alpha}{b'_3}\right)^{y'_2} (p'^2 - o'^2)\; for\ \alpha' \in [0, b'_3] \\ \mu'_{2D}(\alpha') = p'^2 - \left(\dfrac{\alpha}{b'_2}\right)^{y'_3} (q'^2 - p'^2)\; for\ \alpha' \in [0, b'_2] \\ \mu'_{1D}(\alpha') = q'^2 - \left(\dfrac{\alpha}{b'_1}\right)^{y'_4} (r'^2 - q'^2)\; for\ \alpha' \in [0, b'_1] \end{cases}$$

$\mu'_{1B}(\alpha'), \mu'_{2B}(\alpha'), \mu'_{3B}(\alpha'), \mu'_{4B}(\alpha')$ are increasing function with respect to α' .
$\mu'_{4D}(\alpha'), \mu'_{3D}(\alpha'), \mu'_{2D}(\alpha'), \mu'_{1D}(\alpha')$ are decreasing function with respect to α'.

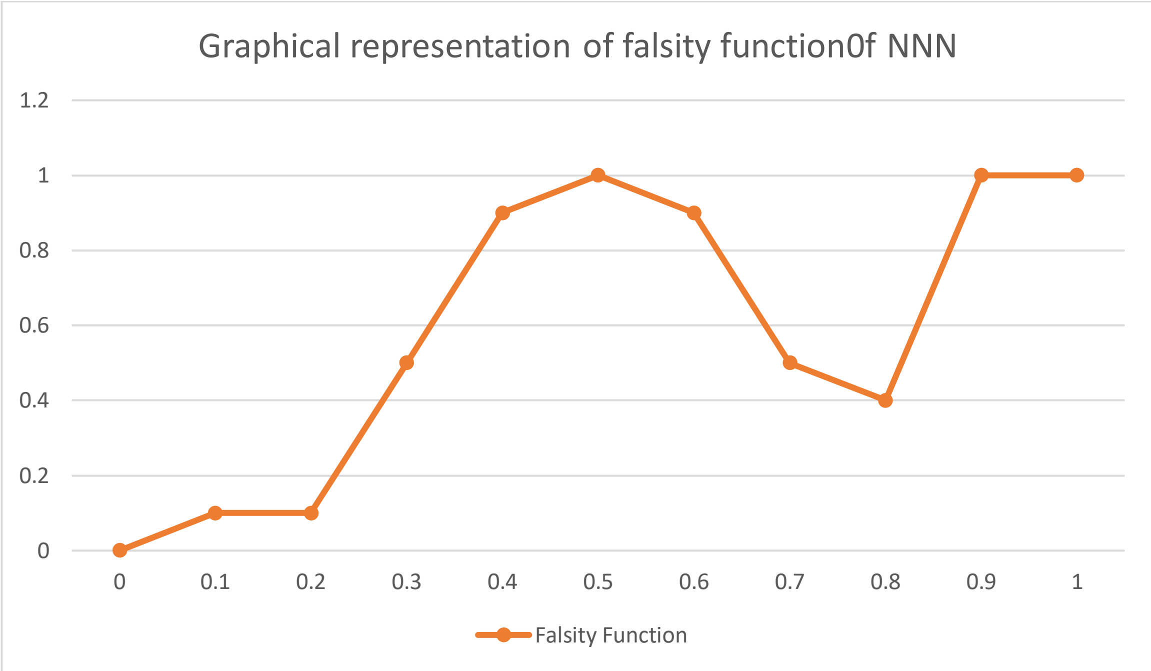
Graphical representation of Nonagonal Neutrosophic Number: Consider Nonagonal Neutrosophic Number for the graphical representation of truth membership function

$$T_L'(x) = \begin{cases} 0 & x < 0.1 \\ k' \left(\dfrac{x - a^*}{b^* - a^*}\right) & 0.1 < x < 0.2 \\ k' \left(\dfrac{(x - b^*)}{c^* - b^8}\right) & 0.2 < x < 0.3 \\ k & 0.3 < x < 0.4 \\ k' + (1 - k')\left(\dfrac{x - d^*}{e^* - d^*}\right) & 0.4 < x < 0.5 \\ 1 & 0.5 < x < 0.6 \\ k' + (1 - k')\left(\dfrac{g^* - x}{g^* - f^*}\right) & 0.6 < x < 0.7 \\ k' & 0.7 < x < 0.8 \\ k' \left(\dfrac{(i^* - x)}{i^* - h^*}\right) & 0.8 < x < 0.9 \\ 0 & x > 0.9 \end{cases}$$



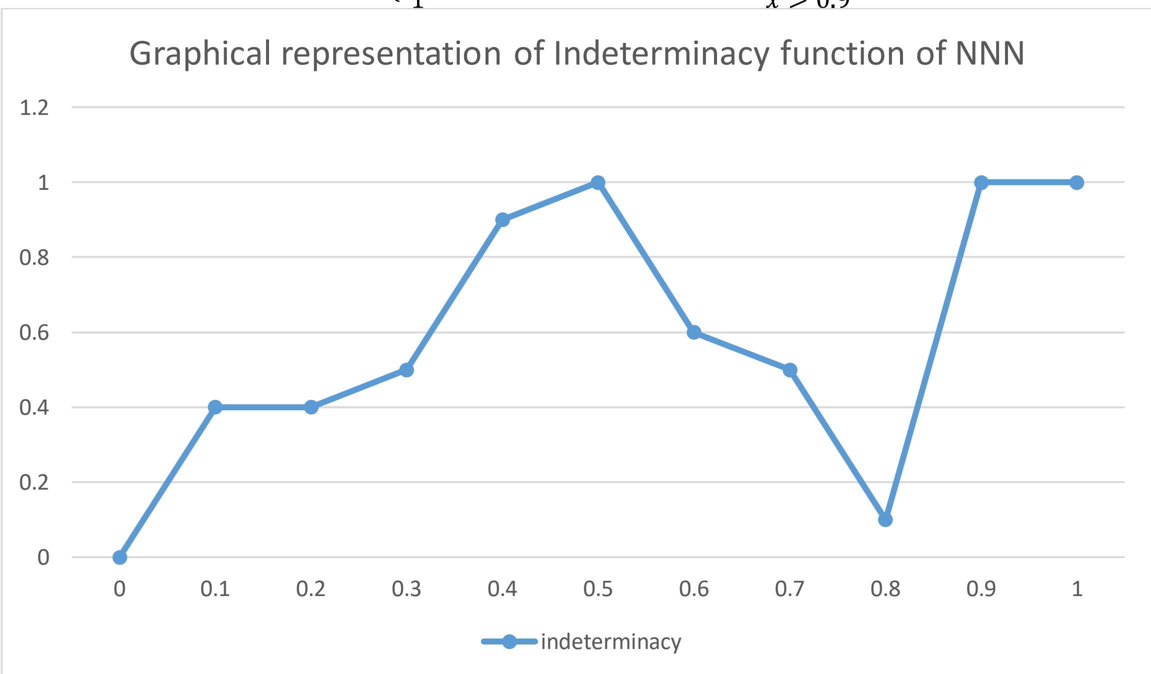
Consider Nonagonal Neutrosophic Numbers for the graphical representation of falsity membership function

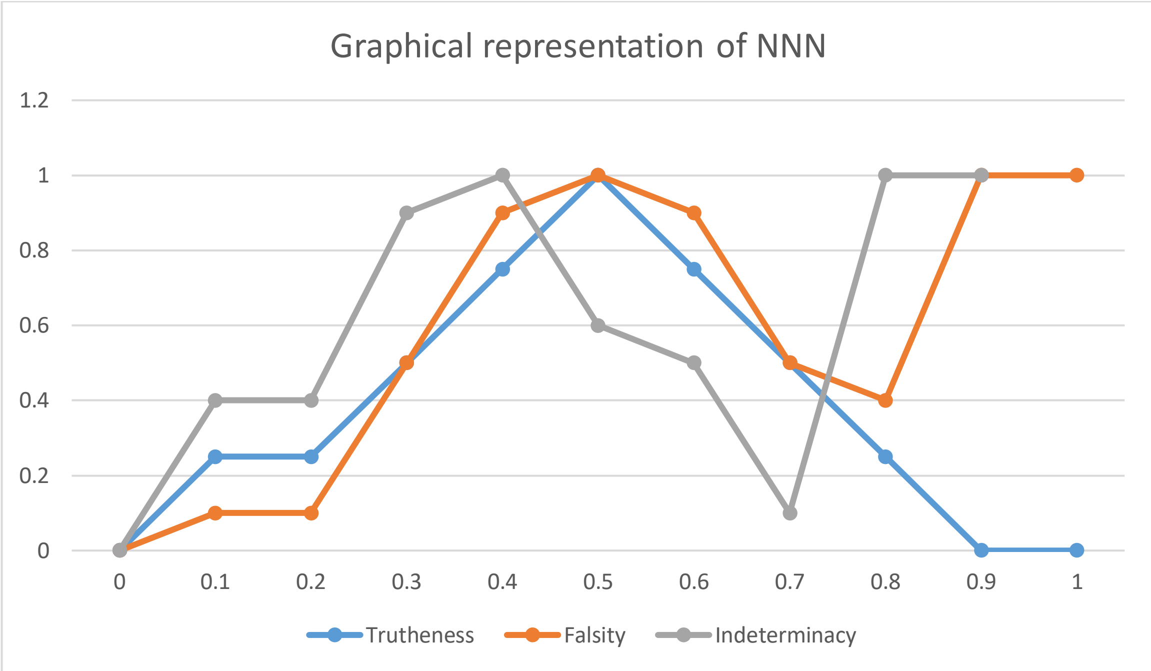
$$F_L'(x) = \begin{cases} 0 & x < 0.1 \\ k' \left(\dfrac{x - a^{*1}}{b^{*1} - a^{*1}}\right) & 0.1 < x < 0.2 \\ k' \left(\dfrac{x - b^{*1}}{c^{*1} - b^{*1}}\right) & 0.2 < x < 0.3 \\ k' & 0.3 < x < 0.4 \\ k' + (1 - k')\left(\dfrac{x - d^{*1}}{e^{*1} - d^{*1}}\right) & 0.4 < x < 0.5 \\ 1 & 0.5 < x < 0.6 \\ k' + (1 - k')\left(\dfrac{g^{*1} - x}{g^{*1} - f^{*1}}\right) & 0.6 < x < 0.7 \\ k' & 0.7 < x < 0.8 \\ k' \left(\dfrac{i^{*1} - x}{i^{*1} - h^{*1}}\right) & 0.8 < x < 0.9 \\ 1 & x > 0.9 \end{cases}$$



Consider Nonagonal Neutrosophic numbers for the graphical representation of indeterminacy membership function

$$I_L'(x) = \begin{cases} 0 & x < 0.1 \\ k\cdot\left(\dfrac{x - a^{*2}}{b^{*2} - a^{*2}}\right) & 0.1 < x < 0.2 \\ k'\left(\dfrac{x - b^{*2}}{c^{*2} - b^{*2}}\right) & 0.2 < x < 0.3 \\ k' & 0.3 < x < 0.4 \\ k' + (1 - k')\left(\dfrac{x - d^{*2}}{e^{*2} - d^{*2}}\right) & 0.4 < x < 0.5 \\ 1 & 0.5 < x < 0.6 \\ k' + (1 - k')\left(\dfrac{g^{*2} - x}{g^{*2} - f^{*2}}\right) & 0.6 < x < 0.7 \\ k' & 0.7 < x < 0.8 \\ k'\left(\dfrac{i^{*2} - x}{i^{*2} - h^{*2}}\right) & 0.8 < x < 0.9 \\ 1 & x > 0.9 \end{cases}$$





CASE STUDY

For checking the production and viability of VIKOR and TOPSIS method, we have a selection of candidates for scholarship.

For selection we have four candidates$\{\Psi, \Omega, \Upsilon, \Gamma\}$.

We select the candidates on the basis of $\{\mathcal{M}: Marks, \mathcal{E}: Members\ taking\ education, \mathcal{F}: Family\ expenses, \mathcal{I}: Income\}$

$$\Psi = \{\mathcal{M} = 995, \mathcal{E} = 4, \mathcal{F} = Rs. 40\ 000, \mathcal{I} = Rs. 50\ 000\}$$
$$\Omega = \{\mathcal{M} = 1045, \mathcal{E} = 3, \mathcal{F} = Rs. 35\ 000, \mathcal{I} = Rs. 50\ 000\}$$
$$\Upsilon = \{\mathcal{M} = 1001, \mathcal{E} = 5, \mathcal{F} = Rs. 50\ 000, \mathcal{I} = Rs. 55\ 000\}$$
$$\Gamma = \{\mathcal{M} = 970, \mathcal{E} = 4, \mathcal{F} = Rs. 45\ 000, \mathcal{I} = Rs. 48\ 000\}$$

Candidates Attributes	$\mathcal{M}$	$\mathcal{E}$	$\mathcal{F}(Rs.)$	$\mathcal{I}(Rs.)$
Ψ	995	4	40 000	50 000
Ω	1045	3	35 000	50 000
Υ	1001	5	50 000	55 000
Γ	970	4	45 000	48 000

Neutrosophic Number assigned by decision maker: Nonagonal Neutrosophic Numbers for candidate Ψ

Sr. #	Attributes	Nonagonal Neutrosophic Numbers
1	$\mathcal{M}$	(0.79,0.83,0.85,0.87,0.78,0.81,0.88,0.82,0.89),(0.77,0.67,0.56,0.98,0.78,0.69,0.72,0.83,0.85),(0.73,0.69,0.84,0.81,0.90,0.49,0.57,0.66,0.56)
2	$\mathcal{E}$	(0.79,0.76,0.81,0.84,0.74,0.88,0.89,0.86,0.82),(0.67,0.81,0.88,0.45,0.56,0.80,0.79,0.55,0.40),(0.66,0.72,0.67,0.81,0.71,0.79,0.77,0.81,0.73)
3	$\mathcal{F}$	(0.81,0.52,0.62,0.74,0.73,0.91,0.66,0.82,0.80),(0.59,0.63,0.91,0.57,0.73,0.81,0.43,0.77,0.74),(0.67,0.86,0.83,0.78,0.90,0.88,0.79,0.67,0.72)
4	$\mathcal{I}$	(0.72,0.76,0.80,0.86,0.73,0.66,0.68,0.71,0.53),(0.69,0.72,0.79,0.81,0.86,0.91,0.83,0.78,0.88),(0.67,0.75,0.88,0.79,0.65,0.73,0.84,0.81,0.90)

Nonagonal Neutrosophic Numbers for candidate Ω

Sr #	Attributes	Nonagonal Neutrosophic Numbers
1	$\mathcal{M}$	(0.93,0.95,0.99,0.94,0.91,0.82,0.96,0.98,0.92),(0.87,0.85,0.79,0.81,0.92,0.78,0.86,0.78,0.95),(0.79,0.86,0.88,0.97,0.76,0.85,0.82,0.90,0.89)
2	$\mathcal{E}$	(0.67,0.74,0.72,0.73,0.81,0.80,0.79,0.78,0.82),(0.66,0.61,0.75,0.81,0.79,0.83,0.63,0.78,0.80),(0.59,0.76,0.82,0.64,0.69,0.78,0.77,0.75,0.66)
3	$\mathcal{F}$	(0.87,0.43,0.90,0.46,0.52,0.67,0.81,0.88,0.91),(0.59,0.66,0.81,0.54,0.49,0.63,0.73,0.80,0.45),(0.79,0.66,0.93,0.85,0.86,0.67,0.56,0.80,0.76)
4	$\mathcal{I}$	(0.72,0.76,0.80,0.86,0.73,0.66,0.68,0.71,0.53),(0.52,0.87,0.65,0.69,0.61,0.73,0.58,0.75,0.81),(0.67,0.57,0.86,0.82,0.64,0.43,0.59,0.66,0.69)

Nonagonal Neutrosophic Numbers for candidate Υ

Sr #	Attributes	Nonagonal Neutrosophic Numbers
1	$\mathcal{M}$	(0.93,0.95,0.99,0.94,0.91,0.82,0.96,0.98,0.92),(0.89,0.95,0.83,0.88,0.85,0.94,0.92,0.84,0.79),(0.86,0.83,0.95,0.99,0.92,0.78,0.85,0.88,0.90)
2	$\mathcal{E}$	(0.94,0.89,0.85,0.91,0.96,0.97,0.88,0.92,0.93),(0.79,0.76,0.88,0.84,0.94,0.86,0.93,0.99,0.85),(0.91,0,83,0.85,0.79,0.88,0.93,0.78,0.84,0.94)
3	$\mathcal{F}$	(0.90,0.89,0.85,0.88,0.94,0.91,0.80,0.79,0.78),(0.81,0.80,0.79,0.90,0.87,0.76,0.69,0.84,0.88),(0.97,0.88,0.76,0.87,0.92,0.80,0.79,0.84,0.90)
4	$\mathcal{I}$	(0.45,0.72,0.80,0.67,0.63,0.75,0.80,0.81,0.53),(0.55,0.58,0.67,0.49,0.76,0.88,0.62,0.67,0.70),(0.79,0.55,0.76,0.49,0.67,0.78,0.69,0.75,0.48)

Nonagonal Neutrosophic Numbers for candidate Γ:

Sr #	Attributes	Nonagonal Neutrosophic Numbers
1	$\mathcal{M}$	(0.65,0.73,0.81,0.75,0.69,0.72,0.77,0.79,0.80),(0.76,0.85,0.73,0.68,0.74,0.82,0.71,0.90,0.77), (0.69,0.85,0.81,0.77,0.73,0.95,0.61,0.65,0.71)
2	$\mathcal{E}$	(0.79,0.76,0.81,0.84,0.74,0.88,0.89,0.86,0.82),(0.68,0.70,0.73,0.78,0.86,0.82,0.79,0.71,0.84), (0.77,0.80,0.74,0.89,0.83,0.72,0.82,0.86,0.71)
3	$\mathcal{F}$	(0.90,0.89,0.85,0.88,0.94,0.91,0.80,0.79,0.78),(0.82,0.79,0.93,0.78,0.72,0.91,0.86,0.82,0.80), (0.79,0.73,0.88,0.84,0.94,0.91,0.85,0.85,0.81)
4	$\mathcal{I}$	(0.65,0.73,0.78,0.86,0.67,0.68,0.77,0.81,0.72),(0.64,0.73,0.69,0.82,0.81,0.74,0.75,0.60,0.82), (0.83,0.74,0.92,0.69,0.71,0.89,0.86,0.75,0.72)

Candidate Ψ	Candidate Ω	Candidate Υ	Candidate Γ
$\mathcal{M}${(0.93,0.95,0.99,0.94,0.91, 0.82,0.96,0.98,0.92), (0.87,0.85,0.79,0.81,0.92,0.78,0.86,0.78,0.95), (0.79,0.86,0.88,0.97,0.76,0.85,0.82,0.90,0.89)}	$\mathcal{M}${(0.93,0.95,0.99,0.94,0.91, 0.82,0.96,0.98,0.92), (0.87,0.85,0.79,0.81,0.92,0.78,0.86,0.78,0.95), (0.79,0.86,0.88,0.97,0.76,0.85,0.82,0.90,0.89)}	$\mathcal{M}${(0.93,0.95,0.99,0.94,0.91, 0.82,0.96,0.98,0.92), (0.89,0.95,0.83,0.88,0.85,0.94,0.92,0.84,0.79), (0.86,0.83,0.95,0.99,0.92,0.78,0.85,0.88,0.90)}	$\mathcal{M}${(0.65,0.73,0.81,0.75,0.69, 0.72,0.77,0.79,0.80), (0.76,0.85,0.73,0.68,0.74,0.82,0.71,0.90,0.77), (0.69,0.85,0.81,0.77,0.73,0.95,0.61,0.65,0.71)}
$\mathcal{E}${(0.79,0.76,0.81,0.84,0.74,0.88,0.89,0.86,0.82), (0.67,0.81,0.88,0.45,0.56,0.80,0.79,0.55,0.40), (0.66,0.72,0.67,0.81,0.71,0.79,0.77,0.81,0.73)}	$\mathcal{E}${(0.67,0.74,0.72,0.73,0.81,0.80,0.79,0.78,0.82), (0.66,0.61,0.75,0.81,0.79,0.83,0.63,0.78,0.80), (0.59,0.76,0.82,0.64,0.69,0.78,0.77,0.75,0.66)}	$\mathcal{E}${(0.94,0.89,0.85,0.91,0.96,0.97,0.88,0.92,0.93), (0.79,0.76,0.88,0.84,0.94,0.86,0.93,0.99,0.85), (0.91,0,83,0.85,0.79,0.88,0.93,0.78,0.84,0.94)}	$\mathcal{E}${(0.79,0.76,0.81,0.84,0.74,0.88,0.89,0.86,0.82), (0.68,0.70,0.73,0.78,0.86,0.82,0.79,0.71,0.84), (0.77,0.80,0.74,0.89,0.83,0.72,0.82,0.86,0.71)}
$\mathcal{F}${(0.81,0.52,0.62,0.74,0.73, 0.91,0.66,0.82,0.80), (0.59,0.63,0.91,0.57,0.73,0.81,0.43,0.77,0.74), (0.67,0.86,0.83,0.78,0.90,0.88,0.79,0.67,0.72)}	$\mathcal{F}${(0.87,0.43,0.90,0.46,0.52, 0.67,0.81,0.88,0.91), (0.59,0.66,0.81,0.54,0.49,0.63,0.73,0.80,0.45), (0.79,0.66,0.93,0.85,0.86,0.67,0.56,0.80,0.76)}	$\mathcal{F}${(0.90,0.89,0.85,0.88,0.94, 0.91,0.80,0.79,0.78), (0.81,0.80,0.79,0.90,0.87,0.76,0.69,0.84,0.88), (0.97,0.88,0.76,0.87,0.92,0.80,0.79,0.84,0.90)}	$\mathcal{F}${(0.90,0.89,0.85,0.88,0.94, 0.91,0.80,0.79,0.78), (0.82,0.79,0.93,0.78,0.72,0.91,0.86,0.82,0.80), (0.79,0.73,0.88,0.84,0.94,0.91,0.85,0.85,0.81)}
$\mathcal{I}${(0.72,0.76,0.80,0.86,0.73,0.66,0.68,0.71,0.53), (0.69,0.72,0.79,0.81,0.86,0.91,0.83,0.78,0.88), (0.67,0.75,0.88,0.79,0.65,0.73,0.84,0.81,0.90)}	$\mathcal{I}${(0.72,0.76,0.80,0.86,0.73,0.66,0.68,0.71,0.53), (0.52,0.87,0.65,0.69,0.61,0.73,0.58,0.75,0.81), (0.67,0.57,0.86,0.82,0.64,0.43,0.59,0.66,0.69)}	$\mathcal{I}${(0.45,0.72,0.80,0.67,0.63,0.75,0.80,0.81,0.53), (0.55,0.58,0.67,0.49,0.76,0.88,0.62,0.67,0.70), (0.79,0.55,0.76,0.49,0.67,0.78,0.69,0.75,0.48)}	$\mathcal{I}${(0.65,0.73,0.78,0.86,0.67,0.68,0.77,0.81,0.72), (0.64,0.73,0.69,0.82,0.81,0.74,0.75,0.60,0.82), (0.83,0.74,0.92,0.69,0.71,0.89,0.86,0.75,0.72)}

VIKOR METHOD

VIKOR method [5, 6] is used for deciphering multi criteria decision problem. VIKOR method give a balance solution that gives maximum group benefits for majority.

Steps of VIKOR method:

STEP 1: Defuzzified the Neutrosophic numbers and assign weights.

STEP 2: Ascertain positive ideal result and negative ideal result.

$$\mathbb{r}^+ = \{\mathbb{r}_{ij} max \;\; for\ benefical\ criteria \qquad \mathbb{r}_{ij} \min \;\; for\ beneficial\ criteria\}$$

$$\mathbb{r}^- = \{\mathbb{r}_{ij} \min \;\; for\ non\ beneficial\ criteria \qquad \mathbb{r}_{ij} \max \;\; for\ non\ beneficila\ criteria\}$$

STEP 3: Calculate group unity $\mathbb{H}_j = \{\mathbb{H}_j^*,\ \mathbb{H}_j^-\}$ and individual regard value $\mathfrak{S}_j = \{\mathfrak{S}_j^*,\ \mathfrak{S}_j^-\}$

$$\mathbb{H}_j = \sum_{j=1}^{m}\left[w_j \frac{(x_j^* - x_i)}{x_j^* - x_j^-}\right] \qquad \mathfrak{S}_j = \max_j\left[w_j \frac{(x_j^* - x_{ij})}{x_j^* - x_j^-}\right]$$

$$\mathbb{H}_j^* = \min_j \mathbb{H}_j \qquad \mathbb{H}_j^- = \max_j \mathbb{H}_j$$

$$\mathfrak{S}_j^* = \min_j \mathfrak{S}_j \qquad \mathfrak{S}_j^- = \max_j \mathfrak{S}_j$$

STEP 4: Now we will calculate the comprehensive sorting index $\mathfrak{Q}_i$.

$$\mathfrak{Q}_i = \wp\frac{\mathbb{H}_i - \mathbb{H}^*}{\mathbb{H}^- - \mathbb{H}^*} + (1-\wp)\frac{\mathfrak{S}_i - \mathfrak{S}^*}{\mathfrak{S}^- - \mathfrak{S}^*}$$

$$\wp: decision\ , mechanism\ index \in [0,1]$$

STEP 5: Rank the values base on the sorting index $\mathfrak{Q}$. Decision maker select the best one according to ranking.
Here we have two conditions;
(a) Adequate advantage:

$$\mathfrak{Q}(\omega^2) - \mathfrak{Q}(\omega^1) \geq \frac{1}{\jmath - 1}$$

$\mathfrak{Q}(\omega^2)$: second ranked sorting index
$\mathfrak{Q}(\omega^1)$: first ranked sorting index
$\jmath$: number of alternates
Adequate stability in verdict where ω as well must ranked by $\mathbb{H}_i$ or/and $\mathfrak{Q}_i$.

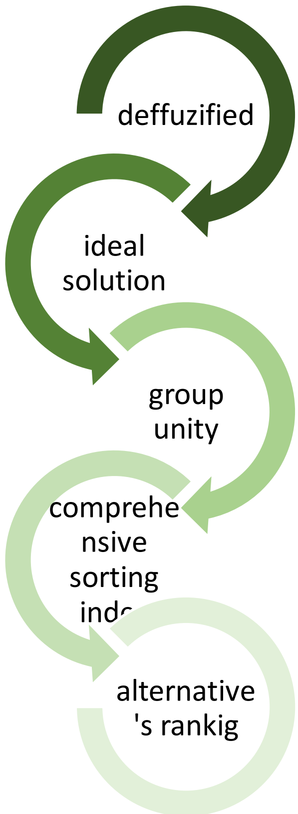


Figure 2. Steps for VIKOR method

TOPSIS METHOD
In this method two synthetic alternate are theorized [4, 14].
Ideal alternative: the one which has the finest level for attributes measured.
Negative ideal alternative: the one which has the poorest attributes values.
STEP 1: Defuzzified the Neutrosophic numbers and construct the weights.
STEP 2: Normalize data using

$$\mathscr{r}_{ij} = \frac{\mathfrak{x}_{ij}}{\left(\sum \mathfrak{x}_{ij}^2\right)^{\frac{1}{2}}}$$

STEP 3: Multiply each column by associative weights

$$v_{ij} = w_j r_{ij}$$

STEP 4: Determine

Positive ideal result $\mathcal{A}^*$
Negative ideal result $\mathcal{A}^-$.

$$\mathcal{A}^* = \{v_1^*, v_2^*, \dots v_n^*\}$$
$$v_j^* = \{v_{ij} \max for\ beneficial\ criteria \quad v_{ij} \min\ for\ beneficial\ criteria\}$$
$$\mathcal{A}^- = \{v_1^-, v_2^-, \dots v_n^-\}$$
$$v_j^- = \{v_{ij} \min\ for\ non\ beneficial\ criteria \quad v_{ij} \min\ for\ non\ beneficial\ criteria\}$$

STEP 5 : In this step we determine separation measure for each alternate.
For positive ideal alternate

$$\mathcal{S}_i^* = \left[\sum \left(v_j^* - v_{ij}\right)^2\right]^{\frac{1}{2}}$$

For negative ideal alternative

$$\mathcal{S}_i^- = \left[\sum \left(v_j^- - v_{ij}\right)^2\right]^{\frac{1}{2}}$$

STEP 6: Now we will calculate the relative closeness to the ideal solution

$$\mathcal{C}_i^* = \frac{\mathcal{S}_i^-}{\mathcal{S}_i^* + \mathcal{S}_i^-} \quad 0 < \mathcal{C}_i^* < 1$$

Pick the option $\mathcal{C}_i^*$ closet to 1.

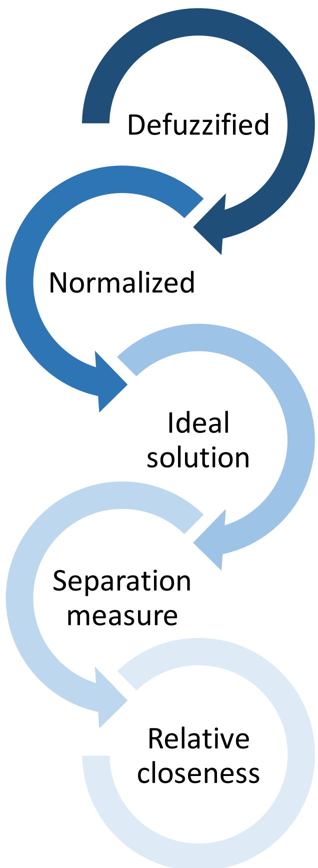


Figure 3. Steps for TOPSIS method

NUMERICAL ANALYSIS

VIKOR Method: First we use VIKOR method for decision making
STEP 1: We defuzzied the NNN by

$$\mathfrak{D}^{t_{NON}} = \left(\frac{a + b + c + d + e + f + g + h + i}{9}\right)$$
$$\mathfrak{D}^{t_{NON}} = \left(\frac{a_1 + b_1 + c_1 + d_1 + e_1 + f_1 + g_1 + h_1 + i_1}{9}\right)$$
$$\mathfrak{D}^{t_{NON}} = \left(\frac{a_2 + b_2 + c_2 + d_2 + e_2 + f_2 + g_2 + h_2 + i_2}{9}\right)$$

	Ψ	Ω	ϒ	Γ

$\mathcal{M}$	(0.83,0.76,0.69)	(0.93,0.84,0.85)	(0.93,0.87,0.88)	(0.75,0.77,0.75)
$\mathcal{E}$	(0.82,0.66,0.74)	(0.76,0.74,0.71)	(0.91,0.87,0.86)	(0.82,0.77,0.79)
$\mathcal{F}$	(0.73,0.68,0.79)	(0.71,0.63,0.76)	(0.86,0.81,0.85)	(0.86,0.82,0.84)
$\mathcal{J}$	(0.71,0.81,0.78)	(0.72,0.69,0.66)	(0.68,0.65,0.66)	(0.74,0.73,0.79)

$$x = \begin{pmatrix} \mathbf{0.2} & \mathbf{0.1} & \mathbf{0.3} & \mathbf{0.4} \\ 0.76 & 0.87 & 0.89 & 0.76 \\ 0.74 & 0.74 & 0.88 & 0.79 \\ 0.73 & 0.70 & 0.84 & 0.84 \\ 0.77 & 0.69 & 0.66 & 0.75 \end{pmatrix}$$

STEP 2: Now we determine the positive ideal solution and negative ideal solution

weightage	0.2	0.1	0.3	0.4
	$\mathcal{M}$	$\mathcal{E}$	$\mathcal{F}$	$\mathcal{J}$
Ψ	0.76	0.74	0.73	0.77
Ω	0.87	0.74	0.70	0.69
ϒ	0.89	0.88	0.84	0.66
Γ	0.76	0.76	0.84	0.75
$\mathbb{r}^{+}$	**0.87**	**0.88**	**0.84**	**0.75**
$\mathbb{r}^{-}$	**0.76**	**0.74**	**0.70**	**0.66**

STEP 3 to 5: In this table we determine $\mathbb{H}_i$, $\mathfrak{S}_i$, $\mathfrak{Q}_i$ and also select the candidate according to step 5.

	$\mathcal{M}$	$\mathcal{E}$	$\mathcal{F}$	$\mathcal{J}$	$\mathbb{H}_i$	$\mathfrak{S}_i$	$\mathfrak{Q}_i$
Ψ	0.2	0.1	0.23	0.08	0.61	0.23	0.5
Ω	0	o.1	0.3	0.27	**0.67**	0.3	0.75
ϒ	0.03	0	0	0.4	0.43	**0.4**	0.69
Γ	0.2	0.08	0	0	**0.28**	**0.2**	0
$\mathbb{H}_i^*$,$\mathfrak{S}_i^*$					**0.28**	**0.2**	
$\mathbb{H}_i^-$,$\mathfrak{S}_i^-$					**0.67**	**0.4**	

According to ranking candidate Γ is selected.

TOPSIS Method: Now we apply TOPSIS method on the same application.

STEP 1:

weightage	0.2	0.1	0.3	0.4
	$\mathcal{M}$	$\mathcal{E}$	$\mathcal{F}$	$\mathcal{J}$
Ψ	0.76	0.74	0.73	0.77
Ω	0.87	0.74	0.70	0.69
ϒ	0.89	0.88	0.84	0.66
Γ	0.76	0.76	0.84	0.75

STEP 2:

$$r_{ij} = \frac{\mathfrak{x}_{ij}}{\left(\sum \mathfrak{x}_{ij}^2\right)^{\frac{1}{2}}}$$

	$\mathcal{M}$	$\mathcal{E}$	$\mathcal{F}$	$\mathcal{J}$
Ψ	0.46	0.47	0.46	0.53
Ω	0.53	0.47	0.45	0.48
ϒ	0.54	0.56	0.53	0.46
Γ	0.46	0.48	0.53	0.52

STEP 3:

$$v_{ij} = w_j r_{ij}$$

	$\mathcal{M}$	$\mathcal{E}$	$\mathcal{F}$	$\mathcal{J}$
Ψ	***0.092***	***0.047***	0.138	***0.212***
Ω	0.106	***0.047***	***0.135***	0.192
ϒ	**0.108**	**0.056**	**0.159**	**0.184**
Γ	***0.092***	0.048	**0.159**	0.208

STEP 4:

$$\mathcal{A}^* = \{0.108, 0.056, 0.159, 0.184\}$$
$$\mathcal{A}^- = \{0.092, 0.047, 0.135, 0.212\}$$

STEP 5 AND 6:

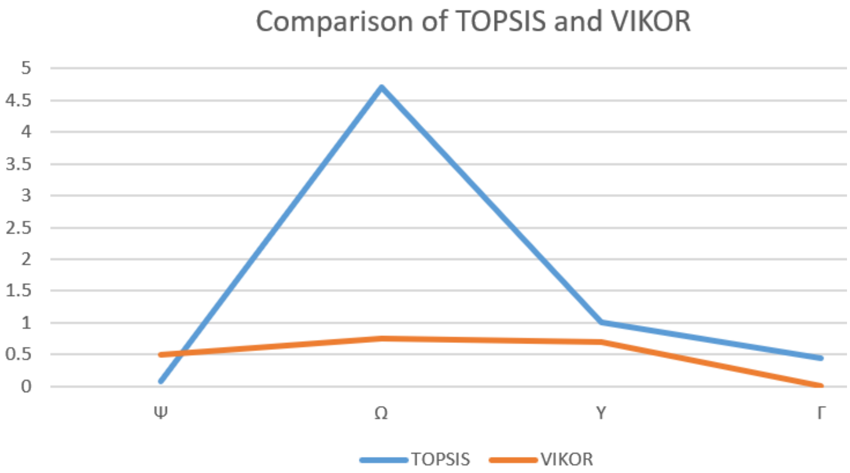
$$\mathcal{S}_i^* = \left[\sum \left(v_j^* - v_{ij}\right)^2\right]^{\frac{1}{2}} \qquad \mathcal{S}_i^- = \left[\sum \left(v_j^- - v_{ij}\right)^2\right]^{\frac{1}{2}}$$

$$\mathcal{C}_i^* = \frac{\mathcal{S}_i^-}{\mathcal{S}_i^* + \mathcal{S}_i^-}$$

	$\mathcal{S}_i^*$	$\mathcal{S}_i^-$	$\mathcal{C}_i^*$
Ψ	0.039	0.003	0.071
Ω	0.027	0.024	4.71
ϒ	0	0.041	1
Γ	0.029	0.042	0.45

According to relative ideal solution candidate **Γ** is selected.

	TOPSIS METHOD				VIKOR METHOD		
	$\mathcal{S}_i^*$	$\mathcal{S}_i^-$	$\mathcal{C}_i^*$		$\mathbb{H}_i$	$\mathfrak{S}_i$	$\mathfrak{Q}_i$
Ψ	0.039	0.003	0.071	V/S	0.61	0.23	0.5
Ω	0.027	0.024	4.71		0.67	0.3	0.75
ϒ	0	0.041	1		0.43	0.4	0.69
Γ	0.029	0.042	0.45		0.28	0.2	0



VIKOR method is better than TOPSIS method because it gives more accurate results in indeterminacy.

CONCLUSION

We work on Nonagonal Neutrosophic Numbers and find out $\alpha - cut$ and graphs of the numbers as well as comparison of TOPSIS and VIKOR method is done on a real-life case study. In comparison graph blue line is for TOPSIS and orange line for VIKOR method, which shows that VIKOR method gives more accurate result in indeterminacy.

REFERENCES

[1] Hwang, C. L., & Yoon, K. "Multiple Attribute Decision Making–Methods and Applications". Springer-Verlag Berlin Heidelberg, New York, 1981, pp. 29-35.
[2] Smarandache, F. "Neutrosophic set-a generalization of the intuitionistic fuzzy set". International journal of pure and applied mathematics, Vol. 24, No. 3, 2005, pp. 287.
[3] Ma, Y. X., Wang, J. Q., Wang, J., & Wu, X. H. "An interval neutrosophic linguistic multi-criteria group decision-making method and its application in selecting medical treatment options". Neural Computing and Applications, Vol. 28, No. 9, 2017, pp. 2745-2765.
[4] Muhammad Naveed Jafar, Ezgi Turkarsalan, Ali Hamza, Sana Farooq, "Nonagonal Neutrosophic Linear Non-Linear Numbers, Alpha Cuts and their Applications using TOPSIS", International Journal of Neutrosophic Science, Vol. 10, 2020, pp. 45-64.
[5] Saqlain, M., & Smarandache, F. Octagonal, "Neutrosophic Number: Its Different Representations, Properties, Graphs and De-neutrosophication with the application of Personnel Selection". Infinite Study, 2021, pp. 23-45.
[6] Opricovic, S., & Tzeng, G. H. "Extended VIKOR method in comparison with outranking methods". European journal of operational research, Vol. 178, No.2, 2007, pp. 514-529.
[7] Saeed, M., Saqlain, M., & Riaz, M. "Application of generalized fuzzy TOPSIS in decision making for neutrosophic soft set to predict the champion of FIFA 2018: A mathematical analysis", Infinite Study. 2019, pp. 42-61.
[8] Jafar, M. N., TÜRKARSALAN, E., Hamza, A., & Farooq, S. "Nonagonal Neutrosophic Linear Non-Linear Numbers, Alpha Cuts and Their Applications using TOPSIS". International Journal of Neutrosophic Science, Vol. 10, No. 1, 2020, pp. 45-64.
[9] Saqlain, M., Hamza, A., & Farooq, S. "Linear and Non-Linear Octagonal Neutrosophic Number: Its Representation, α− *Cut* and Applications". International Journal of Neutrosophic Science, Vol. 3, 2020, pp. 1-17.

[10] Smarandache, F. “A unifying field in logics. neutrosophy: Neutrosophic probability, set and logic”. 1999, pp. 1-141.
[11] Lai, Y. J., Liu, T. Y., & Hwang, C. L. “Topsis for MODM”. European journal of operational research, Vol. 76, No. 3, 1994, pp. 486-500.
[12] Zadeh, L.A. “Fuzzy sets”. Inf. Control, Vol. 8, 1995, pp. 338-353.
[13] Pavić, Z., & Novoselac, V. “Notes on TOPSIS method”. International Journal of Research in Engineering and Science, Vol. 1, No. 2, 2013, pp. 5-12.
[14] Tian, J., Yu, D., Yu, B., & Ma, S. “A fuzzy TOPSIS model via chi-square test for information source selection”. Knowledge-Based Systems, Vol. 37, 2013, pp. 515-527.
[15] Smarandache, F. “Neutrosophy, Neutrosophic Probability, Set and Logic”, Amer Res. Press, Rehoboth, USA, 1998, pp. 105.
[16] Chakraborty, A., Mondal, S. P., Mahata, A., & Alam, S. “Different linear and non-linear form of trapezoidal neutrosophic numbers, de-neutrosophication techniques and its application in time-cost optimization technique, sequencing problem”. RAIRO-Operations Research, Vol. 55, 2021, pp. S97-S118.
[17] Chakraborty, A., Mondal, S. P., Alam, S., Ahmadian, A., Senu, N., De, D., & Salahshour, S. “The pentagonal fuzzy number: its different representations, properties, ranking, defuzzification and application in game problems”. Symmetry, Vol. 11, No. 2, 2019, pp. 248.
[18] Chakraborty, A., Broumi, S., & Singh, P. K. “Some properties of pentagonal neutrosophic numbers and its applications in transportation problem environment”. Neutrosophic Set and System. Vol. 28, 2019, pp. 201-214.
[19] Molodtsov, D. “Soft set theory—first results”. Computers & Mathematics with Applications, Vol. 37, No. 4-5, 1999, pp. 19-31.
[20] Du, Y., & Liu, P. “Extended fuzzy VIKOR method with intuitionistic trapezoidal fuzzy numbers”. INFORMATION-An International Interdisciplinary Journal, Vol. 14, No. 8, 2011, pp. 2575-2583.
[21] Ghadikolaei, A. S. Madhoushi, & Divsalar. “Extension of the VIKOR method for group decision making with extended hesitant fuzzy linguistic information”. Neural Computing and Applications. Vol .30, 2018, pp. 3589–3602.
[22] Opricovic, S., & Tzeng, G. H. “Compromise solution by MCDM methods: A comparative analysis of VIKOR and TOPSIS”. European journal of operational research, Vol. 156, No. 2, 2004, pp. 445-455.
[23] Mahdavi, I., Mahdavi-Amiri, N., Heidarzade, A., & Nourifar, R. “Designing a model of fuzzy TOPSIS in multiple criteria decision making”. Applied Mathematics and Computation, Vol. 206, No. 2, 2008, pp. 607-617.
[24] Li, D. F. “TOPSIS-based nonlinear-programming methodology for multiattribute decision making with interval-valued intuitionistic fuzzy sets”. IEEE Transactions on Fuzzy Systems, Vol. 18, No. 2, 2010, pp. 299-311.
[25] Jia Bao Liu, Sana Akram, Muhammad Javaid, Abdul Raheem, Roslan Hasni, "Bounds of Degree-Based Molecular Descriptors for Generalized -sum Graphs", Discrete Dynamics in Nature and Society, vol. 2021, Article ID 8821020, 17 pages, 2021. https://doi.org/10.1155/2021/8821020
[26] Akram, Sana, Javaid, Muhammad and Jamal, Muhammad. "Bounds on F-index of tricyclic graphs with fixed pendant vertices" Open Mathematics, vol. 18, no. 1, 2020, pp. 150-161. https://doi.org/10.1515/math-2020-0006
[27] Wang, Haiying, Javaid, Muhammad, Akram, Sana, Jamal, Muhammad and Wang, Shaohui. "Least eigenvalue of the connected graphs whose complements are cacti" Open Mathematics, vol. 17, no. 1, 2019, pp. 1319-1331. https://doi.org/10.1515/math-2019-0097
[28] Jia-Bao Liu, Sana Akram, Muhammad Javaid, Zhi-Ba Peng, "Exact Values of Zagreb Indices for Generalized T-Sum Networks with Lexicographic Product", Journal of Mathematics, vol. 2021, Article ID 4041290, 17 pages, 2021. https://doi.org/10.1155/2021/4041290