Stochastic Modeling and Applications
Vol. 26. No. 1 (January-June, 2022)
ISSN: 0972-3641

Received: 16th February 2022 Revised: 09th April 2022 Selected: 30th April 2022

INFLUENCE OF SPACE AND TEMPERATURE DEPENDENT INTERNAL HEAT GENERATION/ABSORPTION ON MHD BOUNDARY LAYER VISCO-ELASTIC FLUID FLOW OVER STRETCHING SURFACE IN POROUS MEDIUM

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Abstract: In the field of fluid mechanics due to large application in engineering and technology of viscoelastic fluid flow and heat transfer in the presence of porous media and space and temperature dependent internal heat generation/absorption. The flow is influenced by linear stretching sheet . The heat transfer analysis has been carried out to get the effect visco-elastic parameter, porosity parameter, magnetic parameter to space and temperature dependent internal heat generation /absorption.

Key words: Viscastic fuid, stretching sheet, MHD, porous media

1. Introduction:

The study of laminar boundary layer flow of non-Newtonian fluids with continuous moving surfaces is an important type of flow occurring in a number of engineering processes. Aerodynamic extrusion of plastic sheets, Cooling of an infinite metallic plate in a cooling path, which may be an electrolyte crystal growing, the boundary layer along a liquid film in condensation processes and a polymer sheet or filament extruded continuously from a die, are examples of practical applications of continuous moving surface. Glass blowing, continuous casting, and spinning of fibers also involve the flow due to a stretching surface. It has several practical applications in the field of metallurgy and chemical engineering such as material manufactured by extrusion process and heat-treated materials traveling between a feed roll and wind-up roll or on conveyor belt possess the feature of a moving continuous surface. And also in the polymer industry where plastic films and artificial fibers are drawn , which increases from almost zero at the orifice up to a maximum value at which it remains constant. The moving fiber produces a boundary layer flow in the surrounding medium. Finally the fiber is cooled and that in turn effect the final product of the yarn.

Flow through a fluid saturated porous medium is important in many technological applications, and has increasing importance with the growth of interest in geothermal energy and astrophysical problems, several other applications may also benefit from a better understanding of the fundamentals of mass, energy and momentum transport in porous media, namely, cooling of nuclear reactors and underground disposal of nuclear waste, petroleum reservoir operations, building insulation, food processing, casting and welding in manufacturing processes, etc. Heat transfer in porous media is a topic of vital importance to these applications, there by generating the need for their full understanding, recent books by Ingham & Pop[1998],Nield and Bejan [1999] and Vafai[2000] demonstrate that flow in porous media are becoming a classical subject , one where earlier developments have been confirmed by a large number of subsequent studies.

Most of the heat conduction studies in porous media have been considered for ambient fluids with constant physical properties. However, it is well known that the viscosity of liquid changes markedly with temperature and this influences the variation of velocity and temperature through the flow. Therefore, when applied to practical heat transfer problems with large temperature differences between the surface and the fluid, the constant property assumption could cause significant errors. For example, the viscosity of water decreases by about 240 percent when the temperature increases from 10 ^{0}C(μ=0.00131 g/cm.s) to 500 ^{0}C (μ=0.00548 g/cms), SeeLing & Dybbs [1987] effect of property variations on heat transfer is a highly complicated task for various reasons. First of all, the variations of properties with temperature differ from one fluid to another, some times it is impossible to express their variations in analytical form. However, for practical applications, a reliable & appropriate correlation equation based on the constant-property assumption can be used so that it may be used when the variable-property effect becomes important.

In 1961, Sakiadis [1961-I, 1961-II] initiated the study of the boundary layer flow over a continuous solid surface moving with constant speed. Due to the entrainment of ambient fluid, the boundary layer flow situation is quite different from the classical Blasius problem of boundary flow over a semi-infinite flat plate. Erickson et al. [1966] extended the work of Sakiadi's to account for mass transfer at this stretched sheet surface.Tsou et al [1967] reported both analytical and experimental results for the flow and heat transfer aspects developed by a continuously moving surface. Soundalgekar and Ramana Murthy [1980] investigated the constant surface velocity case with a power law temperature variation. There are several practical applications in which significant temperature differences between the surface of the body and the ambient fluid exist, these temperature differences cause density gradient in the fluid medium. Thus the study of boundary layer flow of a Visco-elastic fluid has been the main subject of a large number of researchers in the past D.W.Beard et al [1964], I.J.Crane [1998], C.K.Cher et al1998], V.K.Gage et al[1991]. Recently the applications of the Visco-elastic boundary layer flow extrude to the area with additional effects, such as heat transfer in porous media and the effect of magnetic field variable viscosity, the effect of magnetic field, and diffusion of chemical reactive species etc K.Vajravelu et al [991], A.Subhas et al [1998], M.S.Sarma et al[1998], K.Vajravelu et al[1999], M.S.Abel et al[2001], S.Asghar [2002], K.V.Prasad [2003].

All the above investigations are restricted to a continuous surface moving with constant velocity which is not adequate for many practical applications. Since the surface undergoes stretching and cooling or heating that cause surface velocity and temperature variations. Danberg et al [1976] investigated the non-similar solution for the flow in the boundary layer past a wall that is stretched with a velocity proportional to the distance along the wall. Velggaar [1977] studied the momentum and heat transfer to a continuous accelerating surface by assuming linear variation of surface velocity with respect to distance from the slot. Groubka & Bobba [1985] analyzed the stretching problem for surface moving with a linear velocity and with variable surface temperature. Ali [1994] has reported flow and heat transfer characteristics on a stretched surface subject to power-law velocity and temperature distribution. The flow field of a stretching wall with a power law velocity variation was investigated by Banks [25]. Recently, Ali [26] and Elbashbeshy [1998] extended Banks work for a porous stretched surface for different values of the injection parameter. All the above investigations are restricted to hydrodynamic flow and heat transfer problems. However, of late hydro magnetic flows & heat transfer have become more important in recent years because of many important applications.

The use of magnetic field has been also used in the process of purification of molten metals from non-metallic inclusions. Many works have been reported on flow and heat transfer of electrically conducting fluids over a stretched surface in the presence of magnetic field. See for instance Chakrabarti & Gupta [1979] Chiam [2002] Chandran et al [1996].In several practical applications, there exist significant

temperature differences between the surface and the ambient fluid. These necessitates the consideration of temperature dependent heat source or sink which may exert strong influence on the heat transfer characteristics (Vajravelu and Nayfeh [1993].
The study of heat generation or absorption effects is important in view of several physical problems such fluids undergoing exothermic or endothermic chemical reactions (Vajraveluy and Hadjinicalou [1993] although, exact modeling of internal heat generation or absorption is quite difficult, can express its average behavior for most physical situations. Heat generation or absorption has been assumed to be constant, space dependent exponentially decaying heat generation or absorption in their work on flow and heat transfer from a vertical plate.

It is worth mentioning that heat transfer in porous media which is induced by internal heat generation arises in various physical problems such as heat removal from nuclear fuel debris in nuclear reactors, the underground disposal of radioactive waste materials, fire and combustion modeling, the development of metal waste form from spent nuclear fuel, and exothermic chemical reaction in packed-bed reactors.
Exact modeling of internal heat generation/absorption is impossible and hence simple mathematical models considering average behavior in most physical situations have been proposed by Abo-Eladahab and El Aziz [2004]. In the present work we extend the work on Newtonian liquids investigated by Abo-Eldahab & et al [2004] to visco-elastic liquid flows. The main aim of the article is to analyze the effect of space dependent and temperature dependent heat generation /absorption parameters, Prandtl number, magnetic parameter ,visco-elastic parameter, porosity parameter, on a visco-elastic boundary layer flow and heat transfer over stretching sheet with suction/blowing effects.

2.Mathematical Formulation
3. Momentum Transfer

Two-dimensional flow of an incompressible electrically conducting Visco-elastic fluid of the type Walter's liquid B past a porous stretching sheet is considered. The flow is generated due to stretching sheet along x-axis by application of two equal and opposite forces. This flow obeys the rheological equation of state derived by Beard & Walters [1964], further this flow field is exposed under the influence of uniform transverse magnetic field. Hence, the modified form of MHD Visco-elastic boundary layer flow, equations takes the form

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=v\frac{\partial^2 u}{\partial y^2}-\sigma B_0^{\,2}\frac{u}{\rho}-k_0\left\{u\frac{\partial^3 u}{\partial x\partial y^2}+v\frac{\partial^3 u}{\partial y^3}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}-\frac{\partial^2 u}{\partial x\partial y}\frac{\partial u}{\partial y}\right\}-\frac{v}{k'}u \tag{1}$$

Here, B_0 is the applied magnetic field, σ the electrical conductivity of the fluid, k_0 the first moment of the distribution function of relaxation times, ν kinematic viscosity and k' permeability of the porous medium. The magnetic field B_0 is applied in the transverse direction of the sheet and induced magnetic field is assumed to be negligible. The equation of continuity is

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

(2)

We assume the flow is subjected to suction and boundary sheet with velocity v_0 and stretching sheet with stretching rate c.
The boundary conditions for the above flow situation are.

$$u_w(x)=cx,\quad v=-v_0 \ at \quad y=0$$

$$u\to 0 \qquad u_y=0 \qquad as \qquad y\to\infty$$

(3)

4. Solution of Momentum Equation
Equations (1) and (2) admit self-similar solution of the form

$$u=cxf'(\eta), \qquad v=-(c\nu)^{1/2} f(\eta) \qquad where \qquad \eta=\left(c/\nu\right)^{1/2} y \quad (4)$$

Substituting the values of (4) in (1), and (2) we obtain the following equation:

$$f'^2-ff''=f'''-Mnf'-k_1\{2ff'''-f''^2-ff^{iv}\}-k_2 f'$$

(5)

Where $k_1=k_0c/\nu$ is the visco-elastic parameter and

$Mn=\sigma B_0^{\ 2}/\rho$ The magnetic parameter $k_2=\dfrac{\nu}{k'c}$ is the Porosity parameter.

And the corresponding boundary conditions are

$$f'(0)=1, \qquad f(0)=R, \qquad at \qquad \eta=0$$

$$f'(\infty)=0, \qquad f''(\infty)=0 \qquad as \qquad \eta\to\infty$$

(6)

Where $R=\dfrac{v_0}{\sqrt{c\nu}}$ is the suction parameter,

The flow behavior permits us to assume the solution of equation (5) in the form
Which satisfies the basic equation (2) and boundary conditions (3) with

$$f(\eta)=A+B\exp(-\alpha\eta), \qquad \alpha>0$$

(7)

$$A=R+1/\alpha \qquad B=-1/\alpha$$

Here, α is the positive real root of the cubic equation

$$\alpha^3 + \frac{(k_1 - 1)}{Rk_1}\alpha^2 + \frac{1}{k_1}\alpha + \frac{(1 + M_n + k_2)}{Rk_1} = 0 \tag{8}$$

Hence, the resultant solutions of velocity components are

$$u = cx\exp(-\alpha\eta), \qquad v = -(\nu c)^{\frac{1}{2}}\{A + B\exp(-\alpha\eta)\} \tag{9}$$

5. HEAT TRANSFER ANALYSIS

The governing boundary layer equation of energy with space and temperature dependent heat generation or absorption is

$$\rho\, c_p\left\{u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right\} = k\frac{\partial^2 T}{\partial y^2} + q''' \tag{10}$$

Where u,v and T are the fluid x-component of velocity, y-component of velocity and the temperature respectively. ρ ,ν , k, and c_p are the fluid density, kinematic viscosity, thermal conductivity and specific heat at constant pressure of the fluid respectively,

q''' is the rate of internal heat generation (> 0) or absorption (< 0) coefficient,

The plus and minus signs that appears on the right-hand side of equation (10) pertain to assisting or opposing flows respectively.

The internal heat generation or absorption term q''' is modeled according to the following equation:

$$q''' = \left(\frac{ku_w(x)}{x\nu}\right)\left[A^*(T_w - T_\infty)e^{-\alpha\eta} + B^*(T - T_\infty)\right] \tag{11}$$

Where A^* and B^* are coefficients of space and temperature-dependent internal heat generation/absorption, respectively.

In equation (10) , the first term represents the dependence of the internal heat generation or absorption on the space coordinates while the latter represents its dependence on the temperature, Note that both A^*>0 and B^*>0, this case corresponds to internal heat generation while for both A^*<0 and B^*<0, this case corresponds to internal heat absorption.

6. Solution of Heat Transfer Equation

We define non-dimensional temperature profile as

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{12}$$

Substituting equation (12) in equation (11) and also considering the values of u and v from equation (9), equation (10), takes the form

$$\theta'' + \Pr f\theta' + \left(B^* - \Pr lf'\right)\theta = -A^* f' \tag{13}$$

Similarly boundary conditions ( ) , takes the form

$$\theta(0) = 1, \qquad \theta(\infty) = 0 \tag{14}$$

Where $\Pr = \mu c_p / k$ is the Prandtl number and

A^* and B^* are space and temperature dependent internal heat generation/absorption.

Introducing a new independent variable

$$\xi = \Pr B \exp(-\alpha\eta)/\alpha \tag{15}$$

and substituting this in equation (13) and considering the value of f , we obtain

$$\xi\theta'' + \left(1 - {}^{\Pr A}\!/_{\alpha} - \xi\right)\theta' - lB^*\theta = \frac{A^*}{\Pr} \tag{16}$$

The corresponding boundary conditions are

$$\theta(P_r B/\alpha) = 1 \qquad \theta(0) = 0 \tag{17}$$

The solution of (16) subjected to the boundary conditions (16) is

$$\theta(\eta) = c_1\left(e^{-\alpha\eta}\right)^{p_1} M\left[p_1 + lB^*, p_1 + 1,; \Pr Be^{-\alpha\eta}/\alpha\right] + c_2\left(\Pr Be^{-\alpha\eta}/\alpha\right)^2 \tag{18}$$

Here M denotes the Kummer's function with

$$c_1 = \frac{\left(1 - c_2 \left(P_r B / \alpha\right)^2\right)}{\left(P_r B/\alpha\right)^{p_1} \; M\left[p_1 + lB^*, p_1 + 1; P_r B/\alpha\right]}$$

(19)

$$c_2 = \frac{A^*}{P_r\left[4 - 2p_1 - lB^*\right]}$$

(20)

$$p_1 = {P_r A}/{\alpha} \;, \quad A = R + \frac{1}{\alpha} \quad, B = -\frac{1}{\alpha}$$

(21)

The non-dimensional wall temperature gradient derived from equation (5.6.7) is

$$\theta'(0) = c_1\left[-\alpha p_1 M\left[p_1 + lB^*, p_1 + 1; \frac{\Pr B}{\alpha}\right] + \frac{p_1 + lB^*}{p_1 + 1}\frac{\Pr B}{\alpha} M\left[p_1 + lB^* + 1, p_1 + 2; \frac{\Pr B}{\alpha}\right]\right] - c_2 \frac{\Pr^2}{\alpha}$$

Discussion of the results:

A boundary layer problem for momentum and heat transfer in MHD boundary layer visco-elastic fluid flow over a stretching surface in porous media with space dependent and temperature dependent internal heat generation/absorption is examined in this paper. Linear stretching of the porous boundary, temperature dependent, space dependent, heat source/sink and porosity, magnetic parameter are taken into consideration in this study. The basic boundary layer partial differential equations, which are highly non-linear, have been converted into a set of non-linear ordinary differential equations by applying suitable similarity transformations and their analytical solutions are obtained in terms of confluent hyper geometric function (Kummer's function). Different analytical expressions are obtained for non-dimensional temperature profile

In order to have some insight of the flow and heat transfer characteristics, results are plotted graphically for typical choice of physical parameters. Figure (1a) and (1b) are graphical representation of horizontal velocity profiles $f'(\eta)$ for different values of k_1 and k_2 .Fig (1a) provides the information that the increase of visco-elastic parameter leads to the decrease of the horizontal velocity profile. This is because of the fact that introduction of tensile stress due to visco-elasticity causes transverse contraction of the boundary layer and hence velocity decreases. The effect of porosity parameter on the horizontal velocity profile in the boundary layer is shown in Figure (1b) , It is observed that the increase of permeability parameter k_2 leads to the decrease of the horizontal velocity profiles, which leads to the enhanced deceleration of the flow and hence , the velocity decreases.

Figure (1c) illustrates that the effect of magnetic parameter i-e , the introduction of transverse magnetic field normal to the direction have a tendency to create a drag due to horizontal force which tends to resist the flow and, hence the horizontal velocity boundary layer decreases. This result is even true for the presence of porous parameter k_2 .

The presence of magnetic field in an electrically conducting fluid tends to produce a body force against the flow. This type of resistive force tend to slow down the motion of the fluid in the boundary layer which, in turn reduce the rate of heat in the flow and appears in increasing the flow temperature.

Figure (1d) and (1e) depict the influence of suction/blowing parameter R on the velocity profiles in the boundary layer. It is known that imposition of the wall suction (R >0) have the tendency to reduce the momentum boundary layer thickness, This causes reduction in the velocity profiles , However, the opposite behavior is observed by imposition of the wall fluid blowing or injection (R <0).

In figure (2) , temperature distribution $\theta(\eta)$ for different values of visco-elastic parameters k_1 are plotted. Figure (2) reveals that increase of visco-elastic parameter k_1 leads to increase of temperature profile $\theta(\eta)$ in the boundary layer. This is consistent with the fact that thickening of thermal boundary layer occurs due to the increase of non-Newtonian visco-elastic normal stress.

The effect of porosity parameter k_2 on temperature profile is shown in Figure (3). It is observed that the effect of porosity parameter k_2 is to decrease the temperature profile in the boundary layer.

The effect of Magnetic parameter on temperature profile for presence of porosity parameter and heat source/sink parameter is shown in figure(4) . It is observed that the effect of magnetic parameter is to increase the temperature profile in the boundary layer. The Lorentz force has the tendency to increase the temperature profile, also the effect on the flow and thermal fields become more so as the strength of the magnetic field increases. The effect of magnetic parameter is to increase the wall temperature gradient .

Figure (5) depict the influence of suction/blowing parameter R on the temperature profile in the boundary layer. It is observed that imposition of the wall suction(R>0) have the tendency to reduce the thermal boundary layer thickness. This causes reduction in the temperature profile. However, opposite behavior is observed by imposition of the wall fluid blowing or injection (R<0) as shown in Figure (6) .

The influence of the presence of space dependent internal heat generation ($A^* > 0$) or absorption ($A^* < 0$) in the boundary layer on the temperature field is presented in Figure (7) , It is clear from this graph that increasing the value of A^* produces increase in temperature distributions of the fluid. This is expected since the presence of heat source ($A^* > 0$) in the boundary layer generates energy which causes the temperature of the fluid to increase. This increase in the temperature produces an increase in the flow field due to the buoyancy effect. However, as the heat source effect becomes large ($A^* =1$, $A^*=2$) ,a distinctive peak in the temperature profile occurs in the fluid adjacent to the wall. This means that the temperature of the fluid near the sheet is higher than the sheet temperature and consequently, heat is expected to transfer to the wall. On the contrary , heat sink ($A^* < 0$) has the opposite effect , namely cooling of the fluid.

When the internal heat generation is absent or present, it is seen that the effect of the internal heat generation is especially pronounced for high values of A^*. The fluid temperature is greater when internal heat generation exists. This is logical because the increase of the heat transfer close to the plate and this will induce more flow along the plate.

The influence of the temperature-dependent internal heat generation ($B^* > 0$) or absorption ($B^* < 0$) in the boundary layer on the temperature field is the same as that of spacel-dependent internal heat generation or absorption .Namely, for $B^* > 0$ (heat source), the temperature of fluid increase while they decrease for $B^* < 0$ (heat sink). These behaviors are depicted in Figure (8).

In Figure(9) several temperature profiles are drawn . The effect of Prandtl number on heat transfer may be analyzed from these figures. Both graphs implicate that the increase of Prandtl number results in the decrease of temperature distribution at a particular point. This is due to the fact that there would be a decrease of thermal boundary layer thickness with the increasing values of Prandtl number. Temperature distribution in both situations asymptotically approaches to zero in the free stream region.

The values of wall temperature gradient $\theta'(0)$ there would be suction parameter present or absent in the flow in the PST case are recorded in comparison of those published results of Gupta et al [1977] ,Grubka et al [1988] , Ali [1995], Emad M et al [2004] and present result are recorded in the Table–I. From this table it is observed that the effect of increasing values of Prandtl number Pr is to lower the wall temperature. Special cases of our results are in excellent agreement with some of the existing work.

Conclusion:

A mathematical model study on the influence of heat transfer in MHD boundary layer visco -elastic fluid flow over stretching surface in porous media with space and temperature dependent internal heat generation/absorption, where flow is subject to suction/blowing through the porous boundary, are taken in to consideration in this study. Analytical solutions of the governing boundary layer problem have been obtained in terms of confluent hypergeometric function (Kummer's function) and its special form, different analytical expressions are obtained for non-dimensional temperature profile for boundary conditions,. Explicit analytical expressions are also obtained for dimensionless temperature gradient $\theta'(0)$ and heat flux q_w for general cases as well as for special cases of different physical situations. The special conclusions derived from this study can be listed as follows.

(i) Explicit expressions are obtained for various heat transfer characteristics in the form of confluent hypergeometric function (Kummer's function) ,several expressions are also obtained in the form of some other elementary functions as the special cases of Kummer' s function.

(ii) The combined effect of increasing values of visco-elastic parameter k_1 and porosity parameter k_2 is to decrease velocity of the fluid significantly in the boundary layer region, this is because of the fact that introduction of tensile stress due to visco-elasticity causes transverse contraction of the boundary layer and hence velocity decreases, and increasing the values of porosity parameter which leads to enhanced deceleration of the flow and hence, velocity decreases.

(iii) The effect of increasing values of magnetic parameter Mn is to decrease velocity of the fluid, i-e the introduction of transverse magnetic field normal to the direction have a tendency to create a drag due to horizontal force which tends to resist the flow and, hence the horizontal velocity boundary layer decreases.

(iv) The effect of increasing values of suction parameter ($R>0$) is to decrease the velocity, where as it has opposite effect for $R<0$.

(v) The combined effect of increasing values of magnetic parameter Mn and visco-elastic parameter k_1 is to increase temperature distribution in the flow region, as the increasing values of magnetic parameter is to increase the temperature, because Lorentz force has the tendency to increase the temperature profile, also the effect on the flow and thermal fields become more so as the strength of the magnetic field increases. An increasing values of visco-elastic parameter is to increase temperature, this is consistent with the fact that thickening of thermal boundary layer occurs due to the increase of non-Newtonian visco-elastic normal stress.

(vi) The effect of increasing the values of porosity parameter k_2 is to decrease the temperature distribution in the flow region.

(vii) The effect of increasing values of suction parameter R is to decrease the temperature distribution and that of blowing is to increase the same, It is known that imposition of the wall suction ($R >0$) have the tendency to reduce the momentum boundary layer thickness. This causes reduction in the velocity profiles. However the opposite behavior is observed by imposition of the wall fluid blowing or injection ($R <0$).

(viii) The combined effect of increasing values of space dependent and temperature dependent heat generation/absorption parameters A^* and B^* respectively is to increase the temperature distribution in the boundary layer flow region, as increase in the values of A^* is to increase fluid temperature is greater when internal heat generation exists. This is logical because the increase of the heat transfer close to the plate and this will induce more flow along the plate.

(ix) The effect of increasing values of Prandtl number Pr is to reduce the temperature largely in the boundary layer flow region This is due to the fact that there would be a decrease of thermal boundary layer thickness with the increasing values of Prandtl number. Temperature distribution in both situations asymptotically approaches to zero in the free stream region.

(x) The limiting cases of the results of this paper are in excellent agreement with the results of Emad M. et al [2004].

(xi) The values of wall temperature gradient $\theta'(0)$ with absence/presence of suction parameter in the flow are recorded in comparison to those published results of Gupta et al [1977] ,Grubka et al [1988] , Ali [1995], Emad M et al [2004] and the present results are in excellent agreement.

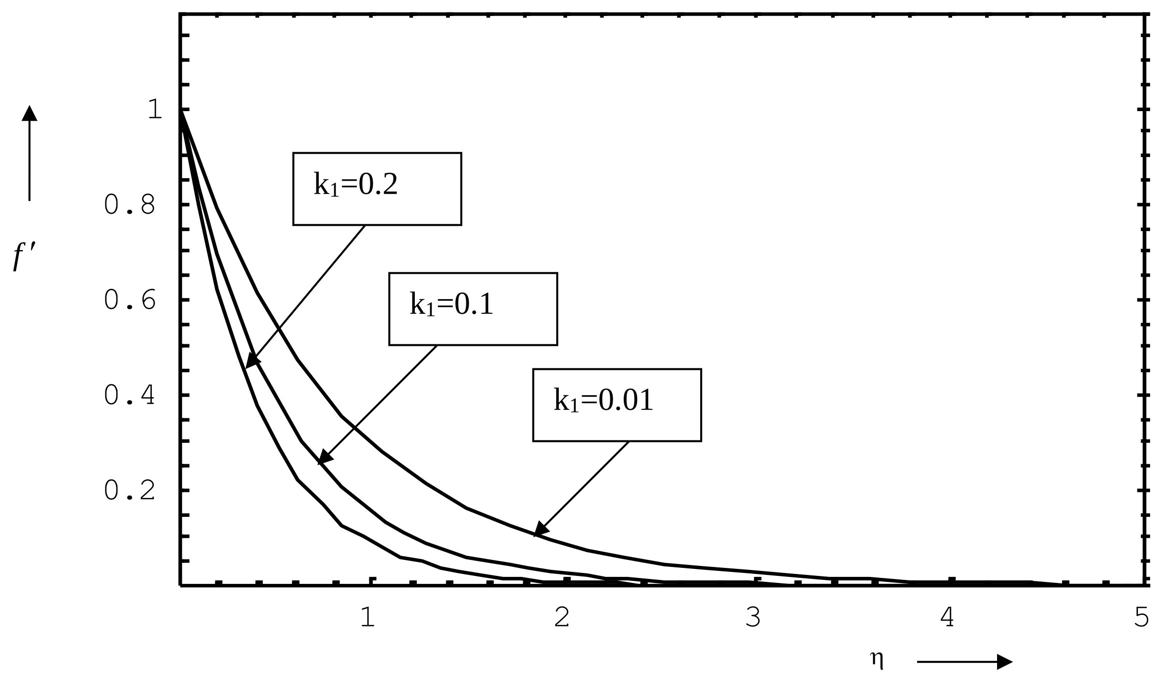

Fig (1a) : Velocity profiles for various values of k_1 with k_2=0.2 , Mn=0.5 , R=0.1.

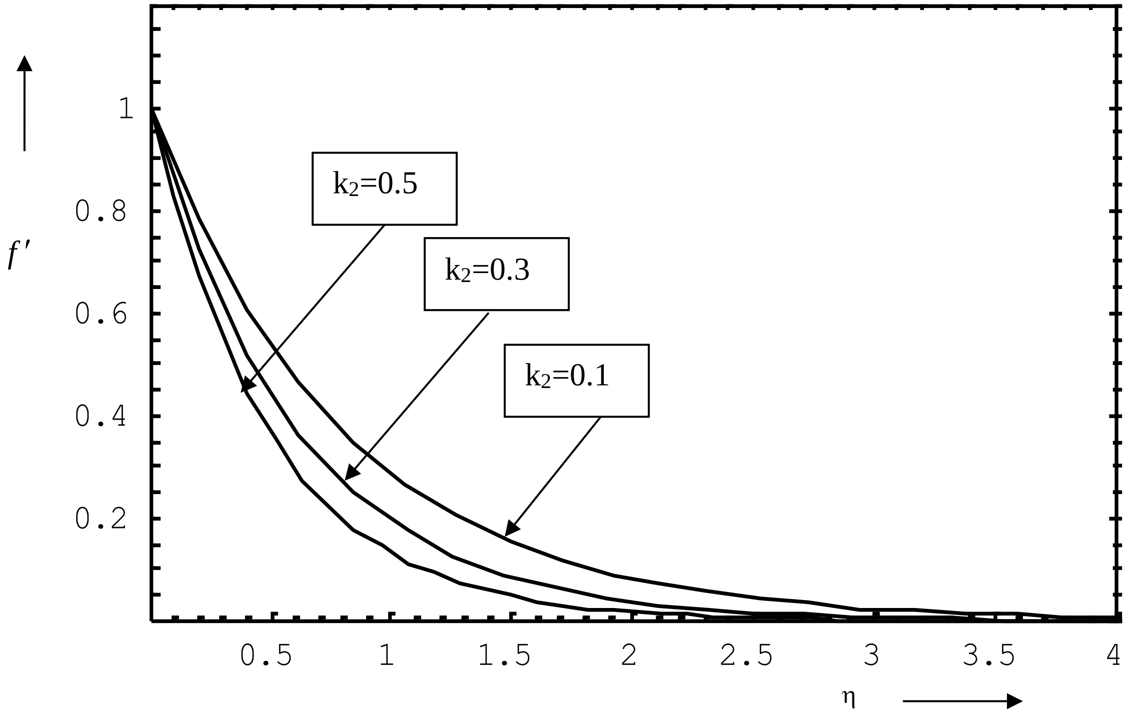

Fig (1b) : Velocity profiles for various values of k_2 with k_1=0.1, Mn=0.5 , R=0.5.

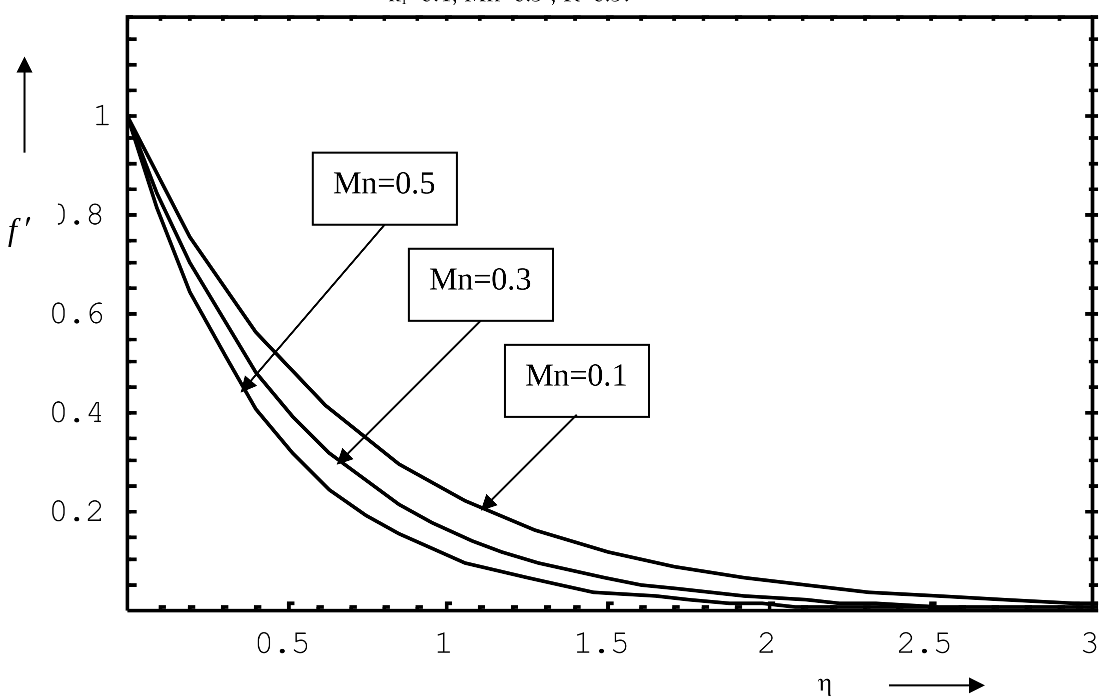

Fig (1c): Velocity profiles for various values of Mn with k_1=0.5,k_2 =0.5, R=0.5.

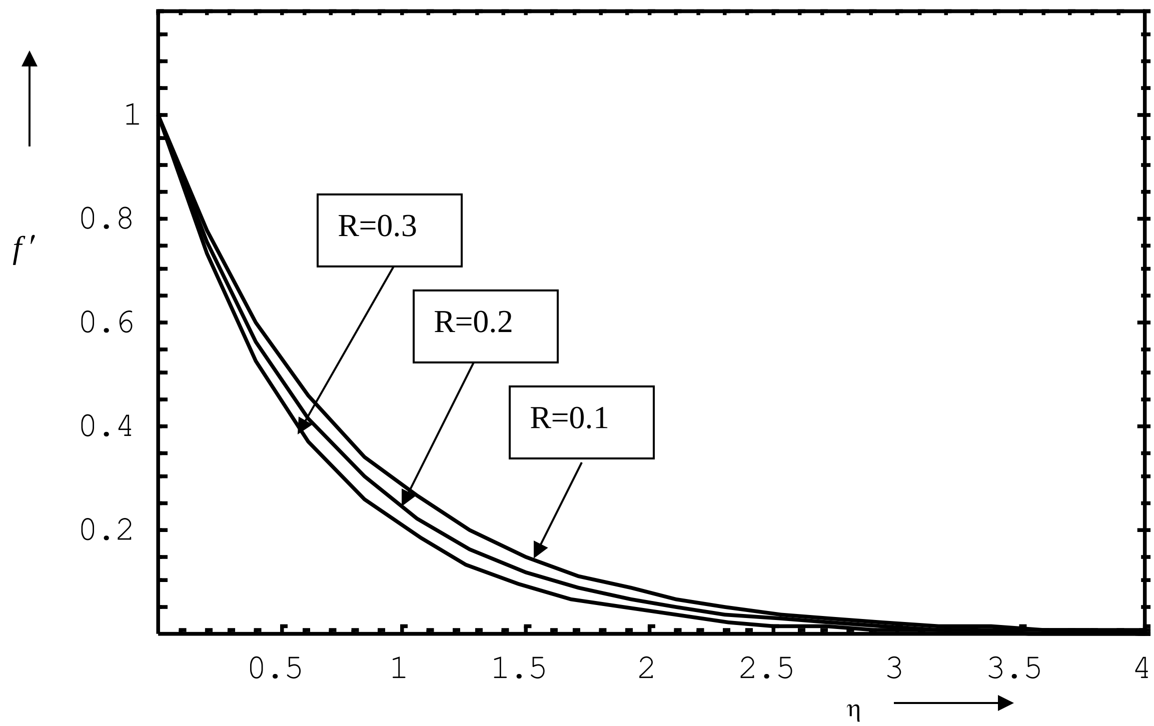

Fig (1d) : Velocity profiles for various values of R (R>0) with k_1=0.1,k_2, =0.2,Mn=0.5.

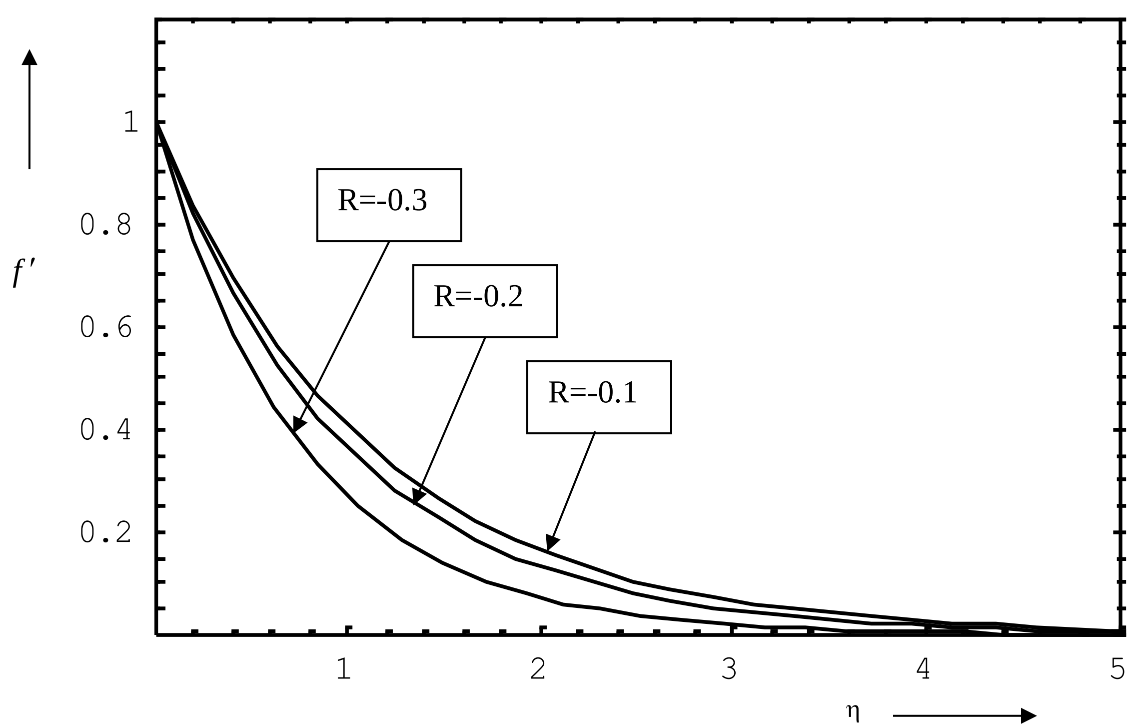

Fig (1e) : Velocity profiles for various values of R (R< 0)with k_1=0.1,k_2 =0.1, Mn=0.5.

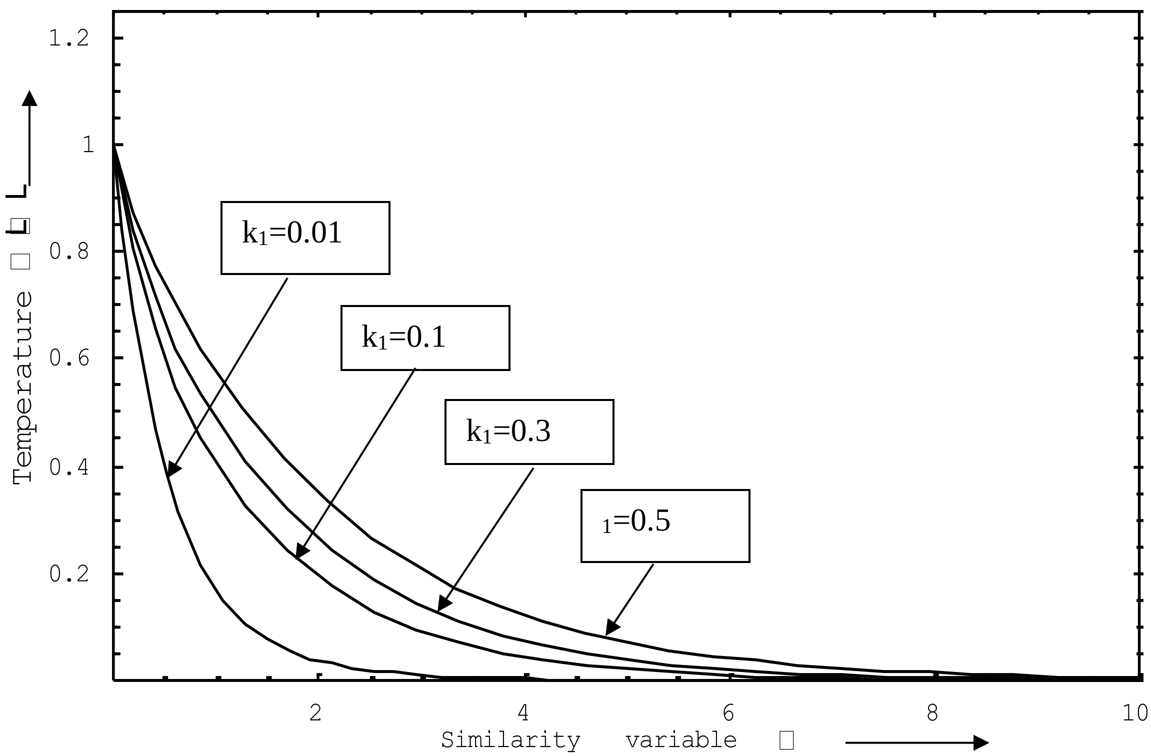

Fig (2a) : Temperature profiles for various values of k_1 with $l=0.1$, $Pr=2.0$, $A^*=1.0$, $B^*=0.01$, $R=-0.5$.

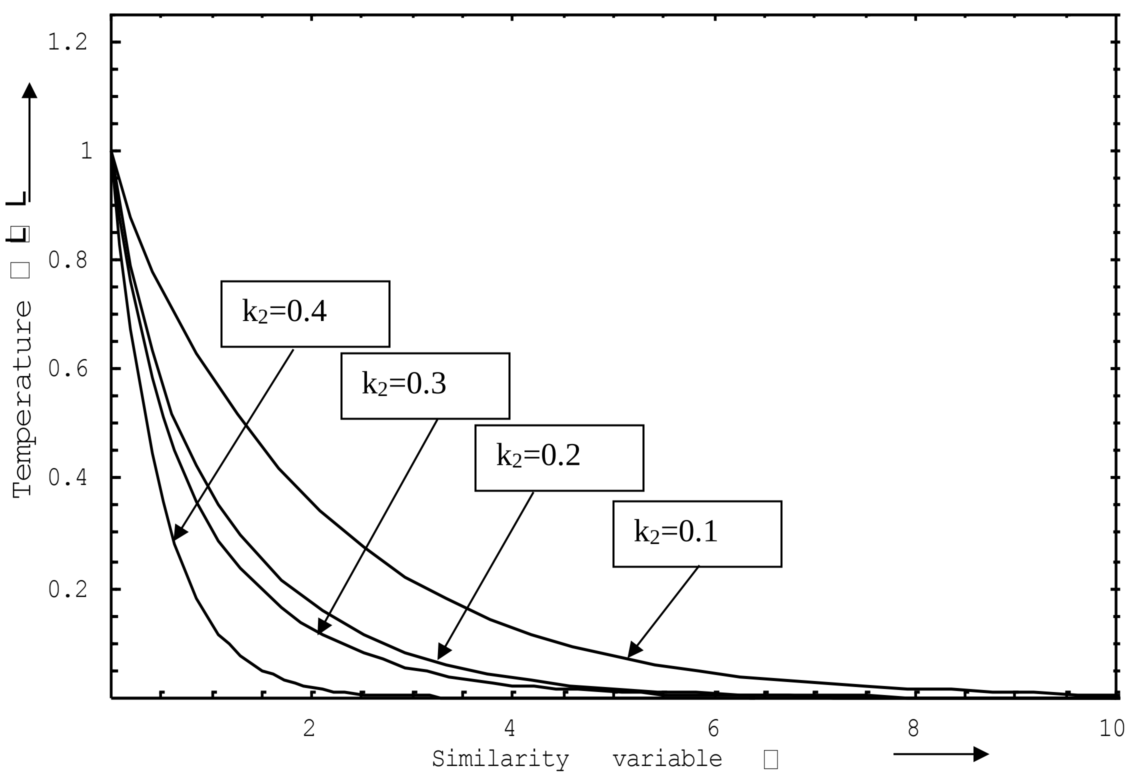

Fig (3a) : Temperature profiles for various values of k_2 with $l=0.1$, $Pr=2.0$, $A^*=1.0$, $B^*=0.01$, $R=-0.5$.

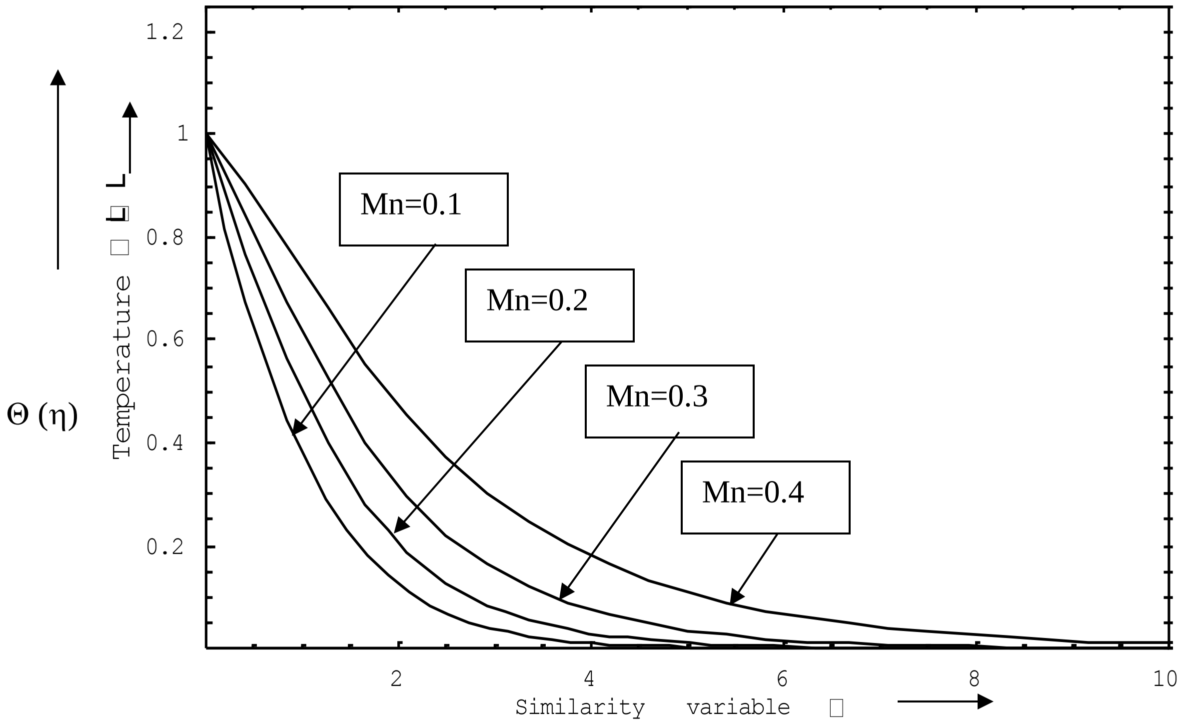

Fig (4a) : Temperature profiles for various values of Mn with
l=0.1, Pr=2.0, A^*=-1.0, B^*=0.01, R=-0.5 .

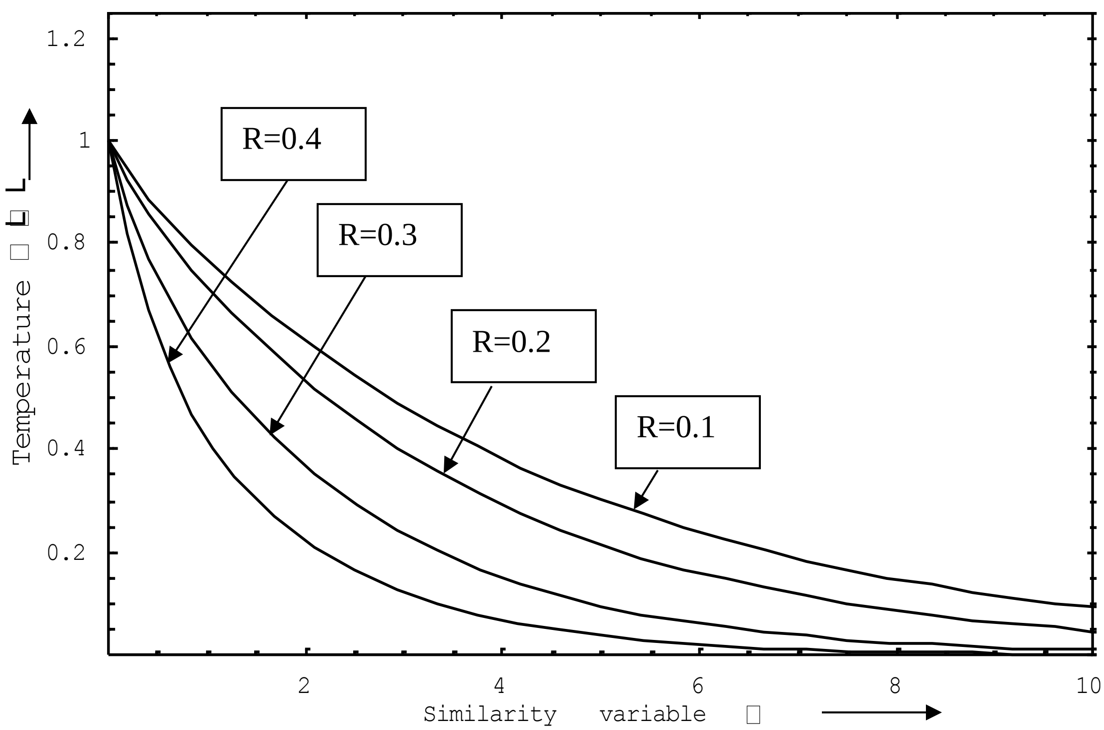

Fig (5) : Temperature profiles for various values of R (R > 0) with
l=0.1, Pr=2.0, A^*=1.0, B^*=0.01 .

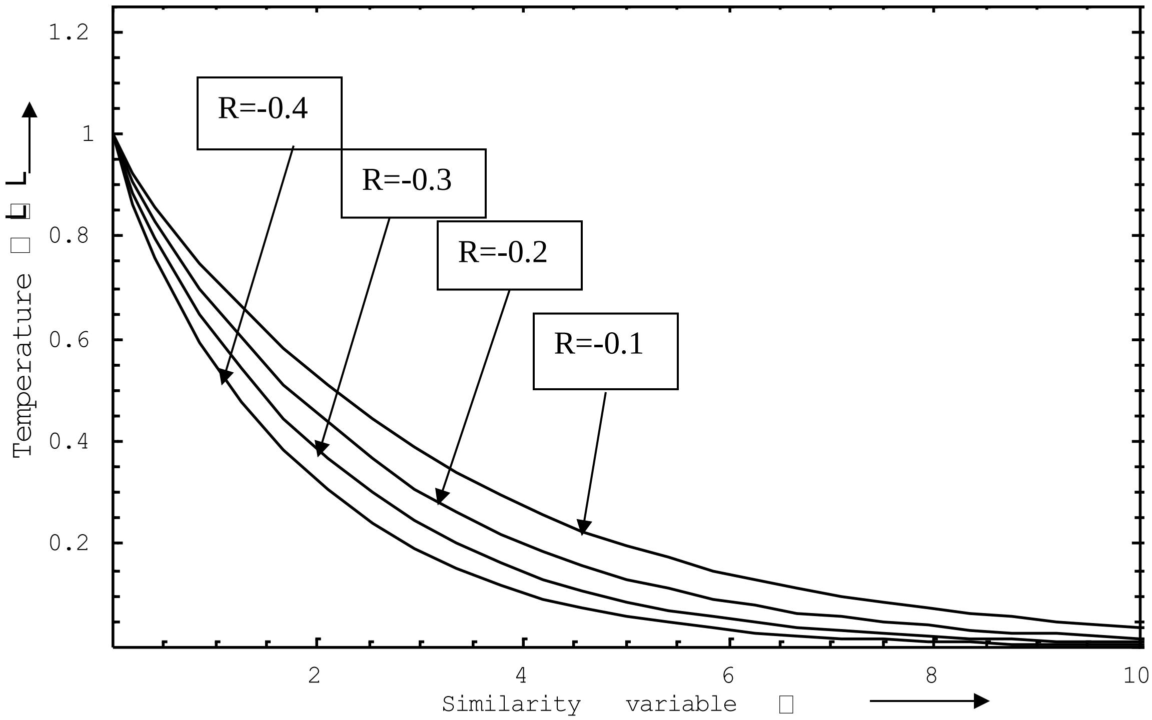

Fig (6) : Temperature profiles for various values of R (R < 0)

with

l=0.1, Pr=2.0, A^*=1.0, B^*=0.01.

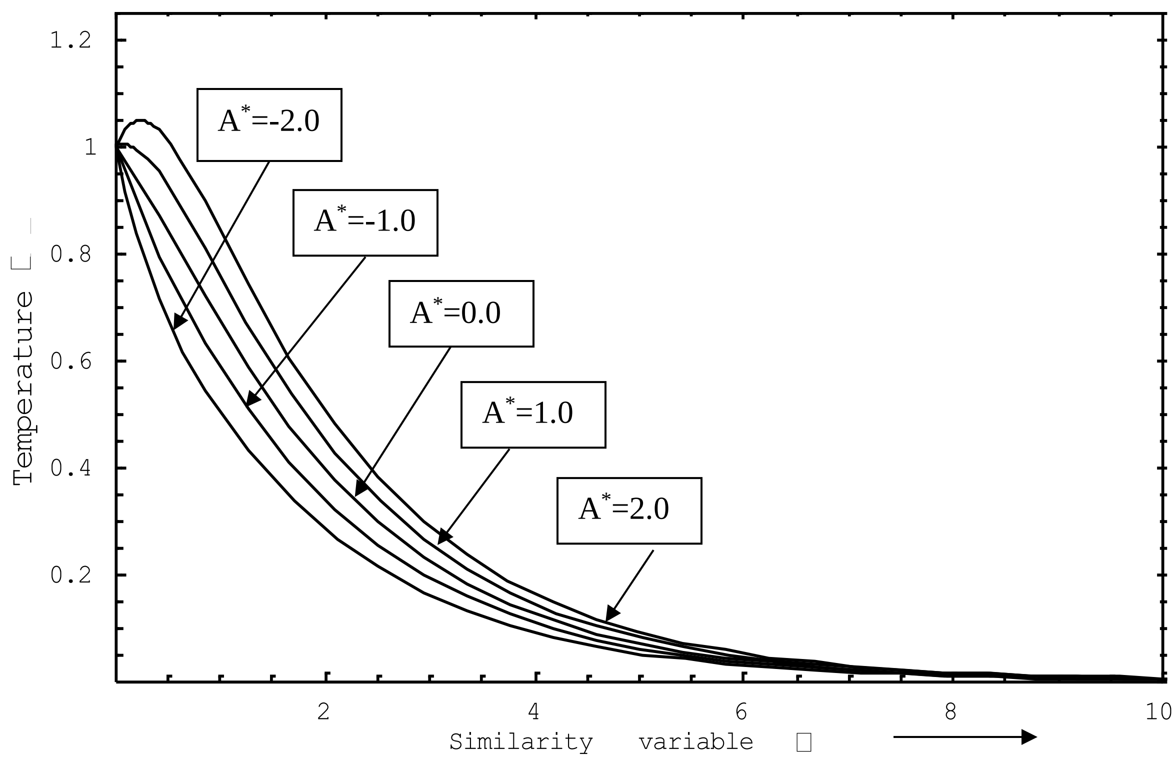

Fig (7) : Temperature profiles for various values of A^* with l=0.1, Pr=2.0, B^*=0.01, R=-0.5.

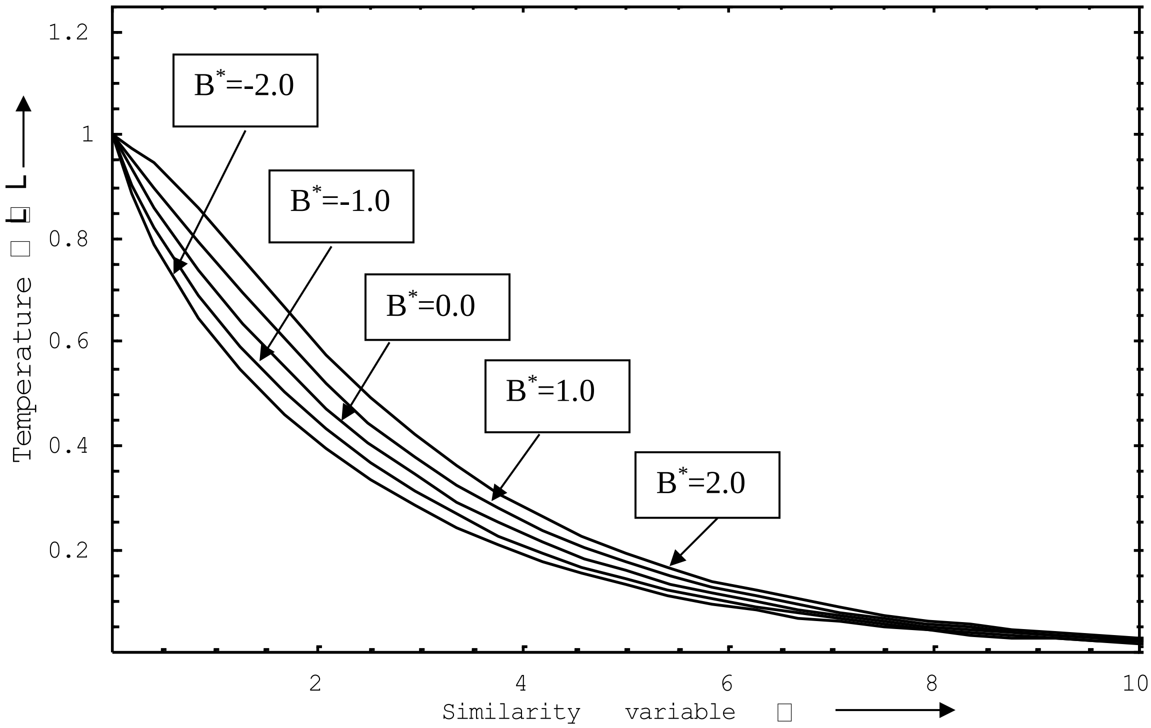

Fig (8) : Temperature profiles for various values of B^* with l=0.1, Pr=2.0, A^*=1.0, R=-0.5, PST Case.

Table-I Copmarision of the values of $\theta'(0)$ for various values of Pr with Mn=R=A^*= B^*=0.

Pr	R	Gupta et al [1977]	Grubka et al [1985]	Lashmishka et al [1988]	Ali [26]	Emad M et al [2004]	**Present study**
0.7	0	-	-	-0.45446	-0.45255	-0.45449	-0.43563
1.0	0	-0.5820	-0.5820	-	-0.59589	-0.58201	-0.61148
10.0	0	-	-	-	-2.09589	-2.30801	-2.34107

1.0	-1.067	-0.1105	-2.3080	-	0.10996	-0.11077	-0.11015

Nomenclature:

C : stretching rate
f : dimensionless stream function
k_1 : viscoelastic parameter
Mn : Magnetic parameter
k_2 : porosity parameter
R : suction parameter
P_r : Prandtl number
A^* : coefficient of space dependent internal heat generation
B^* : coefficient of temperature dependent internal heat absorpion
u : horizontal velocity component [ms]
x : horizontal coordinate[m]
y : vertical coordinate[m]
v : vertical velocity component [ms]
η : similarity variable
τ : shear stress
μ : dynamic viscosity [kg m s]
ρ : density [kg m]
γ : kinematic viscosity [m s]
q''' : internal heat generation /absorption

Superscripts:

′ : first derivative w.r.t η
″ : second derivative w.r.t η

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