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GROUP S_4 DIFFERENCE CORDIAL LABELING FOR CERTAIN GRAPHS

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ABSTRACT. Let $G = (V, E)$ be a graph. Consider the group S_4. A graph G is said to be group S_4 difference cordial labeling if there exist a function $\alpha : V(G) \to S_4$ for each edge xy assign the label 0 when $|o(\alpha(x)) - o(\alpha(y))| = 0$ and 1 otherwise. The number of vertices labeled with λ and μ differ by atmost 1 where λ, μ are the elements in S_4 and the number of edges labeled with 0 and 1 differ by atmost 1. The graph with a group S_4 difference cordial labeling is called group S_4 difference cordial graph.

1. Introduction

Graphs considered here are finite, simple, connected and undirected. To know the origin of labeling we can read the paper written by Rosa [3] in 1967. We follow the basic notation and terminologies of graph theory as in Douglas B. West [4]. Graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. The concept of cordial labeling was introduced by Cahit [1].

Lourdusamy et. al introduced the concept of group S_3 cordial remainder labeling [2]. In this paper we discuss group S_4 difference cordial labeling for star related graphs and snake related graphs.

2. S_4 Difference Cordial Labeling

S_4 is a group of all permutations of 4 element set. It has one 1-order element, nine 2-order elements, eight 3 order elements and six 4 order elements.

$$\begin{bmatrix}1 & 2 & 3 & 4\\1 & 2 & 3 & 4\end{bmatrix} = e, \quad \begin{bmatrix}1 & 2 & 3 & 4\\2 & 1 & 3 & 4\end{bmatrix} = a, \quad \begin{bmatrix}1 & 2 & 3 & 4\\3 & 2 & 1 & 4\end{bmatrix} = b,$$

$$\begin{bmatrix}1 & 2 & 3 & 4\\4 & 2 & 3 & 1\end{bmatrix} = c, \quad \begin{bmatrix}1 & 2 & 3 & 4\\1 & 3 & 2 & 4\end{bmatrix} = d, \quad \begin{bmatrix}1 & 2 & 3 & 4\\1 & 4 & 3 & 2\end{bmatrix} = f,$$

$$\begin{bmatrix}1 & 2 & 3 & 4\\1 & 2 & 4 & 3\end{bmatrix} = g, \quad \begin{bmatrix}1 & 2 & 3 & 4\\2 & 1 & 4 & 3\end{bmatrix} = h, \quad \begin{bmatrix}1 & 2 & 3 & 4\\3 & 4 & 1 & 2\end{bmatrix} = i,$$

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$$\begin{bmatrix}1&2&3&4\\4&3&2&1\end{bmatrix}=j,\quad \begin{bmatrix}1&2&3&4\\2&3&1&4\end{bmatrix}=k,\quad \begin{bmatrix}1&2&3&4\\3&1&2&4\end{bmatrix}=l,$$

$$\begin{bmatrix}1&2&3&4\\1&3&4&2\end{bmatrix}=m,\quad \begin{bmatrix}1&2&3&4\\1&4&2&3\end{bmatrix}=n,\quad \begin{bmatrix}1&2&3&4\\3&2&4&1\end{bmatrix}=o,$$

$$\begin{bmatrix}1&2&3&4\\4&2&1&3\end{bmatrix}=p,\quad \begin{bmatrix}1&2&3&4\\2&4&3&1\end{bmatrix}=q,\quad \begin{bmatrix}1&2&3&4\\4&1&3&2\end{bmatrix}=r,$$

$$\begin{bmatrix}1&2&3&4\\2&3&4&1\end{bmatrix}=s,\quad \begin{bmatrix}1&2&3&4\\2&4&1&3\end{bmatrix}=t,\quad \begin{bmatrix}1&2&3&4\\4&1&2&3\end{bmatrix}=u,$$

$$\begin{bmatrix}1&2&3&4\\3&4&2&1\end{bmatrix}=v,\quad \begin{bmatrix}1&2&3&4\\3&1&4&2\end{bmatrix}=w,\quad \begin{bmatrix}1&2&3&4\\4&3&1&2\end{bmatrix}=x.$$

$o(e)=1$, $o(a)=o(b)=o(c)=o(d)=o(f)=o(g)=o(h)=o(i)=o(j)=2$, $o(k)=o(l)=o(m)=o(n)=o(o)=o(p)=o(q)=o(r)=3$, $o(s)=o(t)=o(u)=o(v)=o(w)=o(x)=4$.

Definition 2.1. Let $\alpha : V(G) \to S_4$ and for all $xy \in E(G)$, $\alpha^*(xy) = 0$ if $|o(\alpha(x)) - o(\alpha(y))| = 0$ and $\alpha^*(xy) = 1$ if $|o(\alpha(x)) - o(\alpha(y))| \neq 0$. α is called group S_4 difference cordial labeling if the difference between number of vertices labeled with λ and μ is at most 1 where λ, μ are the elements in S_4 and the difference between the number of edges labeled with 0 and 1 is at most 1

3. Main Results

Theorem 3.1. *Star graph $K_{1,n}$ is not S_4 difference cordial graph.*

Proof. Let V_1, V_2 be the bipartition of $K_{1,n}$ with $V_1 = z$ and $V_2 = \{z_\beta : 1 \le \beta \le n\}$.
Case 1.
Let $\alpha(z) = e$. To get $|e_\alpha(0) - e_\alpha(1)| \le 1$ we must have e as the label for $\frac{n}{2}$ vertices. Hence it is a contradiction to $|v_\alpha(\lambda) - v_\alpha(\mu)| \le 1$, where λ, μ are the elements of S_4.
Case 2.
Without loss of generality let $\alpha(z) = a$. To get $|e_\alpha(0) - e_\alpha(1)| \le 1$, we must have 2 order elements as the label for $\frac{n}{2}$ vertices. This is a contradiction to $|v_\alpha(\lambda) - v_\alpha(\mu)| \le 1$.
Case 3.
Without loss of generality let $\alpha(z) = k$. To get $|e_\alpha(0) - e_\alpha(1)| \le 1$, we must have 3 order elements as the label for $\frac{n}{2}$ vertices. Hence it is a contradiction to $|v_\alpha(\lambda) - v_\alpha(\mu)| \le 1$.
Case 4.
Without loss of generality let $\alpha(z) = s$. To get $|e_\alpha(0) - e_\alpha(1)| \le 1$, we must have 4 order elements as the label for $\frac{n}{2}$ vertices. This it is a contradiction to $|v_\alpha(\lambda) - v_\alpha(\mu)| \le 1$. ☐

Theorem 3.2. *Bistar $B_{n,n}$ is S_4 difference cordial graph.*

Proof. Let y and z be the apex vertices and $y_1, y_2, \cdots, y_n,\ z_1, z_2, \cdots, z_n$ be the pendent vertices. We define $\alpha : V(B_{n,n}) \to S_4$ as follows:
$\alpha(y) = a,\ \alpha(z) = k;$
for $1 \leq \beta \leq n$,

$$\alpha(y_\beta) = \begin{cases} b & \text{if } \beta \equiv 1\ (mod\ 12) \\ c & \text{if } \beta \equiv 2\ (mod\ 12) \\ d & \text{if } \beta \equiv 3\ (mod\ 12) \\ f & \text{if } \beta \equiv 4\ (mod\ 12) \\ g & \text{if } \beta \equiv 5\ (mod\ 12) \\ h & \text{if } \beta \equiv 6\ (mod\ 12) \\ i & \text{if } \beta \equiv 7\ (mod\ 12) \\ l & \text{if } \beta \equiv 8\ (mod\ 12) \\ n & \text{if } \beta \equiv 9\ (mod\ 12) \\ p & \text{if } \beta \equiv 10\ (mod\ 12) \\ j & \text{if } \beta \equiv 11\ (mod\ 12) \\ k & \text{if } \beta \equiv 0\ (mod\ 12) \end{cases}$$

and

$$\alpha(z_\beta) = \begin{cases} s & \text{if } \beta \equiv 1\ (mod\ 12) \\ t & \text{if } \beta \equiv 2\ (mod\ 12) \\ u & \text{if } \beta \equiv 3\ (mod\ 12) \\ v & \text{if } \beta \equiv 4\ (mod\ 12) \\ w & \text{if } \beta \equiv 5\ (mod\ 12) \\ x & \text{if } \beta \equiv 6\ (mod\ 12) \\ e & \text{if } \beta \equiv 7\ (mod\ 12) \\ m & \text{if } \beta \equiv 8\ (mod\ 12) \\ o & \text{if } \beta \equiv 9\ (mod\ 12) \\ q & \text{if } \beta \equiv 10\ (mod\ 12) \\ r & \text{if } \beta \equiv 11\ (mod\ 12) \\ a & \text{if } \beta \equiv 0\ (mod\ 12) \end{cases}$$

Clearly,

$$e_\alpha(0) = \begin{cases} n+1 & \text{if } n \text{ is multiple of } 11 \\ n & \text{Otherwise} \end{cases}$$

and

$$e_\alpha(0) = \begin{cases} n & \text{if } n \text{ is multiple of } 11 \\ n+1 & \text{Otherwise} \end{cases}$$

Hence α is group S_4 difference cordial labeling.

Theorem 3.3. *$S(K_{1,n})$ is S_4 difference cordial graph.*

Proof. Let the vertex set of $S(K_{1,n})$ be $\{y, y_\beta, z_\beta | 1 \le \beta \le n\}$ and the edge set of $S(K_{1,n})$ be $\{yy_\beta, y_\beta z_\beta | 1 \le \beta \le n\}$. Define $\alpha : V(S(K_{1,n})) \to S_4$ as follows:
$\alpha(y) = a$;
for $1 \le \beta \le n$,

$$\alpha(y_\beta) = \begin{cases} b & \text{if } \beta \equiv 1 \ (mod\ 12) \\ c & \text{if } \beta \equiv 2 \ (mod\ 12) \\ d & \text{if } \beta \equiv 3 \ (mod\ 12) \\ f & \text{if } \beta \equiv 4 \ (mod\ 12) \\ g & \text{if } \beta \equiv 5 \ (mod\ 12) \\ h & \text{if } \beta \equiv 6 \ (mod\ 12) \\ i & \text{if } \beta \equiv 7 \ (mod\ 12) \\ j & \text{if } \beta \equiv 8 \ (mod\ 12) \\ s & \text{if } \beta \equiv 9 \ (mod\ 12) \\ u & \text{if } \beta \equiv 10 \ (mod\ 12) \\ w & \text{if } \beta \equiv 11 \ (mod\ 12) \\ a & \text{if } \beta \equiv 0 \ (mod\ 12) \end{cases}$$

and

$$\alpha(z_\beta) = \begin{cases} k & \text{if } \beta \equiv 1 \ (mod\ 12) \\ l & \text{if } \beta \equiv 2 \ (mod\ 12) \\ m & \text{if } \beta \equiv 3 \ (mod\ 12) \\ n & \text{if } \beta \equiv 4 \ (mod\ 12) \\ o & \text{if } \beta \equiv 5 \ (mod\ 12) \\ p & \text{if } \beta \equiv 6 \ (mod\ 12) \\ q & \text{if } \beta \equiv 7 \ (mod\ 12) \\ r & \text{if } \beta \equiv 8 \ (mod\ 12) \\ t & \text{if } \beta \equiv 9 \ (mod\ 12) \\ v & \text{if } \beta \equiv 10 \ (mod\ 12) \\ x & \text{if } \beta \equiv 11 \ (mod\ 12) \\ e & \text{if } \beta \equiv 0 \ (mod\ 12) \end{cases}$$

It is easy to see that $e_\alpha(0) = n = e_\alpha(1)$. Hence α is group S_4 difference cordial labeling. □

Theorem 3.4. *$S(B_{n,n})$ is S_4 difference cordial graph.*

Proof. Let $V(S(K_{1,n})) = \{y, y', z, y'_\beta, y_\beta, z'_\beta, z_\beta | 1 \le \beta \le n\}$ and $E(S(K_{1,n})) = \{yy', y'z\} \bigcup \{yy'_\beta, y'_\beta y_\beta, zz'_\beta, z'_\beta z_\beta | 1 \le \beta \le n\}$. Define $\alpha : V(S(B_{n,n})) \to S_4$ by,
$\alpha(y) = e,\ \alpha(y') = a,\ \alpha(z') = b$;
for $1 \le \beta \le n$,

$$\alpha(y_\beta^{'}) = \begin{cases} c & \text{if } \beta \equiv 1 \ (mod\ 6) \\ f & \text{if } \beta \equiv 2 \ (mod\ 6) \\ h & \text{if } \beta \equiv 3 \ (mod\ 6) \\ q & \text{if } \beta \equiv 4 \ (mod\ 6) \\ w & \text{if } \beta \equiv 5 \ (mod\ 6) \\ j & \text{if } \beta \equiv 0 \ (mod\ 6) \end{cases}$$

$$\alpha(y_\beta) = \begin{cases} d & \text{if } \beta \equiv 1 \ (mod\ 6) \\ g & \text{if } \beta \equiv 2 \ (mod\ 6) \\ i & \text{if } \beta \equiv 3 \ (mod\ 6) \\ r & \text{if } \beta \equiv 4 \ (mod\ 6) \\ x & \text{if } \beta \equiv 5 \ (mod\ 6) \\ a & \text{if } \beta \equiv 0 \ (mod\ 6) \end{cases}$$

$$\alpha(z_\beta^{'}) = \begin{cases} k & \text{if } \beta \equiv 1 \ (mod\ 6) \\ m & \text{if } \beta \equiv 2 \ (mod\ 6) \\ o & \text{if } \beta \equiv 3 \ (mod\ 6) \\ s & \text{if } \beta \equiv 4 \ (mod\ 6) \\ u & \text{if } \beta \equiv 5 \ (mod\ 6) \\ b & \text{if } \beta \equiv 0 \ (mod\ 6) \end{cases}$$

$$\alpha(z_\beta) = \begin{cases} l & \text{if } \beta \equiv 1 \ (mod\ 6) \\ n & \text{if } \beta \equiv 2 \ (mod\ 6) \\ p & \text{if } \beta \equiv 3 \ (mod\ 6) \\ t & \text{if } \beta \equiv 4 \ (mod\ 6) \\ v & \text{if } \beta \equiv 5 \ (mod\ 6) \\ e & \text{if } \beta \equiv 0 \ (mod\ 6) \end{cases}$$

Here $e_\alpha(0) = 2n + 1 = e_\alpha(1)$. Hence $S(B_{n,n})$ is group S_4 difference cordial graph. □

Theorem 3.5. *The graph obtained by duplication of each vertex by an edge in P_n is S_4 difference cordial graph.*

Proof. Let $z_1, z_2, \cdots, z_n$ be the vertices of the path P_n and G be the graph obtained by duplication of each vertex z_β of the path P_n by an edge $z_\beta^{'} z_\beta^{''}$ for $1 \le \beta \le n$ at a time. Let $V(G) = \{z_\beta, z_\beta^{'}, z_\beta^{''} | 1 \le \beta \le n\}$ and $E(G) = \{z_\beta z_\beta^{'}, z_\beta z_\beta^{''}, z_\beta^{'} z_\beta^{''} | 1 \le \beta \le n\} \bigcup \{z_\beta z_{\beta+1} | 1 \le \beta \le n-1\}$ Define $\alpha : V(G) \to S_4$ as follows:
$\alpha(y) = a$;
for $1 \le \beta \le n$,

$$\alpha(z_\beta) = \begin{cases} a & \text{if } \beta \equiv 1 \ (mod\ 8) \\ c & \text{if } \beta \equiv 2 \ (mod\ 8) \\ d & \text{if } \beta \equiv 3 \ (mod\ 8) \\ f & \text{if } \beta \equiv 4 \ (mod\ 8) \\ g & \text{if } \beta \equiv 5 \ (mod\ 8) \\ h & \text{if } \beta \equiv 6 \ (mod\ 8) \\ i & \text{if } \beta \equiv 7 \ (mod\ 8) \\ j & \text{if } \beta \equiv 0 \ (mod\ 8) \end{cases}$$

$$\alpha(z'_\beta) = \begin{cases} b & \text{if } \beta \equiv 1 \ (mod\ 8) \\ k & \text{if } \beta \equiv 2 \ (mod\ 8) \\ m & \text{if } \beta \equiv 3 \ (mod\ 8) \\ o & \text{if } \beta \equiv 4 \ (mod\ 8) \\ q & \text{if } \beta \equiv 5 \ (mod\ 8) \\ s & \text{if } \beta \equiv 6 \ (mod\ 8) \\ u & \text{if } \beta \equiv 7 \ (mod\ 8) \\ w & \text{if } \beta \equiv 0 \ (mod\ 8) \end{cases}$$

$$\alpha(z''_\beta) = \begin{cases} e & \text{if } \beta \equiv 1 \ (mod\ 8) \\ l & \text{if } \beta \equiv 2 \ (mod\ 8) \\ n & \text{if } \beta \equiv 3 \ (mod\ 8) \\ p & \text{if } \beta \equiv 4 \ (mod\ 8) \\ r & \text{if } \beta \equiv 5 \ (mod\ 8) \\ t & \text{if } \beta \equiv 6 \ (mod\ 8) \\ v & \text{if } \beta \equiv 7 \ (mod\ 8) \\ x & \text{if } \beta \equiv 0 \ (mod\ 8) \end{cases}$$

We observe that $e_\alpha(0) = 2n-1$ and $e_\alpha(1) = 2n$. Therefore G is group S_4 difference cordial graph. □

Theorem 3.6. *Double alternate quadrilatral snake is group S_4 difference cordial graph.*

Proof. Let $z_1, z_2, \cdots, z_n$ be the vertices of P_n then $V(DAQ_n) = V(P_n) \bigcup \{y'_\beta, y''_\beta, z'_\beta, z''_\beta | 1 \leq \beta \leq \lfloor \frac{n}{2} \rfloor\}$ and $E(DAQ_n) = E(P_n) \bigcup \{z_{2\beta-1}y'_\beta, z_{2\beta-1}z'_\beta, z_{2\beta}y''_\beta, z_{2\beta}z''_\beta, y'_\beta y''_\beta, z'_\beta z''_\beta : 1 \leq \beta \leq \lfloor \frac{n}{2} \rfloor\}$.
Define α as follows:

$$\alpha(z_\beta) = \begin{cases} k & \text{if } \beta \equiv 1 \ (mod\ 8) \\ c & \text{if } \beta \equiv 2 \ (mod\ 8) \\ e & \text{if } \beta \equiv 3 \ (mod\ 8) \\ u & \text{if } \beta \equiv 4 \ (mod\ 8) \\ x & \text{if } \beta \equiv 5 \ (mod\ 8) \\ i & \text{if } \beta \equiv 6 \ (mod\ 8) \\ p & \text{if } \beta \equiv 7 \ (mod\ 8) \\ j & \text{if } \beta \equiv 0 \ (mod\ 8) \end{cases}$$

$$\alpha(y'_\beta) = \begin{cases} a & \text{if } \beta \equiv 1 \ (mod\ 4) \\ s & \text{if } \beta \equiv 2 \ (mod\ 4) \\ v & \text{if } \beta \equiv 3 \ (mod\ 4) \\ n & \text{if } \beta \equiv 0 \ (mod\ 4) \end{cases}$$

$$\alpha(y''_\beta) = \begin{cases} b & \text{if } \beta \equiv 1 \ (mod\ 4) \\ t & \text{if } \beta \equiv 2 \ (mod\ 4) \\ w & \text{if } \beta \equiv 3 \ (mod\ 4) \\ o & \text{if } \beta \equiv 0 \ (mod\ 4) \end{cases}$$

$$\alpha(z'_\beta) = \begin{cases} l & \text{if } \beta \equiv 1 \ (mod\ 4) \\ d & \text{if } \beta \equiv 2 \ (mod\ 4) \\ g & \text{if } \beta \equiv 3 \ (mod\ 4) \\ q & \text{if } \beta \equiv 0 \ (mod\ 4) \end{cases}$$

and

$$\alpha(z''_\beta) = \begin{cases} m & \text{if } \beta \equiv 1 \ (mod\ 4) \\ f & \text{if } \beta \equiv 2 \ (mod\ 4) \\ h & \text{if } \beta \equiv 3 \ (mod\ 4) \\ r & \text{if } \beta \equiv 0 \ (mod\ 4) \end{cases}$$

We observe that

$$e_\alpha(0) = \begin{cases} 2n-2 & \text{if } n \text{ is odd} \\ 2n \text{ or } 2n-1 & \text{if } n \text{ is even} \end{cases}$$

$$e_\alpha(1) = \begin{cases} 2n-2 & \text{if } n \text{ is odd} \\ 2n \text{ or } 2n-1 & \text{if } n \text{ is even} \end{cases}$$

Therefore α is group S_4 difference cordial labeling.

Theorem 3.7. *Quadrilatral snake is group S_4 difference cordial graph.*

Proof. Let the vertex set of $Q_n = \{z_\beta | 1 \leq \beta \leq n\} \bigcup \{z_\beta^{'}, z_\beta^{''} | 1 \leq \beta \leq n-1\}$. Let edge set of $Q_n = \{z_\beta z_{\beta+1}, z_\beta z_\beta^{'}, z_{\beta+1} z_\beta^{''}, z_\beta^{'} z_\beta^{''} : 1 \leq \beta \leq n-1\}$.
Define α as follows:
For $1 \leq \beta \leq n$,

$$\alpha(z_\beta) = \begin{cases} a & \text{if } \beta \equiv 1 \ (mod\ 8) \\ c & \text{if } \beta \equiv 2 \ (mod\ 8) \\ m & \text{if } \beta \equiv 3 \ (mod\ 8) \\ d & \text{if } \beta \equiv 4 \ (mod\ 8) \\ h & \text{if } \beta \equiv 5 \ (mod\ 8) \\ o & \text{if } \beta \equiv 6 \ (mod\ 8) \\ t & \text{if } \beta \equiv 7 \ (mod\ 8) \\ v & \text{if } \beta \equiv 0 \ (mod\ 8) \end{cases}$$

For $1 \leq \beta \leq n-1$,

$$\alpha(z_\beta^{'}) = \begin{cases} b & \text{if } \beta \equiv 1 \ (mod\ 8) \\ k & \text{if } \beta \equiv 2 \ (mod\ 8) \\ n & \text{if } \beta \equiv 3 \ (mod\ 8) \\ g & \text{if } \beta \equiv 4 \ (mod\ 8) \\ i & \text{if } \beta \equiv 5 \ (mod\ 8) \\ p & \text{if } \beta \equiv 6 \ (mod\ 8) \\ u & \text{if } \beta \equiv 7 \ (mod\ 8) \\ w & \text{if } \beta \equiv 0 \ (mod\ 8) \end{cases}$$

and

$$\alpha(z_\beta^{''}) = \begin{cases} e & \text{if } \beta \equiv 1 \ (mod\ 8) \\ l & \text{if } \beta \equiv 2 \ (mod\ 8) \\ f & \text{if } \beta \equiv 3 \ (mod\ 8) \\ s & \text{if } \beta \equiv 4 \ (mod\ 8) \\ j & \text{if } \beta \equiv 5 \ (mod\ 8) \\ q & \text{if } \beta \equiv 6 \ (mod\ 8) \\ r & \text{if } \beta \equiv 7 \ (mod\ 8) \\ x & \text{if } \beta \equiv 0 \ (mod\ 8) \end{cases}$$

We observe that

$$e_\alpha(0) = 2n - 2 = e_\alpha(1)$$

Hence Q_n is group S_4 difference cordial graph. □

Theorem 3.8. *Alternate quadrilatral snake is group S_4 difference cordial graph.*

Proof. Let $V(AQ_n) = \{z_\beta | 1 \leq \beta \leq n\} \bigcup \{z_\beta^{'}, z_\beta^{''} | 1 \leq \beta \leq \lfloor \frac{n}{2} \rfloor\}$. Let $E(AQ_n) = \{z_\beta z_{\beta+1} | 1 \leq \beta \leq n-1\} \bigcup \{z_{2\beta-1} z_\beta^{'}, z_{2\beta} z_\beta^{''}, z_\beta^{'} z_\beta^{''} : 1 \leq \beta \leq \lfloor \frac{n}{2} \rfloor\}$.
Define α as follows:
For $1 \leq \beta \leq n$,

$$\alpha(z_\beta) = \begin{cases} c & \text{if } \beta \equiv 1 \ (mod\ 12) \\ e & \text{if } \beta \equiv 2 \ (mod\ 12) \\ m & \text{if } \beta \equiv 3 \ (mod\ 12) \\ s & \text{if } \beta \equiv 4 \ (mod\ 12) \\ v & \text{if } \beta \equiv 5 \ (mod\ 12) \\ n & \text{if } \beta \equiv 6 \ (mod\ 12) \\ w & \text{if } \beta \equiv 7 \ (mod\ 12) \\ q & \text{if } \beta \equiv 8 \ (mod\ 12) \\ r & \text{if } \beta \equiv 9 \ (mod\ 12) \\ g & \text{if } \beta \equiv 10 \ (mod\ 12) \\ x & \text{if } \beta \equiv 11 \ (mod\ 12) \\ j & \text{if } \beta \equiv 0 \ (mod\ 12) \end{cases}$$

$$\alpha(z_\beta^{'}) = \begin{cases} a & \text{if } \beta \equiv 1 \ (mod\ 6) \\ k & \text{if } \beta \equiv 2 \ (mod\ 6) \\ t & \text{if } \beta \equiv 3 \ (mod\ 6) \\ o & \text{if } \beta \equiv 4 \ (mod\ 6) \\ d & \text{if } \beta \equiv 5 \ (mod\ 6) \\ h & \text{if } \beta \equiv 0 \ (mod\ 6) \end{cases}$$

$$\alpha(z_\beta^{''}) = \begin{cases} b & \text{if } \beta \equiv 1 \ (mod\ 6) \\ l & \text{if } \beta \equiv 2 \ (mod\ 6) \\ u & \text{if } \beta \equiv 3 \ (mod\ 6) \\ p & \text{if } \beta \equiv 4 \ (mod\ 6) \\ f & \text{if } \beta \equiv 5 \ (mod\ 6) \\ i & \text{if } \beta \equiv 0 \ (mod\ 6) \end{cases}$$

It is clear that $|v_\alpha(\lambda) - v_\alpha(\mu)| \leq 1$ and $|e_\alpha(0) - e_\alpha(1)| \leq 1$. Hence α is group S_4 difference cordial labeling. □

Theorem 3.9. *Alternate triangular snake is group S_4 difference cordial graph.*

Proof. Let $z_1, z_2, \cdots, z_n$ be the vertices of P_n. Then $V(AT_n) = V(P_n) \bigcup \{y_\beta | 1 \leq \beta \leq \lfloor \frac{n}{2} \rfloor\}$ and $E(AT_n) = E(P_n) \bigcup \{z_{2\beta-1} y_\beta, z_{2\beta} y_\beta : 1 \leq \beta \leq \lfloor \frac{n}{2} \rfloor\}$.
Define α as follows:
For $1 \leq \beta \leq n$,

$$\alpha(z_\beta) = \begin{cases} a & \text{if } \beta \equiv 1\ (mod\ 16) \\ b & \text{if } \beta \equiv 2\ (mod\ 16) \\ c & \text{if } \beta \equiv 3\ (mod\ 16) \\ l & \text{if } \beta \equiv 4\ (mod\ 16) \\ m & \text{if } \beta \equiv 5\ (mod\ 16) \\ t & \text{if } \beta \equiv 6\ (mod\ 16) \\ u & \text{if } \beta \equiv 7\ (mod\ 16) \\ f & \text{if } \beta \equiv 8\ (mod\ 16) \\ g & \text{if } \beta \equiv 9\ (mod\ 16) \\ w & \text{if } \beta \equiv 10\ (mod\ 16) \\ x & \text{if } \beta \equiv 11\ (mod\ 16) \\ o & \text{if } \beta \equiv 12\ (mod\ 16) \\ p & \text{if } \beta \equiv 13\ (mod\ 16) \\ q & \text{if } \beta \equiv 14\ (mod\ 16) \\ r & \text{if } \beta \equiv 15\ (mod\ 16) \\ j & \text{if } \beta \equiv 0\ (mod\ 16) \end{cases}$$

For $1 \leq \beta \leq \lfloor \frac{n}{2} \rfloor$,

$$\alpha(y_\beta) = \begin{cases} e & \text{if } \beta \equiv 1\ (mod\ 8) \\ k & \text{if } \beta \equiv 2\ (mod\ 8) \\ s & \text{if } \beta \equiv 3\ (mod\ 8) \\ d & \text{if } \beta \equiv 4\ (mod\ 8) \\ v & \text{if } \beta \equiv 5\ (mod\ 8) \\ n & \text{if } \beta \equiv 6\ (mod\ 8) \\ h & \text{if } \beta \equiv 7\ (mod\ 8) \\ i & \text{if } \beta \equiv 0\ (mod\ 8) \end{cases}$$

It is clear that
$e_\alpha(0) = n - 1$ and

$$e_\alpha(1) = \begin{cases} n-1 & \text{if } n \text{ is odd} \\ n & \text{if } n \text{ is even} \end{cases}$$

Therefore triangular snake is group S_4 difference cordial labeling.

Theorem 3.10. *Double alternate triangular snake is group S_4 difference cordial graph.*

Proof. Let $z_1, z_2, \cdots, z_n$ be the vertices of P_n. Then $V(DA(T_n)) = V(P_n) \bigcup \{z'_\beta, z''_\beta : 1 \leq \beta \leq \lfloor \frac{n}{2} \rfloor\}$ and $E(DA(T_n)) = E(P_n) \bigcup \{z_{2\beta-1}z'_\beta, z_{2\beta-1}z''_\beta, z_{2\beta}z'_\beta, z_{2\beta}z''_\beta : 1 \leq \beta \leq \lfloor \frac{n}{2} \rfloor\}$.
Define α as follows:

For $1 \leq \beta \leq n$,

$$\alpha(z_\beta) = \begin{cases} a & \text{if } \beta \equiv 1 \ (mod\ 12) \\ b & \text{if } \beta \equiv 2 \ (mod\ 12) \\ l & \text{if } \beta \equiv 3 \ (mod\ 12) \\ m & \text{if } \beta \equiv 4 \ (mod\ 12) \\ f & \text{if } \beta \equiv 5 \ (mod\ 12) \\ g & \text{if } \beta \equiv 6 \ (mod\ 12) \\ t & \text{if } \beta \equiv 7 \ (mod\ 12) \\ u & \text{if } \beta \equiv 8 \ (mod\ 12) \\ p & \text{if } \beta \equiv 9 \ (mod\ 12) \\ q & \text{if } \beta \equiv 10 \ (mod\ 12) \\ w & \text{if } \beta \equiv 11 \ (mod\ 12) \\ x & \text{if } \beta \equiv 0 \ (mod\ 12) \end{cases}$$

For $1 \leq \beta \leq \left\lfloor \frac{n}{2} \right\rfloor$,

$$\alpha(z'_\beta) = \begin{cases} e & \text{if } \beta \equiv 1 \ (mod\ 6) \\ k & \text{if } \beta \equiv 2 \ (mod\ 6) \\ h & \text{if } \beta \equiv 3 \ (mod\ 6) \\ s & \text{if } \beta \equiv 4 \ (mod\ 6) \\ o & \text{if } \beta \equiv 5 \ (mod\ 6) \\ v & \text{if } \beta \equiv 0 \ (mod\ 6) \end{cases}$$

and

$$\alpha(z''_\beta) = \begin{cases} c & \text{if } \beta \equiv 1 \ (mod\ 6) \\ d & \text{if } \beta \equiv 2 \ (mod\ 6) \\ n & \text{if } \beta \equiv 3 \ (mod\ 6) \\ i & \text{if } \beta \equiv 4 \ (mod\ 6) \\ j & \text{if } \beta \equiv 5 \ (mod\ 6) \\ r & \text{if } \beta \equiv 0 \ (mod\ 6) \end{cases}$$

It is clear that

$$e_\alpha(0) = \begin{cases} n + \left\lfloor \frac{n}{2} \right\rfloor - 1 & \text{if } n \text{ is odd} \\ n + \frac{n}{2} & \text{if } n \text{ is even} \end{cases}$$

$$e_\alpha(1) = \begin{cases} n + \left\lfloor \frac{n}{2} \right\rfloor - 1 & \text{if } n \text{ is odd} \\ n + \frac{n}{2} - 1 & \text{if } n \text{ is even} \end{cases}$$

Therefore double alternate triangular snake is group S_4 difference cordial labeling.

□

Theorem 3.11. *Double triangular snake is group S_4 difference cordial graph.*

Proof. Let $z_1, z_2, \cdots, z_n$ be the vertices of P_n. Then $V(D(T_n)) = V(P_n) \bigcup \{z_\beta^{'}, z_\beta^{''} : 1 \leq \beta \leq n-1\}$ and $E(D(T_n)) = E(P_n) \bigcup \{z_\beta z_\beta^{'}, z_{\beta+1} z_\beta^{'}, z_\beta z_\beta^{''}, z_{\beta+1} z_\beta^{''} : 1 \leq \beta \leq n-1\}$.
Define α as follows:
For $1 \leq \beta \leq n$,

$$\alpha(z_\beta) = \begin{cases} k & \text{if } \beta \equiv 1 \ (mod\ 8) \\ l & \text{if } \beta \equiv 2 \ (mod\ 8) \\ a & \text{if } \beta \equiv 3 \ (mod\ 8) \\ b & \text{if } \beta \equiv 4 \ (mod\ 8) \\ u & \text{if } \beta \equiv 5 \ (mod\ 8) \\ v & \text{if } \beta \equiv 6 \ (mod\ 8) \\ g & \text{if } \beta \equiv 7 \ (mod\ 8) \\ h & \text{if } \beta \equiv 0 \ (mod\ 8) \end{cases}$$

For $1 \leq \beta \leq n-1$,

$$\alpha(z_\beta^{'}) = \begin{cases} s & \text{if } \beta \equiv 1 \ (mod\ 8) \\ n & \text{if } \beta \equiv 2 \ (mod\ 8) \\ p & \text{if } \beta \equiv 3 \ (mod\ 8) \\ t & \text{if } \beta \equiv 4 \ (mod\ 8) \\ e & \text{if } \beta \equiv 5 \ (mod\ 8) \\ f & \text{if } \beta \equiv 6 \ (mod\ 8) \\ q & \text{if } \beta \equiv 7 \ (mod\ 8) \\ r & \text{if } \beta \equiv 0 \ (mod\ 8) \end{cases}$$

and

$$\alpha(z_\beta^{''}) = \begin{cases} m & \text{if } \beta \equiv 1 \ (mod\ 8) \\ o & \text{if } \beta \equiv 2 \ (mod\ 8) \\ c & \text{if } \beta \equiv 3 \ (mod\ 8) \\ d & \text{if } \beta \equiv 4 \ (mod\ 8) \\ w & \text{if } \beta \equiv 5 \ (mod\ 8) \\ x & \text{if } \beta \equiv 6 \ (mod\ 8) \\ i & \text{if } \beta \equiv 7 \ (mod\ 8) \\ j & \text{if } \beta \equiv 0 \ (mod\ 8) \end{cases}$$

It is clear that

$$e_\alpha(0) = \begin{cases} 2n + \frac{n+1}{2} & \text{if } n \text{ is odd} \\ 2n + \frac{n}{2} & \text{if } n \text{ is even} \end{cases}$$

$$e_\alpha(1) = \begin{cases} 2n + \frac{n+1}{2} - 1 & \text{if } n \text{ is odd} \\ 2n + \frac{n}{2} & \text{if } n \text{ is even} \end{cases}$$

Therefore α is group S_4 difference cordial labeling.

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