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MODIFIED EVEN ORDERED CANTOR SETS

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Abstract: In Cantor ternary sets middle third is removed and the pattern of removal continues indefinitely. Taking the number of divisions as order here, even ordered Cantor sets are considered. Unlike Cantor sets here lengths of unequal intervals are removed. In this pattern of removal middle interval in successive iteration follows a geometric sequence of powers of two. The intervals equally spaced from the middle to the left and right follows different nature as the iteration increases. Its characteristics are studied in this paper. Also, the diagrammatic representation of modified even ordered Cantor sets has been exhibited.

2010 AMS Classification: 26A30

Keywords: Cantor set

1. Introduction

The Cantor ternary set is a set of rational numbers in the closed interval [0, 1] obtained by dividing the interval into 3 parts successfully after removing the middle third. There are many publications describing various properties of Cantor middle sets. The Cantor middle sets are considered only for odd integers. The analysis is done only for C_{2m-1} middle sets $(2 \leq m < \infty)$. Cantor sets for even integers are not so far studied in detail.

Unlike Cantor ternary sets, in even ordered sets the intervals of lengths one and two are removed successively. Again, if intervals of lengths two are taken away the formulas for retaining terms are given. In this paper Cantor sets of even numbers are considered. Contrary to the procedure followed by Cantor, intervals of various lengths are removed in different patterns. These various patterns of removal of intervals are analyzed here.

2. Preliminaries

Throughout this paper we study the modified Cantor even ordered sets.

Definition 1: Cantor Hexnary Set

Divide the closed interval $[0,1]$ into six equal intervals. Remove the second and last but one of the six intervals of length $\left(\frac{1}{6}\right)$**.** The middle interval of length $\left(\frac{2}{6}\right)$ is only retained. Now for the first, last and middle intervals continue the procedure indefinitely. The set obtained is known as **Cantor Hexnary Set.**

Definition 2: Cantor Octanary Set

The closed interval [0, 1] is divided into eight equal parts. By removing the second part, last but one part and middlemost part, the open intervals $\left(\frac{1}{8},\frac{2}{8}\right)$, $\left(\frac{3}{8},\frac{5}{8}\right)$ and $\left(\frac{6}{8},\frac{7}{8}\right)$ are removed. The middlemost removable interval is of length $\left(\frac{2}{8}\right)$. Each retained intervals are of length $\left(\frac{1}{8}\right)$.

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Continue the process indefinitely and the set obtained is known as the **Cantor Octanary Set.**

3. Modified Cantor $\left(\frac{1}{6}\right)$ Sets

Theorem 3.1:

In C_6 , if the middle interval of length $^2/_6$ is retained and subdivided successfully as in Cantor then in the successive iterations the middlemost interval is retained that follow a series of the form $^2/_6, (^2/_6)^2, (^2/_6)^3, \ldots.$ for all these intervals. The general term of the middlemost interval is given by $\left[\frac{k}{6^n}, \frac{k+2^n}{6^n}\right]$ were k can be represented by the series $2(6)^{n-1}(2)^0 + 2(6)^{n-2}(2)^1 + 2(6)^{n-3}(2)^2 + \cdots + 2(6)^0(2)^{n-1}$

Proof:

The closed interval $[0,1]$ is divided into six equal parts. Following the theory of Cantor set, the open intervals $\left(\frac{1}{6}, \frac{2}{6}\right)$ and $\left(\frac{4}{6}, \frac{5}{6}\right)$ are removed. In first iteration the number of parts removed is $2*3^0$.The remaining parts $\left[\frac{0}{6} = 0, \frac{1}{6}\right]$, $\left[\frac{2}{6}, \frac{4}{6}\right]$ and $\left[\frac{5}{6}, \frac{6}{6} = 1\right]$ are again subdivided as follows. The length of the middlemost part is 2/6. For the 2nd iteration, the parts $\left[\frac{0}{6} = 0, \frac{1}{6}\right]$ and $\left[\frac{5}{6}, \frac{6}{6}\right]$ are each divided into six equal parts thereby giving six parts $\left[\frac{0}{36} = 0, \frac{1}{36}\right]$, $\left[\frac{1}{36}, \frac{2}{36}\right]$, $\left[\frac{2}{36}, \frac{3}{36}\right]$, $\left[\frac{3}{36}, \frac{4}{36}\right]$, $\left[\frac{4}{36}, \frac{5}{36}\right]$, $\left[\frac{5}{36}, \frac{6}{36}\right]$. The open intervals $\left(\frac{1}{36}, \frac{2}{36}\right)$ and $\left(\frac{4}{36}, \frac{5}{36}\right)$ are removed. Applying the removal pattern of the middle part $\left[\mathbf{\frac{2}{6}}, \mathbf{\frac{4}{6}}\right]$ again give rise to $\left[\frac{12}{36}, \frac{14}{36}\right]$, $\left[\frac{14}{36}, \frac{16}{36}\right]$, $\left[\mathbf{\frac{16}{36}}, \mathbf{\frac{20}{36}}\right]$, $\left[\frac{20}{36}, \frac{22}{36}\right]$, $\left[\frac{22}{36}, \frac{24}{36}\right]$. The open intervals $\left(\frac{14}{36}, \frac{16}{36}\right)$ and $\left(\frac{20}{36}, \frac{22}{36}\right)$ are removed. The length of the middlemost part is 4/36. In second iteration the number of parts removed is $2*3^1$. The third iteration the middle part $\left[\mathbf{\frac{16}{36}}, \mathbf{\frac{20}{36}}\right]$ is again subdivided into six equal parts are $\left[\frac{96}{216}, \frac{100}{216}\right]$, $\left[\frac{100}{216}, \frac{104}{216}\right]$, $\left[\frac{104}{216}, \frac{108}{216}\right]$, $\left[\frac{108}{216}, \frac{112}{216}\right]$, $\left[\frac{112}{216}, \frac{116}{216}\right]$, $\left[\frac{116}{216}, \frac{120}{36}\right]$. The intervals $\left(\frac{100}{216}, \frac{104}{216}\right)$ and $\left(\frac{112}{36}, \frac{116}{36}\right)$ are removed. The retained intervals are $\left[\frac{96}{216}, \frac{100}{216}\right]$, $\left[\frac{100}{216}, \frac{104}{216}\right]$, $\left[\mathbf{\frac{104}{216}}, \mathbf{\frac{112}{216}}\right]$, $\left[\frac{112}{216}, \frac{116}{216}\right]$, $\left[\frac{116}{216}, \frac{120}{36}\right]$. The length of the middlemost part is 8/216. In third iteration the number of parts removed is $2*3^2$.

Here it is noted that in the successive iterations every interval which are equally spaced from the middle interval are subdivided into six equal parts whose length $\frac{1}{6^1}, \frac{1}{6^2}, \frac{1}{6^3}, \frac{1}{6^4}, \cdots\cdots$. The middlemost part when subdivided into six equal parts are of length is $\frac{2}{6^1}, \frac{2^2}{6^2}, \frac{2^3}{6^3}, \frac{2^4}{6^4}, \cdots\cdots$. In each iteration the number of parts removed are $(2*3^0)$, $(2*3^1)$, $(2*3^2)$, $(2*3^3)$ … $(2*3^n)$ successively.

Therefore in C_6 , if the middle interval of length $^2/_6$ is retained and subdivided successfully as in Cantor then in successive iterations the middlemost intervals retained follow a series of the form $^2/_6, (^2/_6)^2, (^2/_6)^3, \ldots$ for all these intervals. The general representation of the middlemost term at the nth iteration is given by $\left[\frac{k}{6^n}, \frac{k+2^n}{6^n}\right]$ where

k = $2(6)^{n-1}(2)^0 + 2(6)^{n-2}(2)^1 + 2(6)^{n-3}(2)^2 + \cdots + 2(6)^0(2)^{n-1}$

The Following **Figure 1** shows the graphical representation of modified Cantor $\left(\frac{1}{6}\right)$ sets and **Figure 3** shows the tree representation of middlemost part of modified Cantor $\left(\frac{1}{6}\right)$ sets.

First iteration:

The closed interval $[0,1]$ is subdivided into 6 equal sub- intervals

$$\left[\frac{0}{6}=0,\frac{1}{6}\right],\left[\frac{1}{6},\frac{2}{6}\right],\left[\frac{2}{6},\frac{3}{6}\right],\left[\frac{3}{6},\frac{4}{6}\right],\left[\frac{4}{6},\frac{5}{6}\right],\left[\frac{5}{6},\frac{6}{6}=1\right]$$

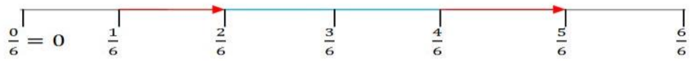

Figure 1: Modified Cantor $\left(\frac{1}{6}\right)$ sets

The Removed intervals are $\left(\frac{1}{6},\frac{2}{6}\right),\left(\frac{4}{6},\frac{5}{6}\right)$.

The remaining intervals are $\left[\frac{0}{6}=0,\frac{1}{6}\right]\cup\left[\frac{2}{6},\frac{4}{6}\right]\cup\left[\frac{5}{6},\frac{6}{6}=1\right]$

$$\therefore\ C_{6^1}=\left\{\frac{0}{6}=0,\ \frac{1}{6},\frac{2}{6},\ \frac{4}{6},\ \frac{5}{6},\frac{6}{6}=1\ \right\} \tag{3.1}$$

Second iteration:

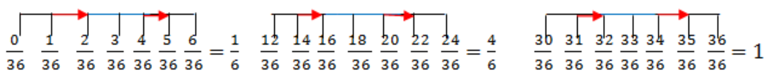

Figure 2: Second iteration - Modified Cantor $\left(\frac{1}{6}\right)$ sets

The Removable intervals are

$\left(\frac{1}{36},\frac{2}{36}\right),\left(\frac{5}{36},\frac{6}{36}\right),\left(\frac{14}{36},\frac{16}{36}\right),\left(\frac{20}{36},\frac{22}{36}\right),\left(\frac{31}{36},\frac{32}{36}\right),\left(\frac{34}{36},\frac{35}{36}\right)$ from each of the subintervals will result in modified cantor set.

$$\therefore\ C_{6^2}=\left\{\frac{0}{36},\frac{1}{36},\frac{2}{36},\frac{4}{36},\frac{5}{36},\frac{6}{36},\frac{12}{36},\frac{14}{36},\frac{16}{36},\frac{20}{36},\frac{22}{36},\frac{24}{36},\frac{30}{36},\frac{31}{36},\frac{32}{36},\frac{34}{36},\frac{35}{36},\frac{36}{36}\right\} \tag{3.2}$$

This procedure proceeds in every iteration to get the entire modified cantor set.

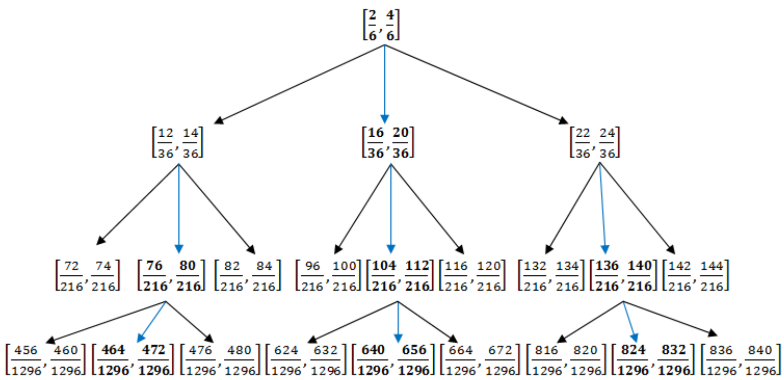

Figure 3: Middlemost part of modified Cantor $\left(\frac{1}{6}\right)$ sets

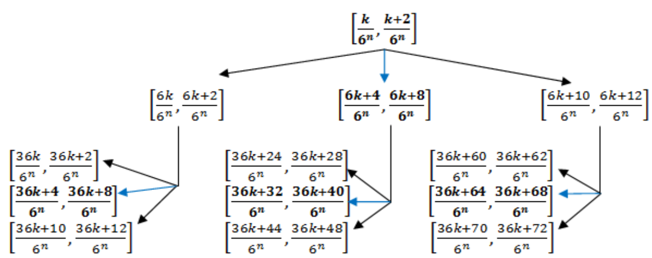

Figure 4: General form of middlemost part of modified Cantor $\left(\frac{1}{6}\right)$ sets

3.1 MODIFIED CANTOR $\left(\frac{1}{10}\right)$ SETS:

The above (Theorem 3.1) same pattern followed by modified Cantor $\left(\frac{1}{10}\right)$ sets.

First iteration:

The closed interval $[0,1]$ is subdivided into 10 equal sub- intervals $\left[\frac{0}{10}=0,\frac{1}{10}\right],\left[\frac{1}{10},\frac{2}{10}\right],\left[\frac{2}{10},\frac{3}{10}\right],\left[\frac{3}{10},\frac{4}{10}\right],\left[\frac{4}{10},\frac{5}{10}\right],\left[\frac{5}{10},\frac{6}{10}\right],\left[\frac{7}{10},\frac{8}{10}\right],\left[\frac{9}{10},\frac{10}{10}\right]$

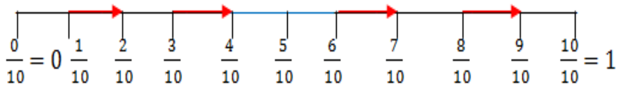

Figure 5: Modified Cantor $\left(\frac{1}{10}\right)$ sets

The Removed intervals are $\left(\frac{1}{10},\frac{2}{10}\right),\left(\frac{3}{10},\frac{4}{10}\right),\left(\frac{6}{10},\frac{7}{10}\right),\left(\frac{8}{10},\frac{9}{10}\right)$.

The remaining intervals are $\left[\frac{0}{10}=0,\frac{1}{10}\right]\cup\left[\frac{2}{10},\frac{3}{10}\right]\cup\left[\frac{\mathbf{4}}{\mathbf{10}},\frac{\mathbf{6}}{\mathbf{10}}\right]\cup$ $\left[\frac{7}{10},\frac{8}{10}\right],\left[\frac{9}{10},\frac{10}{10}=1\right]$

$$\therefore\ C_{10^1}=\left\{\frac{0}{10}=0,\ \frac{1}{10},\frac{2}{10},\frac{3}{10},\frac{4}{10},\frac{6}{10}\frac{7}{10},\frac{8}{10},\frac{9}{10},\frac{10}{10}=1\right\} \qquad (3.3)$$

This procedure proceeds in every iteration to get the entire modified cantor set.

4. Modified Cantor $\left(\frac{1}{8}\right)$ Sets

Theorem 4.1:

In C_8 , the intervals of lengths $\frac{1}{8^n},\frac{2}{8^n},\frac{1}{8^n}$ are successively removed and only $4*4^{n-1}$ intervals of length each $1/8$ is retained. For each interval $\left[\frac{k}{6^n},\frac{k+1}{6^n}\right]=\left(\frac{1}{(2s)^n}[(k-1)(2s)^{n-1}+(8)^{n-1}]\right),\left(\frac{1}{(2s)^n}[(k-1)(2s)^{n-1}+(8)^{n-1}+1]\right)$.

Proof:

The closed interval $[0,1]$ is divided into eight equal parts. The open interval $\left(\frac{1}{8},\frac{2}{8}\right)$, $\left(\frac{3}{8},\frac{5}{8}\right)$ and $\left(\frac{6}{8},\frac{7}{8}\right)$ are removed. In first iteration the number of parts removed is $4*4^0$.The length of the first iteration is 1/8. The remaining parts $\left[\frac{0}{8}=0,\frac{1}{8}\right],\left[\frac{2}{8},\frac{3}{8}\right],\left[\frac{5}{8},\frac{6}{8}\right]$ and $\left[\frac{7}{8},\frac{8}{8}=1\right]$ are again subdivided as follows for the 2nd iteration. The part $\left[\frac{0}{8}=0,\frac{1}{8}\right]$ are subdivided into eight equal parts thereby giving eight parts$\left[\frac{0}{64}=0,\frac{1}{64}\right],\left[\frac{1}{64},\frac{2}{64}\right],\left[\frac{2}{64},\frac{3}{64}\right]\left[\frac{3}{64},\frac{4}{64}\right],\left[\frac{4}{64},\frac{5}{64}\right],\left[\frac{5}{64},\frac{6}{64}\right],\left[\frac{6}{64},\frac{7}{64}\right]\left[\frac{7}{64},\frac{8}{64}\right]$, the open intervals $\left(\frac{1}{64},\frac{2}{64}\right),\left(\frac{3}{64},\frac{5}{64}\right)$ and$\left(\frac{6}{64},\frac{7}{64}\right)$ are removed. Now the part $\left[\frac{2}{8},\frac{3}{8}\right]$ are subdivided into eight equal parts thereby giving eight parts $\left[\frac{16}{64},\frac{17}{64}\right],\left[\frac{17}{64},\frac{18}{64}\right],\left[\frac{18}{64},\frac{19}{64}\right],\left[\frac{19}{64},\frac{20}{64}\right],\left[\frac{20}{64},\frac{21}{64}\right],\left[\frac{21}{64},\frac{22}{64}\right],\left[\frac{22}{64},\frac{23}{64}\right],\left[\frac{23}{64},\frac{24}{64}\right]$, the open intervals $\left(\frac{17}{64},\frac{18}{64}\right),\left(\frac{19}{64},\frac{21}{64}\right)$ and$\left(\frac{22}{64},\frac{23}{64}\right)$ are removed. Next parts$\left[\frac{5}{8},\frac{6}{8}\right]$ are divided into eight equal parts namely $\left[\frac{40}{64},\frac{41}{64}\right],\left[\frac{41}{64},\frac{42}{64}\right],\left[\frac{42}{64},\frac{43}{64}\right],\left[\frac{43}{64},\frac{44}{64}\right],\left[\frac{44}{64},\frac{45}{64}\right],\left[\frac{45}{64},\frac{46}{64}\right],\left[\frac{46}{64},\frac{47}{64}\right],\left[\frac{47}{64},\frac{48}{64}\right]$, the open intervals $\left(\frac{41}{64},\frac{42}{64}\right),\left(\frac{43}{64},\frac{45}{64}\right)$ and$\left(\frac{46}{64},\frac{47}{64}\right)$ are removed. Last part $\left[\frac{7}{8},\frac{8}{8}=1\right]$ is again subdivided into eight equal parts namely $\left[\frac{56}{64},\frac{57}{64}\right],\left[\frac{57}{64},\frac{58}{64}\right],\left[\frac{58}{64},\frac{59}{64}\right],\left[\frac{59}{64},\frac{60}{64}\right],\left[\frac{60}{64},\frac{61}{64}\right],\left[\frac{61}{64},\frac{62}{64}\right],\left[\frac{62}{64},\frac{63}{64}\right],\left[\frac{63}{64},\frac{64}{64}\right]$, the open intervals $\left(\frac{57}{64},\frac{58}{64}\right),\left(\frac{59}{64},\frac{61}{64}\right)$ and$\left(\frac{62}{64},\frac{63}{64}\right)$ are removed.

Here it is noted that in the successive iterations every interval which are equally spaced. Left out parts for partitioned into eight equal parts whose length $\frac{1}{8^1}, \frac{1}{8^2}, \frac{1}{8^3}, \frac{1}{8^4}, \cdots\cdots$. In every cycle the number of parts eliminated $(4*4^0)$, $(4*4^1)$, $(4*4^2)$, $(4*4^3)$ … The n[th] iteration is $(4*4^n)$.

If any part is of the form $\left[\frac{K}{8^n}, \frac{K+1}{8^n}\right]$. When the n[th] iteration the end parts of the iteration are of the form $\left(\frac{1}{(2s)^n}\left[(k-1)(2s)^{n-1} + 8^{n-1}\right]\right), \left(\frac{1}{(2s)^n}\left[(k-1)(2s)^{n-1} + 8^{n-1}\right]\right) + 1$.

The Following **Figure 6** shows the graphical representation of modified Cantor $\left(\frac{1}{8}\right)$ sets and **Figure 8** shows the tree representation of interval of length is $1/8$ retained part of modified Cantor $\left(\frac{1}{8}\right)$ sets.

First iteration:

The closed interval $[0,1]$ is subdivided into 8 equal sub- intervals

$$\left[\frac{0}{8}=0, \frac{1}{8}\right], \left[\frac{1}{8}, \frac{2}{8}\right], \left[\frac{2}{8}, \frac{3}{8}\right], \left[\frac{3}{8}, \frac{4}{8}\right], \left[\frac{4}{8}, \frac{5}{8}\right], \left[\frac{5}{8}, \frac{6}{8}\right], \left[\frac{6}{8}, \frac{7}{8}\right], \left[\frac{7}{8}, \frac{8}{8}=1\right]$$

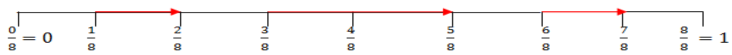

Figure 6: Modified Cantor $\left(\frac{1}{8}\right)$ sets

The Removed intervals are $\left(\frac{1}{8}, \frac{2}{8}\right), \left(\frac{3}{8}, \frac{5}{8}\right), \left(\frac{6}{8}, \frac{7}{8}\right)$.

The remaining intervals are $\left[\frac{0}{8}=0, \frac{1}{8}\right], \left[\frac{2}{8}, \frac{3}{8}\right], \left[\frac{5}{8}, \frac{6}{8}\right]$ and $\left[\frac{7}{8}, \frac{8}{8}=1\right]$

$$\therefore\ C_{8^1} = \left\{\frac{0}{8}=0,\ \frac{1}{8}, \frac{2}{8},\ \frac{3}{8},\ \frac{5}{8}, \frac{6}{8}, \frac{7}{8}, \frac{8}{8}=1\right\} \qquad (4.1)$$

Second iteration:

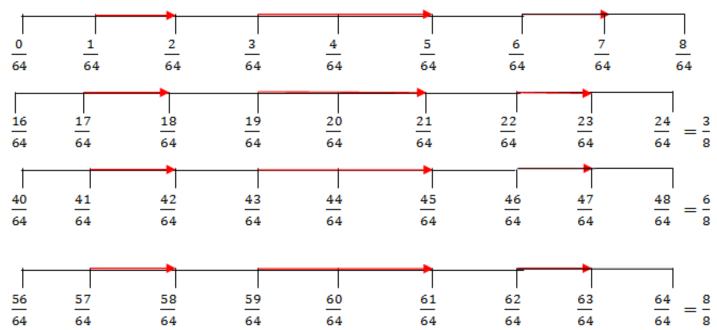

Figure 7: Second iteration - Modified Cantor $\left(\frac{1}{8}\right)$ sets

The Removable intervals are

$\left(\frac{1}{64}, \frac{2}{64}\right), \left(\frac{3}{64}, \frac{5}{64}\right), \left(\frac{6}{64}, \frac{7}{64}\right), \left(\frac{17}{64}, \frac{18}{64}\right), \left(\frac{19}{64}, \frac{21}{64}\right), \left(\frac{22}{64}, \frac{23}{64}\right), \left(\frac{41}{64}, \frac{42}{64}\right)$ $\left(\frac{43}{64}, \frac{45}{64}\right), \left(\frac{46}{64}, \frac{47}{64}\right), \left(\frac{57}{64}, \frac{58}{64}\right), \left(\frac{59}{64}, \frac{61}{64}\right), \left(\frac{62}{64}, \frac{63}{64}\right)$ from each of the subintervals will result in modified cantor set.

Therefore

$$C_{8^2} = \left\{\begin{array}{c}\frac{0}{64}=0, \frac{1}{64}, \frac{2}{64}, \frac{3}{64}, \frac{5}{64}, \frac{6}{64}, \frac{7}{64}, \frac{8}{64}, \frac{16}{64}, \frac{17}{64}, \frac{18}{64}, \frac{19}{64}, \frac{21}{64}, \frac{22}{64}, \frac{23}{64}, \frac{24}{64}, \\ \frac{40}{64}, \frac{41}{64}, \frac{42}{64}, \frac{43}{64}, \frac{45}{64}, \frac{46}{64}, \frac{47}{64}, \frac{48}{64}\end{array}\right\} \qquad (4.2)$$

This procedure proceeds in every iteration to get the entire modified cantor set.

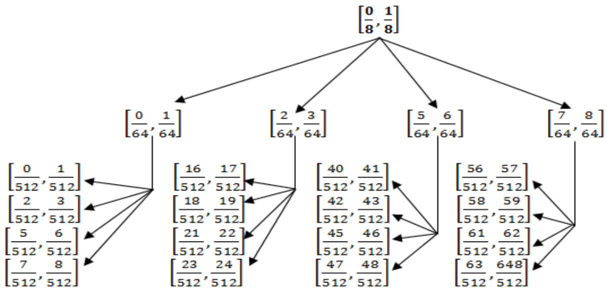

Figure 8: Retained part of modified Cantor $\left(\frac{1}{8}\right)$ sets.

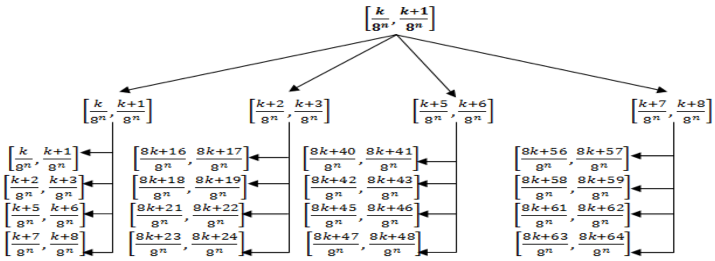

Figure 9: General form of Retained part of modified Cantor $\left(\frac{1}{8}\right)$ sets.

4.1 MODIFIED CANTOR $\left(\frac{1}{12}\right)$ SETS:

The above (Theorem 4.1) same pattern followed by modified Cantor $\left(\frac{1}{12}\right)$ sets.

First iteration:

The closed interval $[0,1]$ is subdivided into 12 equal sub- intervals $\left[\frac{0}{12}=0,\frac{1}{12}\right],\left[\frac{1}{12},\frac{2}{12}\right],\left[\frac{2}{12},\frac{3}{12}\right],\left[\frac{3}{12},\frac{4}{12}\right],\left[\frac{4}{12},\frac{5}{12}\right],\left[\frac{5}{12},\frac{6}{12}\right],\left[\frac{7}{12},\frac{8}{12}\right],$ $\left[\frac{9}{12},\frac{10}{12}\right],\left[\frac{11}{12},\frac{12}{12}=1\right]$

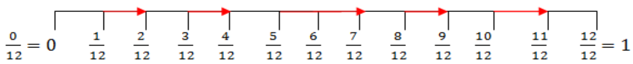

Figure 10: Modified Cantor $\left(\frac{1}{12}\right)$ sets

The Removed intervals are $\left(\frac{1}{12},\frac{2}{12}\right),\left(\frac{3}{12},\frac{4}{12}\right),\left(\frac{5}{12},\frac{7}{12}\right),\left(\frac{8}{12},\frac{9}{12}\right),\left(\frac{10}{12},\frac{11}{12}\right)$.

The remaining intervals are $\left[\frac{0}{12}=0,\frac{1}{12}\right]\cup\left[\frac{2}{12},\frac{3}{12}\right]\cup\left[\frac{4}{12},\frac{5}{12}\right]\cup$ $\left[\frac{7}{12},\frac{8}{12}\right],\left[\frac{9}{12},\frac{10}{12}\right],\left[\frac{11}{12},\frac{12}{12}=1\right]$

$$\therefore\ C_{12^1}=\left\{\frac{0}{12}=0,\ \frac{1}{12},\frac{2}{12},\frac{3}{12},\frac{4}{12},\frac{5}{12},\frac{7}{12},\frac{8}{12},\frac{9}{12},\frac{10}{12},\frac{11}{12},\frac{12}{12}=1\ \right\} \qquad (4.3)$$

This procedure proceeds in every iteration to get the entire modified cantor set.

5. Conclusion

We have established modified cantor $\left(\frac{1}{6}\right)$ and $\left(\frac{1}{8}\right)$ sets. Unlike usual Cantor sets having removal sets of equal lengths here two cases are considered. One is usual away of removing intervals of lengths 1/2n and the middle most intervals of lengths 2/2n. The process is continued successively, so that the general portion of removable intervals can be identified.

In analyzing Cantor sets of even order it is obtained that more than one pattern of removal intervals can be considered. Also it is noted that starting with six and eight every increment of four gives the same mode of removal exists respectively. In this paper the general formula for the existing intervals have been given. The other modes may be considered for future work.

Remark:

Another way of forming Cantor modified set may be given as follows for Cantor Octanary Set.

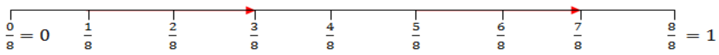

Figure 11: Another way of Cantor $\left(\frac{1}{8}\right)$ sets.

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