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FM CONTRACTION CONDITIONS OF EIGHT SELF MAPS BY OCCASIONALLY WEAKLY COMPATIBILITY

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Abstract. A novel type of contraction known as F_M-contraction is proposed and used it for many self-mappings of a metric space. F_M-contraction is investigated with the self-mappings associated with coincidence and common fixed point theorems. Conditions of F_M-contraction are established through common property (E.A.) and common limit range property without continuity of the maps and completeness of the space. Our results extend, justify and generalize state of the art with examples.

KEYWORDS: F_M-contraction, Common Property (E.A), Common Limit Range Property, Common Fixed Point, Coincidence Point

1. Introduction

Fixed point theory has proved to be very powerful aspect in the study of nonlinear analysis with its plethora of applications catering to several fields in the real world. As explored in [9], the work in this research area are inspired by the principle of contractive mapping. There are diversified variants possible to meet the requirements of different applications. Different metrics spaces are found in the literature such as probabilistic metric (PM) spaces [1], ordered metric spaces [6], cone metric spaces [8] and b-metric spaces [11]. The "coupled coincidence and common fixed point theorems" are investigated in [2] while multi-valued maps are used in [4]. In [10] the usage of ordered cone metric spaces associated with w*-compatible mappings to investigate the utility of the "coupled common fixed point theorems". B-metric spaces with (E: A) property is considered in [11] while cone metric spaces with "coincidence and common fixed and periodic point theorems" is investigated in [12]. Contracting mappings, arbitrary binary relationships and cyclic contractive mappings are investigated in [15]. Fuzzy metric spaces with weakly compatible mappings to ascertain the utility of common fixed point theorems are investigated in [19].

Menger PM spaces and weakly compatible mappings in relation with common fixed point theorems is studied in [22]. B-metric spaces and expansive conditions to investigate the fixed point theorems are considered in [25]. In the process, they extended many existing theorems. From the literature it is understood the coincidence and fixed point theorems in different metric spaces are very useful in many applications. In this paper, we used eight self-maps that are in tune with F_M-contraction condition considering "coincidence and common fixed point theorems". The remainder of the paper is structured as follows. Section 2 reviews existing literature on fixed point theorems and coincidence. Section 3

2. RELATED WORK

This section reviews literature on common fixed point theorems and the related works. Imdad et al. [1] focused on probabilistic metric (PM) spaces that are associated with coincidence and common fixed point theorems. Without any continuity need, they defined four self-mapping functions and different alternative natural completeness conditions. It is on the contrary to the completeness of spaces. With respect to Menger, they have made many generalized results including metric spaces. Abbas et al. [2] investigated on "coupled coincidence and common fixed point theorems" and used them for hybrid pair of mappings and multi-valued mappings. Their work involves hybrid pairs consisting of multi-valued and single-valued maps that satisfy contractive conditions that are generalized. They also defined two suitable examples to support their theorems. Shukla et al. [3] considered "ordered Preši´c-Reich type contractions" and defined theorems on coincidence and common fixed points. Their work is associated with metric spaces with illustrations that prove the theorems. Damjanovic [4] investigated on multi-values maps and proposed "common fixed point theorems" for them. Their results are able to generalize the ones provided by Gordiji et al. [26]. Abdeljawad [5] introduced Meir-Keeler α-contractive functions in a generalized fashion. They also investigated on them and derived new sufficient conditions.

Radenovic [6] investigated on the partially ordered metric space along with theorems on tripled common fixed points and tripled coincidence. Their results could extend and generalize those theorems with ordered metric spaces. Esmaily et al. [7] considered ordered metric spaces and sequence of mappings to work with common fixed point theorems and coincidence. The sequence of mappings is explored with generalization of weakly contractive conditions sans continuity considerations. Cvetkovic et al. [8] studied cone metric type space in order to have common fixed point theorems and associated them with four mappings. It considers generalization of the metric space to relate the theorems for the mappings in the metric space. Roldán-López-de-Hierro and Shahzad [9] investigated on the theorems of common fixed point in presence of "(R, S)-contractivity conditions". They also used a binary relation with respect to metric space without expecting it to be a partial order. Their theorems are established with an example. Nashine et al. [10] considered the usage of ordered cone metric spaces associated with w*-compatible mappings to investigate the utility of the "coupled common fixed point theorems".

Ozturk and Turkoglu [11] investigated on b-metric spaces, particularly (E: A)property, to know whether "common fixed points for mappings" can satisfy the property of the b-metric spaces. They also generalized the notions of both cone metric spaces and b-metric spaces. Yang et al. [12] studied cone metric spaces with "coincidence and common fixed and periodic point theorems". Sintunavarat and Kumam [13] used invariant approximations and JH-operator classes that are generalized with common fixed point theorems. Their results show that they are able to extend and unify several existing ones. Cho et al. [14] studied symmetric spaces for the application of coincidence and fixed point theorems. Berzig [15] investigated on contracting mappings associated with metric spaces that have applications and arbitrary binary relations. They defined theorems of coincidence and common fixed pint and applied them to cyclic contractive mappings as well. Ali et al. [16] investigated on Menger spaces that exhibit common property (E.A) to ascertain the utility of common fixed point theorems on them. Their study encapsulates both metric and Merger spaces. Chauhan et al. [17] studied "modified intuitionistic fuzzy metric spaces" for the application of common fixed point theorems. Their results showed improvement and generalization of many existing fixed point theorems. Eshi et al. [18] investigated on metric space with graph in order to have application of couple common fixed point theorems and coupled coincidence. In order to prove their approach, they introduced the notion of G-g contraction mapping.

Sintunavarat and Kumam [19] considered fuzzy metric spaces with weakly compatible mappings to ascertain the utility of common fixed point theorems. Jungck et al. [20] used cone metric spaces in order to prove several theorems associated with common fixed point theorems. Weakly compatible pairs in the given metric spaces are used for the study. Chauhan et al. [21] proposed "hybrid coincidence and common fixed point theorems" considering Menger probabilistic metric spaces that are to satisfy a strict contractive condition. It is supported by an example illustration. Imdad et al. [22] investigated the relation between Menger PM spaces and weakly compatible mappings in relation with common fixed point theorems. Imdad et al. [23] investigated on the theorems proposed by Bouhadjera and Godet-Thobie [27] and found certain flaws but observed that the flaws can be rectified. Sintunavarat and Kumam [24] extended single-valued mappings related tangential to be suitable for multi-valued mappings to prove the utility of the fixed point theorems of Gregus-type. Jain et al. [25] considered b-metric spaces and expansive conditions to investigate the fixed point theorems. In the process, they extended many existing theorems. From the literature it is understood the coincidence and fixed point theorems in different metric spaces are very useful in many applications. In this paper, we used eight self-maps that are in tune with F_M-contraction condition considering "coincidence and common fixed point theorems".

3. PRELIMINARIES

Let ℝ be the set of Real numbers, $\mathbb{R}^+$ be the set of all positive numbers and ℕ the set of positive integers.

F-contraction concept given by Wardowski [24] is stated as follows.

$\mathcal{F}$is a family of functions F: $\mathbb{R}^+ \longrightarrow \mathbb{R}$ sustaining to the given conditions:

(H1) H is firmly increasing, for all $a, b \epsilon (0, \infty)$ such that $a < b, H(a) < H(b)$;

(H2) Any given sequence $\beta_n$of positive numbers $\lim_{n \to \infty} \beta_n = 0$ iff $\lim_{n \to \infty} H(\beta_n) = -\infty$;

(H3) there exists k$\in (0,1)$ such that $\lim_{\alpha \to 0^+} \alpha^k F(\alpha) = 0$

Definition 3.1 [24] Let F: Y→ Y is a mapping in (Y, d) where d is the metric then F satisfies F-contraction principle for f∈ $\mathcal{F}$ and there is some $\tau > 0$ that

$$for\ all\ x, y \in X, d(Tx, Ty) > 0 \Rightarrow \tau + F\big(d(Hx, Hy)\big) \leq F\big(d(x, y)\big) \qquad (3.1)$$

F contraction F is contractive From (H1) and (1), i.e d (Hx, Hy) < d(x, y) for all x≠y∈ X, so it is necessarily continuous. Taking several functions F, we obtain various F-contractions.

Remark 3.1. Most of the Banach contraction ratio r$\in (0,1)$ also is a F-contraction. And G:$\mathbb{R}^+ \to \mathbb{R}$,

G (t) =ln t, and$\tau = -\ln r$. There are also few F-contractions that are not Banach contractions (see e.g. [24], [17]).

The below lemma is taken form secelean [17]

Lemma 3.1. [17, Lem.3.2] let $F: \mathbb{R}^+ \to \mathbb{R}$ be an increasing mapping and $\{\alpha_n\}$ be a sequence of positive real numbers. Then the following conditions hold:

(i) If $\lim_{n \to \infty} F(\alpha_n) = -\infty$, then $\lim_{n \to \infty} \alpha_n = 0$;

(ii) If inf F= - ∞ and $\lim_{n \to \infty} \alpha_n = 0$,then $\lim_{n \to \infty} F(\alpha_n) = -\infty$

Secelan indicated the condition (F2) in definition 3.1 can be substituted with a corresponding simpler condition by proving the Lemma 3.1:

(H2') $\inf F =$ -∞ or,

(H2'') $\{\beta_n\}$ is a sequence of non-negative real numbers such that $\lim_{n\to\infty} F(\beta_n) = -\infty$

The condition (H3) in 2.1 definition is replaced by [13] Piri and Kumam as follows

(H3') F is continuous on $\mathbb{R}^+$

$\mathcal{F}$ is represented by set of all functions sustaining the conditions (H1), (H2') and (H3')

the existence and uniqueness of fixed points of F-contractions is proved by Wardowski [1, Th.2.1] and Piri and Kuman [13, Th.2.1], where F∈ $\mathcal{F}$ and F∈ $\mathfrak{F}$ respectively.

In 2014, G.Minak, A.Helvaci and I. Altum [12] extended the work of Wardowski, Piri and Kumam and introduced the concept of generalized ciric-type F-contractions, where F∈ $\mathcal{F}$, for a self-map H of a metric space(X, d) there is $\tau > 0$ which is

$\tau + F\big(d(Hx, Hy)\big) \le F\left(max\left\{d(x,y), d(Hx,x), d(Hy,y), \frac{1}{2}\big(d(Hx,y) + d(Hy,x)\big)\right\}\right)$,(3.2)for x, y∈ X, Hx≠Hy.

Theorem 3.1. [12.Th.2.2] Take (Y, d) as a complete metric space and H: X→ X is a Ciric type generalized F-contraction. If H or F is continuous, then H has a unique fixed point in X.

Let us take $\mathcal{F}_M$ as the entire family of continuous functions F:$\mathbb{R}^+ \to \mathbb{R}$

Definition 3.2 let A and B be a pair of self-maps from a metric space (Y, d) having a coincidence point y ∈ Y ,if Ay=By. And also a point y∈ Y is common fixed point of A and B if A y=B y=y. (A, B), (C, D) be the self-maps of a metric space (X, d) they are possessing a common coincidence point if there exists y∈ Y such that Ay = By = Cy = Dy.

Definition 3.3[21] A pair(S, T) on a metric space (X, d) is said to be:

(i) compatible, if $\lim_{n\to\infty} d(STx_n, TSx_n) = 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t$,for some t∈ X;

(ii) weakly compatible, if the pair commutes on the set of their coincidence points, i. e for x∈ X, Sx =Tx implies STx=TSx.

Definition 3.4[8] A pair of self-maps on a metric space (X, d) is occasionally weakly compatible (OWC) if and only if there is a point x∈ X which is a coincidence point of S and T at which S and T commute i.e, there exists a point x∈ X such that Sx =Tx and STx=TSx

Lemma 3.2[8] Let X be a set, S and T be occasionally weakly compatible self-maps on X. If S and T have a unique point of coincidence w = Sx= Tx for x$\in X$, then w is the unique common fixed point of S and T.

Definition 3.5. [1] In a metric space (X, d) a pair (S, T) has:

(i) the property(E.A), if there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t$,for some t$\in X$;

(ii) (CLR_S)which denotes the common limit property is given by , if there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t$,for some t$\in S(X)$

Definition 3.6

Two pairs (A, S) and (B, T) of self-maps of a metric space(X, d) are said to satisfy:

(i) the common property(E.A), if there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = z$, for some z$\in X$

(ii) the common limit range property with respect to S and T, denoted by (CLR_{ST}), if there exist two sequences$\{x_n\} and \{y_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = z$, for some z$\in s(X) \cap T(X)$

Definition 3.7. We say that a pair of self-maps (A, S) of a metric space (X, d) constitutes a ciric type F_M – contraction if there exist $F \in \mathcal{F}_M \; and \; \tau > 0$ such that, for all x, y$\in X$ with d(Ax, Ay)>0,

$$\tau + F\big(d(Ax, Ay)\big) \le F\left(max\left\{d(Ax, Sx), d(Ay, Sy), d(Sx, Sy), \frac{d(Ax, Sy) + d(Ay, Sx)}{2}\right\}\right)$$

(3.2)

Definition 2.8. (A, S) and (B, T) are two self-maps of a metric space (X, d) satisfying a ciric type F_M – contraction if there exist $G \in F_M \; and \; \tau > 0$ such that, for all x, y$\in X$ with d (Ax, By)>0,

$$\tau + G\big(d(Ax, By)\big) \le G\left(max\left\{d(Ax, Sx), d(By, Ty), d(Sx, Ty), \frac{d(Sx, By) + d(Ax, Ty)}{2}\right\}\right)$$

(3.3)

Proposition 3.1([11]). Let P, Q, R, S, T and U be self-maps of a metric space (X, d) satisfying the following conditions:

$$(\alpha)T(x) \subseteq RS(x)\big(resp. (\alpha')M(x) \subseteq AB(x)\big);$$

$(\beta)$the pair (T, PQ) satisfies the (CLR_{PQ}) property (resp.(β') the pair (U, RS) satisfies the (CLR_{RS}) property);

$$(\gamma)RS(x) isaclosedsubsetof X (resp(\gamma')PQ(x) isaclosedsubsetof X);$$

$(\delta) thereexists \tau > 0 and G\colon \mathbb{R}^+ \to \mathbb{R}$such that for all x, y$\in X$

$withd(Lx, My) > 0,$

$$\tau + G\big(d(Tx, Uy)\big) \le G\left(max\left\{\begin{matrix} d(Tx, PQx), d(Uy, RSy), d(PQx, RSy), \\ \frac{d(PQx,Uy)+d(Tx,RSy)}{2} \end{matrix}\right\}\right) \quad (3.4)$$

Then the pairs (T, PQ) and (U, RS) share the ($CLR_{(PQ)(RS)}$) property

As T(X) ⊆ ST(X) and from the property (E.A.) the two pairs (T, PQ) and (U,RS) has common property of (E.A) of the pair(T,PQ), the following result is drawn and its proof is similar to preposition 3.1

Proposition 3.2 ([11])Let D,E,F,G,H,I be self-maps of a metric space (X, d).let $\tau > 0$ *and* $G: \mathbb{R}^+ \to \mathbb{R}$ is defined in a way that equation (4)and succeeding hypothesis hold:

(1)H(x) ⊆ FG(x);

(2) (H, DE) fulfills the E.A. property, FG(x) is closed. Then (H, DE) and (I, FG) fulfill the common property (E.A.)

Remark 3.2.([10])the proposition 3.2 guarantees that common property(E.A.) condition of two pairs (H,DE)and (I,FG) is weaker than the E.A property of (H, DE) , inclusion of H(x) ⊆ FG(x)

4.KEY RESULTS

The key result is proved by using Ciric type F_M-contraction in eight self-maps taking (CLR $_{(PQ)\,(RS)}$) property and occasionally weakly compatible.

THEOREM 4.1

Let A_1, B_1,S_1,T_1,L_1,M_1,P_1 and Q_1 be the self-maps of (Y, d) the metric space. Assume that the pairs (L_1P_1, A_1B_1) and (M_1Q_1, S_1T_1) satisfy $CLR_{(PQ)(RS)}$ property and establish a ciric type F_M – contraction, i.e. there exist f $\in F_M$ and $\tau > 0$ for all x, y$\in Y$ with d(L_1P_1x,M_1Q_1y)>0

$$\tau + F(d(\ L_1P_1x, M_1Q_1y)) \le F\left(max\left\{d(L_1P_1x, A_1B_1x), d(M_1Q_1y, S_1T_1y)\ , d(A_1B_1x, S_1T_1y), \frac{d(A_1B_1x, M_1Q_1y)+d(L_1P_1x, S_1T_1y)}{2}\right\}\right)$$

(4.1)

then (L_1P_1, A_1B_1) and (M_1Q_1, S_1T_1) have a common fixed point

Moreover, if

(i) Both pairs (L_1P_1, A_1B_1) and (M_1Q_1, S_1T_1) are occasionally weakly compatible
(ii) A_1B_1=B_1A_1,L_1P_1=P_1L_1,$L_1P_1A_1$=$A_1L_1P_1$
(iii) S_1T_1=T_1S_1,M_1Q_1=Q_1M_1,$M_1Q_1S_1$=$S_1M_1Q_1$
(iv) P_1x =$P_1{}^2$x, Q_1x=$Q_1{}^2$x for all xϵX

Then A_1, B_1,S_1,T_1,L_1,M_1,P_1 and Q_1 have a unique common fixed point in X

PROOF:

(L_1P_1, A_1B_1) and (M_1Q_1, S_1T_1) fulfill the (CLR $_{(PQ)(RS)}$) property is equal to the survival of two sequences $\{x_n\},\{y_n\}$ in Y such that

$$\lim_{n\to\infty} L_1P_1x_n = \lim_{n\to\infty} A_1B_1x_n = \lim_{n\to\infty} S_1T_1y_n = \lim_{n\to\infty} M_1Q_1y_n = t$$

Where $t \in A_1B_1(x) \cap S_1T_1(x)$ (7)

As t $\in A_1B_1(X)$, there be a u$\in X$ such that A_1B_1u =t. also since t$\in S_1T_1(x)$, there is a point v$\in X$ such that S_1T_1v =t

Let us assume that d (t, M_1Q_1v) = 0, supposing on the contrary that d (t, M_1Q_1v) = c >0 then there occur $\varepsilon > 0, \varepsilon < c$ and n $\in N$ so that d ($L_1P_1$$x_n$, M_1Q_1v) >ε for all n$\geq N$ assuming x=x_n and y=v in (1) we get

$$\tau + F\big(d(L_1P_1x_n, M_1Q_1v)\big) \leq F\left(max \left\{ \begin{matrix} d(L_1P_1x_n, A_1B_1x_n), d(S_1T_1v, M_1Q_1v)\,, d(A_1B_1x_n, S_1T_1v), \\ \dfrac{d(A_1B_1x_n, M_1Q_1v) + d(L_1P_1x_n, S_1T_1v)}{2} \end{matrix} \right\} \right)$$

Every n$\geq N$. From equation (2) and passing limit to the inequality and the continuity of F at C, we get

$$\tau + F(c) \leq F\left(max\left\{0, c, 0, \tfrac{c}{2}\right\}\right) = F(c)$$

Which is a conflict hence d (t, M_1Q_1v) = 0 which shows that t=M_1Q_1v
Hence t=S_1T_1v = M_1Q_1 v which proves that v is a coincident point of pair (M_1Q_1, S_1T_1)
Hence, we can get t=L_1P_1u = A_1B_1 u, therefore u is a coincident point of (L_1P_1, A_1B_1)
Since the pair (L_1P_1, A_1B_1) are occasionally weakly compatible so by definition there exists a point

$u \in X$ such that L_1P_1u $=A_1B_1$u and L_1P_1 (A_1B_1) u = $(A_1B_1)L_1P_1$u, since the pair (M_1Q_1, S_1T_1) are occasionally weakly compatible so by definition there exists a point v$\in X$ such that

M_1Q_1v = S_1T_1 v and

M_1Q_1 (S_1T_1)v = S_1T_1 (M_1Q_1)v

Hence L_1P_1u=A_1B_1u=M_1Q_1v=S_1T_1v

Moreover, if there is another point z such that L_1P_1z=A_1B_1z then using (6) to show that L_1P_1z=A_1B_1z=M_1Q_1v=S_1T_1v

we assert that d (L_1P_1z, M_1Q_1v) = 0

hence on the contrary d (L_1P_1z, M_1Q_1v) >0

$$\tau + F\big(d(L_1P_1z, M_1Q_1v)\big)$$
$$\leq F\left(max\left\{d(L_1P_1z, A_1B_1z), d(M_1Q_1v, S_1T_1v)\ , d(A_1B_1z, S_1T_1v), \frac{d(A_1B_1z, M_1Q_1v) + z(L_1P_1x, S_1T_1v)}{2}\right\}\right)$$

$$\tau + F\big(d(L_1P_1z, M_1Q_1v)\big) \leq F(max\{0,0, d(L_1P_1z, M_1Q_1v), d(L_1P_1z, M_1Q_1v)\})$$

$$\tau + F\big(d(L_1P_1z, M_1Q_1v)\big) \leq F(d(L_1P_1z, M_1Q_1v))$$

This is a conflict, hence d (L_1P_1z, M_1Q_1v) = 0 which shows that L_1P_1z = M_1Q_1v

Hence L_1P_1z=A_1B_1z, M_1Q_1v=S_1T_1v

So, L_1P_1u=L_1P_1z and w= L_1P_1u=A_1B_1u is a unique point of coincidence of L_1P_1 and A_1B_1

By the lemma 2.2, w is the unique common fixed point of $L_1P_1$and A_1B_1 i. e w= L_1P_1w=A_1B_1w

Similarly there is a unique point z$\in X$ such that z=M_1Q_1z=S_1T_1z

5. UNIQUENESS

Suppose that w$\neq z$ $d(w,z) \neq 0$ using inequality (6) with x=w, y=z we get

$$\tau + F\big(d(L_1P_1w, M_1Q_1z)\big)$$
$$\leq F\left(max\left\{d(L_1P_1w, A_1B_1w), d(M_1Q_1z, S_1T_1z), d(A_1B_1w, S_1T_1z), \frac{d(A_1B_1w, M_1Q_1z) + d(L_1P_1w, S_1T_1z)}{2}\right\}\right)$$

$$\leq F(max\{0,0, d(L_1P_1w, M_1Q_1z), d(L_1P_1w, M_1Q_1z)\})$$

$\tau + F\big(d(w,z)\big) \leq F(d(w,z)$a conflict, therefore d (w, z) =0 hence w=z

Hence z is a unique common fixed point of the mappingsL_1P_1, M_1Q_1, $A_1B_1$and S_1T_1

Finally we need to show that z is a common fixed point of A_1, B_1,S_1,T_1,L_1,M_1,P_1 and Q_1

Let us take x=z, y= S_1z in (3.5) with the supposition that d (L_1P_1z ,M_1Q_1 S_1z)$\neq 0$,

Condition (iii) shows that

F (d (L_1P_1z, M_1Q_1 S_1z)) = F (d (z, S_1z))

$$\leq F\left(max\begin{Bmatrix} d(A_1B_1z, L_1P_1z), d(M_1Q_1S1z, S_1T_1S1z), d(A_1B_1z, S_1T_1S1z), \\ \dfrac{d(A_1B_1z, M_1Q_1s_1z) + d(L_1P_1z, S_1T_1Sz)}{2} \end{Bmatrix}\right) - \tau$$

= F (max$\{0,0, d(z, S_1z), d(z, S_1z)\}) - \tau$

= F (d (z, S_1z))- $\tau < F\big(d(z, S_1z)\big)$

It is a conflict

Therefore d (L_1P_1z, M_1Q_1 S_1z) = d (z, S_1z) = 0, i.e., z= S_1z

Hence z= S_1z= S_1T_1z=T_1 S_1z=T_1Z

Hence we prove that

z= A_1z =A_1B_1z=BA_1z=, B_1z

Since P_1z=$P_1{}^2z$, Q_1z=$Q_1{}^2z$ and L_1P_1=P_1 L_1, M_1Q_1=Q_1 M_1

It has z=L_1P_1z=L_1P_1Pz=pL_1P_1z=$P_1z \Longrightarrow L_1z = z$

z=M_1Q_1z=$M_1Q_1Q_1z$=QM_1Q_1z=$Q_1z \Longrightarrow M_1z = z$

Therefore, in the view of above foresaid, we have

z=A_1z =, B_1z = S_1z =T_1z= M_1z=P_1z=Q_1z

It gives that A_1, B_1,S_1,T_1,L_1,M_1,P_1 and Q_1 has a common fixed point z in X

We will prove the uniqueness of the fixed point

Take W as one more common fixed point of A_1, B_1,S_1,T_1,L_1,M_1,P_1 and Q_1 with w$\neq$ z.It follows that

w=A_1w = , B_1w = L_1w = M_1w = S_1w =T_1w = Pw = Q_1w

Taking x=z and y=w in (1) we have

F (d (L_1P_1z, M_1Q_1w)) = F (d (z, w))

$$\leq F\left(max\begin{Bmatrix} d(A_1B_1z, L_1P_1z), d(M_1Q_1w, S_1T_1w), d(A_1B_1z, S_1T_1w), \\ \frac{d(A_1B_1z, M_1Q_1w) + d(L_1P_1z, S_1T_1w)}{2} \end{Bmatrix}\right) - \tau$$

=F (max {0, 0, d (z, w), d (z, w)}) – τ

= F (d (z, w)) - τ

<f (d (z, w))

It is a conflict

As d (z, w) =0, i. e z=w

Therefore A_1, B_1,S_1,T_1,L_1,M_1,P_1 and Q_1has fixed point z in X which is unique

Completeness of the proof is explained

The results from theorem 3.1 can be obtained without function F satisfying (H2), (H3), respectively, (H2), (H3').

Assume g=I_X (or f=I_X) in theorem4.1, we can acquire common fixed point results in seven maps and its coincidence

Corollary 5.1. (Y, d) be a metric space from which self-maps P, Q, R, S, T , U and V are taken let the pairs (TV, PQ) and (U, TL) fulfill (CLR $_{(PQ)\ (TL)}$) property , Ciric type F_M-contraction, i.e., we have F∈ $\mathcal{F}_M$ and $\tau > 0$ and for every x, y∈ Y $with$ $d($Tfx, Uy) >0,

$$\tau + F\big(d(Tfx, Uy)\big) \leq F\left(\max\begin{Bmatrix} d(Tfx, PQx), d(TLy, Uy), d(PQx, TLy), \\ \frac{d(PQx, Uy) + d(Tfx, TLy)}{2} \end{Bmatrix}\right) \quad (5.1)$$

Then a common fixed point exists for (T, PQ) and (U, TL)

And also

(a) (TV, PQ) and (U, TL) are occasionally weakly compatible;

(b)PQ = QP, Tf=fT, TfP=PTf;

(c)TL=LT, UgL=LUg;

(d)Vx =V^2x, for all x∈ Y;

henceP, Q, R, S, T, U and V will have a unique common fixed point in Y.

common fixed point for six self-maps can be attained by taking V=g=I_x in theorem 3.1

Corollary 5.2.

In a metric space (Y, d) we take the self-maps as P, Q, R, S, T, U. (T, PQ) and (U, RS) satisfy (CLR $_{(PQ)(RS)}$) property and they are Ciric F_M-contraction, i.e , we have $\mathcal{F} \in F_M$ and $\tau > 0$ such that, for all x, y$\in Y$ $with$ d(Tx, Uy) >0,

$$\tau + F\big(d(Tx, Uy)\big) \leq F(\max\left\{\begin{matrix} d(Tx, PQx), d(RSy, Uy), d(PQx, RSy), \\ \frac{d(PQx,Uy)+d(Tx,TLy)}{2} \end{matrix}\right\}) \quad (5.2)$$

Then both pairs (T, PQ) and (U, RS) have a common fixed point.

and

(a) (T, PQ) and (U, RS) are occasionally weakly compatible;
(b) PQ = QP, TP=PT;
(c) RS=SR, UR=RS;

thenP, Q, R, S, T, U have a unique common fixed point in Y.

coincidence and common fixed point of five self-maps can be achieved if we take T=I_x in corollary 5.2,

Corollary 5.3.

In a metric space (Y, d) we take the self-maps as P, Q, R, S, T, U. (T, PQ) and (U, RS) satisfy (CLR $_{(PQ)(RS)}$) property and they are Ciric F_M-contraction, i.e, we have $\mathcal{F} \in F_M$ and $\tau > 0$ such that, for all x, y$\in Y$ $with$ d(Tx, Uy) >0,

$$F\big(d(Tx, Uy)\big) \leq F\left(\max\left\{\begin{matrix} d(Tx, PQx), d(Ry, Uy), d(PQx, Ry), \\ \frac{d(PQx,Uy)+d(Tx,RTy)}{2} \end{matrix}\right\}\right) - \tau \quad (5.3)$$

Then both pairs (S, PQ) and (U, R) have a common fixed point.

and

(a) (S, PQ) and (U, R) are occasionally weakly compatible;
(b) PQ = QP, SP=PS;
(c) RU=UR;

thenP, Q, R, S, T, U have a unique common fixed point in Y.

outcomes for four self-maps, can be obtained if B =T=I_X in corollary 3.3which is listed as follows:

Corollary5.4.

In (Y, d) metric space the self-maps P, Q, R, S, T are taken, (S, P) and (T, R) fulfill (CLR $_{(PR)}$) property, Ciric type FM-contraction. (S, P) and (T, R) have a common fixed point and are weakly compatible, SP=PS, TR=RT, then P, Q, R, S, T have a unique common fixed point in Y.

The following examples are given to support our key results.

Example 5.1. Let X = [1,∞) , d Euclidean metric is given as d(x, y) =$|x-y|$, for all x, y$\in X$. Define A, B, S, T, L and M: X → X by

$$Ax = \begin{cases} 2, \text{if x} = 1,2 \\ 5, \text{if x } \in [1,3) - \{1,2\} \\ 1, \text{if x} \geq 3 \end{cases}, Bx = \begin{cases} 2, \text{if x} = 1,2 \\ 4, \text{if x } \in [1,3) - \{1,2\} \\ 1, if\ x \geq 3 \end{cases}$$

$$Sx = \begin{cases} 2, if\ x = 1,2 \\ 8, if\ x\ \in [1,3) - \{1,2\} \\ 1, if\ x \geq 3 \end{cases} Tx = \begin{cases} 2, \text{if x} = 1,2 \\ 3, \text{if x } \in [1,3) - \{1,2\}, \\ 1, \text{if x} \geq 3 \end{cases}$$

$$Lx = \begin{cases} 2, \text{if x} = 1,2 \\ 5, \text{if x } \in [1,3) - \{1,2\}, \\ 1, \text{if x} \geq 3 \end{cases} Mx = \begin{cases} 2, \text{if x} = 1,2 \\ 9, \text{if x } \in [1,3) - \{1,2\}, \\ 1, \text{if x} \geq 3 \end{cases} gx$$

$$= \begin{cases} 2, \text{if x} = 1,2 \\ 7, \text{if x } \in [1,3) - \{1,2\}, \\ 1, \text{if x} \geq 3 \end{cases} fx = \begin{cases} 2, \text{if x} = 1,2 \\ 6, \text{if x } \in [1,2) - \{1,2\}, \\ 1, \text{if x} \geq 3 \end{cases}$$

Hence,

$$Lfx = \begin{cases} 2, \text{if x} = 1,2 \text{ and x } \geq 3 \\ 1, \text{if x } \in [1,3) - \{1,2\} \end{cases} Mgx = \begin{cases} 2, \text{if x} = 1,2 \text{ and x } \geq 3 \\ 1, \text{if x } \in [1,3) - \{1,2\} \end{cases}$$

sequences are$\{x_n\}$ and $\{y_n\}$ in X and $x_n=3+\frac{1}{n}$,y_n =2 and

$$\lim_{n\to\infty} Lfx_n = \lim_{n\to\infty} Lf(3+\tfrac{1}{n}) = 2,$$

$$\lim_{n\to\infty} ABx_n = \lim_{n\to\infty} AB(3+\tfrac{1}{n}) = 2,$$

And

$$\lim_{n\to\infty} Mgy_n = \lim_{n\to\infty} Mg(2) = 2,$$

$$\lim_{n\to\infty} STy_n = \lim_{n\to\infty} ST(2) = 2,$$

Hence, $\lim_{n\to\infty} Lfx_n = \lim_{n\to\infty} ABx_n = \lim_{n\to\infty} Mgy_n = \lim_{n\to\infty} STy_n = 2$, $2 \in AB(X) \cap ST(X)$, i. e (Lf, AB) and (Mg, ST) share the ($CLR_{(AB)(ST)}$) property.

Also, Lfx = ABx =2, Mgx=STx=2, where $x \in \{1,2,\}$ and $x \geq 3, i.e,$ (Lf, AB) and (Mg, ST) has coincidence points in X.

And , LfABx= ABLfx, MgSTx = STMgx, where $x \in \{1,2\}$ and $x \geq 3, i.e$ (Lf, AB) and (Mg, ST) are occasionally weakly compatible and AB=BA, fL=Lf, Mg=gM, ST=TS, LfA= ALf, MgS=SMg, f^2=f and g^2=g. also, A, B, S, T, L, M, f and g satisfy ciric type F_M-contraction assumption (5) for $\tau = \ln 3$ and F(∝) = ln ∝

Therefore, x=2 is the unique common fixed point of A, B, S, T, L, M, f, g

6. CONCLUSION

The key results with our F_M-contraction used for many self-mappings of a metric space with different properties demonstrate the usage of common fixed point and coincidence. F_M-contraction is investigated with the self-mappings associated with coincidence and common fixed point theorems. Conditions of F_M-contraction are established through common property (E.A.) and common limit range property without continuity of the maps and completeness of the space. Our results extend, justify and generalize state of the art with examples.

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