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CONE METRIC WITH FM CONTRACTION BY EIGHT SELF BY MAPS

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Abstract . In here we study the Fmcontraction condition on cone metric taking occasionally weakly compatibility condition, common limit range property, and weakly compatibility for eight self-maps.

KEYWORDS: Coincident Point, Comon Fixed Point, Occasionally Weakly Compatible, Common (CLR $_{(AB)}$ $_{(ST)}$) Property, Common Property (E.A), Weakly Compatible,.

1. Introduction

In [6] Huaang and Zhaang defined cone metric spaces by generalising concept of metric spaces by replacing the set of real numbers with an orderly Banach space. They also developed the idea of completion in the cone metric spaces and explained sequence convergence. With the assumption of a cone's normalcy, they showed certain common fixed point theorems on contractive mappings of complete cone metric space. Various scholars have since generalisedHuaang and Zhaang'sfindings , researched on normal and non-normal cones for fixed point theorems . The Banach contraction principle has been applied in a number of studies using fixed points. Various authors have developed this principle in the form of contraction mappings. There has been a new generalisation of contraction mapping introduced, and Beiranvand [2] defined metric spaces on f contraction mappings on that are dependent on another function. The authors have then established Banach pair of mappings and some common fixed point theorems for a satisfying the f HardyRogers contraction condition in cone metric spaces in [10]. Moralles and Rojaas [8-9] had also proved f Kannan fixed point theorems , f Zamfirescu, and contraction mappings in f weakly cone metric spaces, extending the idea of f contraction mappings to cone metric spaces. In the next section, we'll require a definition given by Subrahmanyam [24] and known as the of type k Banach operator. Chenn and Lii expanded this idea to the pair of Banach operator, proving standard fixed theorems various best approximation results using for f non expansive mappings [4]. Major aim of this study is to develop common fixed point theorems for eight self mappings that meet the fm contraction condition on a cone metric space beneath the premise that cone is occasionally weakly compatible, and uniqueness is also checked. To back up our findings, we provided counter examples.

Here we define cone metric spaces , some properties.

Defniton2.1 A be a non empty set. let the mapping $h: A \times A \longrightarrow E$ satisfies
(e1) $0 < h(x, y)$ for all $x, y \in A$ and $e(x, y) = 0$ iff $x = y$;
(e2) $h(x, y) = h(y, x)$ for every $x, y \in A$;
(e3) $h(x, y) \leq h(x, z) + h(y, z)$ for every $x, y, z \in A$. hence h is called a cone metric on A, and (A, h) is called as cone metric space.
Defniton 2.2 F-contraction concept given by Wardowski [24] is stated as follows.

$\mathcal{F}$, a family of functions and $G: \mathbb{R}^{+} \longrightarrow \mathbb{R}$ sustain to following conditions:

(H1) H strictly non decreasing, for all $a, b \epsilon (0, \infty)$ such that $a < b, H(a) < H(b)$;
(H2) Any given sequence $\beta_n$of positive numbers $\lim_{n \to \infty} \beta_n = 0$ iff $\lim_{n \to \infty} H(\beta_n) = -\infty$;
(H3) there occurs m∈ $(0,1)$ so that $\lim_{\alpha \to 0^+} \alpha^m F(\alpha) = 0$

Defniton 2.3 Let F: Y→ Y is a mapping in (Y, d) where d is the metric then F satisfies F-contraction principle for f∈ $\mathcal{F}$ and there is some $\tau > 0$ that

$$for\ all\ x, y \in X, d(Tx, Ty) > 0 \Rightarrow \tau + F\big(d(Hx, Hy)\big) \leq F\big(d(x, y)\big) \qquad (2.1)$$

From (H1) and (2.1), F contraction H is contractive such that d(x, y)>d (Hx, Hy) for every x≠y∈ X, so it is necessarily continuous. Considering several functions F, we obtain various F-contractions.

Theorem 2.1. [12.Th.2.2] let (Y, d) be cone metric space, G: Y→ Y is Ciric type genralized F-contraction. If H or F is continuous, then H has a unique fixed point in X.
Let us take $\mathcal{F}_M$ as the entire family of continuous functions F:$\mathbb{R}^+ \to \mathbb{R}$

Definition 2.4 consider A and B as a pair of self maps in a cone metric space (Y, d) having a coincidence point y ∈ Y , if Ay=By. And also a point y∈ Y is common fixed point of A and B if A y=B y=y. (A, B), (C, D) be the self maps in a cone metric space (X, d) they are possessing a common coincident point if there exist y∈ Y thus Ay = By = Cy = Dy.

Definition 2.5[21] A pair(S, T) in a cone metric space (X, d) called as

(i) compatible, if $\operatorname{lt}_{n\to\infty} d(STx_n, TSx_n) = 0$, and $\{x_n\}$ is a sequence in X such that $\operatorname{lt}_{n\to\infty} Sx_n = \operatorname{lt}_{n\to\infty} Tx_n = t$,for all t∈ X;
(ii) weakly compatible, if the pair commutes on the set of their coincidence points, i. e for
x∈ X, Sx =Tx then STx=TSx.

Definition 2.6[8] Two self maps in a cone metric space (X, d) are occasionally weakly compatible (OWC) iff there exists a point x∈ X which is a coincidence point of S and T at which S and T commute that is there is a point x∈ X that Sx =Tx and STx=TSx

Lemma 2.2[8] Let X be a set, S and T are occasionally weakly compatible self maps in X . If S and T has an unique point of coincidence w = Sx= Tx for x∈ X , then w is a unique common fixed point of S and T.

Definition 2.7. [1] On a cone metric space (X, d) a pair (S, T) has:

(i) the property(E.A), if there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t$,for some t∈ X;
(ii) (CLR_S)which denotes the common limit property is given by , if there exists a sequence $\{x_n\}$ in X such that $\operatorname{Lt}_{n\to\infty} Sx_n = \operatorname{Lt}_{n\to\infty} Tx_n = t$,for some t∈ $S(X)$

Definition 2.8
Two pairs (A, S) and (B, T) of self maps of a cone metric space(X, d) are said to satisfy:
(i) The common property(E.A) is defined as, if there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that $\operatorname{Lt}_{n\to\infty} Ax_n = \operatorname{Lt}_{n\to\infty} Sx_n = \operatorname{Lt}_{n\to\infty} By_n = \operatorname{Lt}_{n\to\infty} Ty_n = z$, for some z∈ X
(ii) The common limit range property with respect to S and T is denoted by the (CLR_{ST}), if there exists two sequences$\{x_n\} and \{y_n\}$ in X hence that $\operatorname{Lt}_{n\to\infty} Ax_n = \operatorname{Lt}_{n\to\infty} Sx_n = \operatorname{Lt}_{n\to\infty} By_n = \operatorname{Lt}_{n\to\infty} Ty_n = z$, for some z∈ $s(X) \cap T(X)$

Definition 2.9. Two self maps (A, S) in a cone metric space (X, d) establishes a ciric type F_M – contraction if there is $F \in \mathcal{F}_M$ and $\tau > 0$ for all x, y$\in X$ with d(Ax, Ay)>0,

$$\tau + F\big(d(Ax, Ay)\big) \leq F\left(max\left\{d(Ax, Sx), d(Ay, Sy), d(Sx, Sy), \frac{d(Ax,Sy)+d(Ay,Sx)}{2}\right\}\right)$$

(2.3)

Definition 2.10. (A, S) and (B, T) are two self maps of a cone metric space (X, d) satisfying a ciric type F_M – contraction if there exist $G \in F_M$ and $\tau > 0$ such that, for all x, y$\in X$ with d (Ax, By)>0,

$$\tau + G\big(d(Ax, By)\big) \leq G\left(max\left\{d(Ax, Sx), d(By, Ty), d(Sx, Ty), \frac{d(Sx,By)+d(Ax,Ty)}{2}\right\}\right)$$

(2.4)

Proposition 2.1([14]).Let P_1, Q_1, R_1, S_1, T_1 and U_1 be the self maps in a cone metric space (X, e) satisfying the following conditions:

$$(\alpha)T(x) \subseteq RS(x)\big(resp.(\alpha')M(x) \subseteq AB(x)\big);$$

(β) The pair (T, PQ) satisfy the (CLR_{PQ}) property [resp(β') pair (U, RS) satisfy the (CLR_{RS}) property];

$$(\gamma)RS(x) is a closed subset of X (resp(\gamma')PQ(x) is a closed subset of X);$$

(δ) there exists $\tau > 0$ and $G: \mathbb{R}^+ \to \mathbb{R}$ hence for all x, y$\in X$

$$with\, e(Lx, My) > 0,$$

$$\tau + G\big(d(Tx, Uy)\big) \leq G\left(max\left\{\begin{matrix}e(Tx, PQx), e(Uy, RSy), e(PQx, RSy), \\ \frac{e(PQx,Uy)+e(Tx,RSy)}{2}\end{matrix}\right\}\right) \quad (2.5)$$

Then the pairs (T, PQ) and (U, RS) share the ($CLR_{(PQ)(RS)}$) property

As T(X) ⊆ ST (X) and from the property (E.A.) the two pairs (T, PQ) and (U, RS) has common property of (E.A) of the pair (T,PQ), the following result is drawn and its proof is similar to preposition 2.1

Proposition 2.2 ([11])Let D_1,E_1,F_1,G_1,H_1,I_1 are self maps of a cone metric space (Y, d).let $\tau > 0$ and $G: \mathbb{R}^+ \to \mathbb{R}$ is defined in a way that equation (2.4)and succeeding hypothesis hold:

(1)$H_1(x) \subseteq F_1G_1(x)$;

(2) (H_1, D_1E_1) fulfills the E.A. property, FG(x) is closed. Then (H_1, D_1E_1) and (I_1, F_1G_1) fulfill the common property (E.A.)

Remark 2.2.([13])the propsition 2.2 guarantees comon property (E.A.) condition of two pairs (H,DE), (I,FG) weaker than E.A property (H, DE) , inclusion of H(x) ⊆ FG(x)

3. **MAIN RESULT**

The main result is proved by using Ciric type F_M-contraction in 8 self map taking ($CLR_{(AB)(ST)}$) propertey , occasionally weakly compatible.

THEOREM 3.1

Let A_1, B_1, S_1, T_1, L_1, M_1, P_1 and Q_1 are self mappings of the cone metric space (Y, e) .let the pairs (L_1P_1, A_1B_1) , (M_1Q_1, S_1T_1) satisfy $CLR_{(A1B1)(S1T1)}$ property and also constitute a ciric type F_M – contraction condition , hence there exist $f \in F_M$, $\tau > 0$ for all x, y$\in Y$ with $d(L_1P_1x, M_1Q_1y) > 0$

$$\tau + F\big(d(\, L_1P_1x, M_1Q_1y)\big) \le$$
$$F\left(max\begin{Bmatrix} d(L_1P_1x, M_1Q_1y), d(A_1B_1x, M_1Q_1y)\,, d(A_1B_1x, S_1T_1y), d(L_1P_1x, A_1B_1X), d(M_1Q_1y, S_1T_1y), \\ d(L_1P_1x, S_1T_1y) \end{Bmatrix}\right)$$

(3.1)

Then there will be a fixed point common for both pairs (L_1P_1, A_1B_1) ,(M_1Q_1, S_1T_1)

and, if

(i) The pairs (L_1P_1, A_1B_1) and (M_1Q_1, S_1T_1) are occasionally weakly compatible
(ii) $A_1B_1 = B_1A_1$, $L_1P_1=P_1L_1$,$L_1P_1A_1=A_1L_1P_1$
(iii) $S_1T_1=T_1S_1$,$M_1Q_1=Q_1M_1$,$M_1Q_1S_1=S_1M_1Q_1$
(iv) $P_1x = P_1^2x$,$Q_1x=Q_1^2x$ for all $x\epsilon X$

Then A_1, B_1, S_1, T_1, L_1, M_1, P_1 and Q_1 will have a single common fixed point in X

EVIDENCE:

The fact that the pairs (L_1P_1, A_1B_1) , (M_1Q_1, S_1T_1) satisfy the (CLR (AB) (ST)) property equivalient to the existence of two sequences $\{x_m\}, \{y_m\}$ in Y such that

$$\lim_{m\to\infty} L_1P_1x_m = \lim_{m\to\infty} A_1B_1x_m = \lim_{m\to\infty} S_1T_1y_m = \lim_{m\to\infty} M_1Q_1y_m = t$$

And $t \in A_1B_1(Y) \cap S_1T_1(Y)$ (3.2)

As g$\in A_1B_1(Y)$, exist a u$\in Y$ that A_1B_1u =g. also since g$\in S_1T_1(Y)$, a point w$\in Y$ is that S_1T_1w =g

We claim that $d\,(t, M_1Q_1v) = 0$, on the contrary suppouse that d (t, M_1Q_1v) = c >0 then there exist $\varepsilon > 0, \varepsilon < c$, n $\in N$ hence that d ($L_1P_1x_n$, M_1Q_1v) >ε for every n$\geq N$ take x=x_n and y=v in (3.1) we get

$$\tau + F\big(d(L_1P_1x_n, M_1Q_1v)\big)$$
$$\le F\left(max\begin{Bmatrix} d(L_1P_1x_n, M_1Q_1V), d(A_1B_1x_n, M_1Q_1v)\,, d(A_1B_1x_n, S_1T_1v), d(L_1P_1x_n, A_1B_1x_n), d(M_1Q_1V, S_1T_1v), \\ d(L_1P_1x_n, S_1T_1v) \end{Bmatrix}\right)$$

For every n$\geq N$. By applying the limit into the above inequality, using (3.2) and the continuity of F at C, we get

$$\tau + F(c) \le F(max\{0, c, 0, c\}) = F(c)$$

that is a contradiction

hence d (t, M_1Q_1v) = 0 which gives that=M_1Q_1v

hence t=S_1T_1v = M_1Q_1v that shows that v is a coincident point of pair (M_1Q_1, S_1T_1)

Similarily, we can also obtain t=L_1P_1u = A_1B_1 u, so u is a coincident point of pair (L_1P_1, A_1B_1)

Since the pair (L_1P_1, A_1B_1) is occasionally weakly compatible hence by the definition there exists a point u$\in X$ such that L_1P_1u =A_1B_1u and L_1P_1 (A_1B_1) u = (A_1B_1)L_1P_1u, since the pair (M_1Q_1,S_1T_1) are occasionally weakly compatible so by definition there exists a point v$\in X$ such that

M_1Q_1v = S_1T_1 v and

M_1Q_1 (S_1T_1)v = S_1T_1 (M_1Q_1)v
Hence L_1P_1u=A_1B_1u=M_1Q_1v =S_1T_1v
Hence, if there is one more point z such that L_1P_1z=A_1B_1z then using (1) it follow that
L_1P_1z=A_1B_1z=M_1Q_1v =S_1T_1v

we assert that d (L_1P_1z, M_1Q_1v) = 0

Suppouse on the contradicton assume that d (L_1P_1z, M_1Q_1v) >0

$$\tau + F\big(d(L_1P_1z, M_1Q_1v)\big)$$
$$\leq F\left(max\begin{Bmatrix} d(L_1P_1z, M_1Q_1v), d(A_1B_1z, M_1Q_1v), d(A_1B_1z, S_1T_1v), d(L_1P_1z, A_1B_1z)d(M_1Q_1v, S_1T_1v), \\ d(L_1P_1z, S_1T_1v), \end{Bmatrix}\right)$$

$$\tau + F\big(d(L_1P_1z, M_1Q_1v)\big) \leq F(max\{d(L_1P_1z, M_1Q_1v), 0,0,0,0,0\})$$

$$\tau + F\big(d(L_1P_1z, M_1Q_1v)\big) \leq F(d(L_1P_1z, M_1Q_1v))$$

that is a contradiction, hence d (L_1P_1z, M_1Q_1v) = 0 which shows that L_1P_1z = M_1Q_1v

henceL_1P_1z=MQz, M_1Q_1v =S_1T_1v

So, L_1P_1u=L_1P_1z and w= L_1P_1u=A_1B_1u is a single point of coincident of L_1P_1 and A_1B_1

From the lemma , w is a unique common fixed point of L_1P_1 , A_1B_1 i. e. w= L_1P_1w=A_1B_1w

Similarly there is a unique point z$\in X$ such that z=M_1Q_1z=S_1T_1z

Distinctiveness:

Assume that w$\neq z$ taking inequality (1) for x=w, y=z we obtain

$$\tau + F\big(d(L_1P_1w, M_1Q_1z)\big)$$
$$\leq F\left(max\begin{Bmatrix} d(L_1P_1w, M_1Q_1z), d(A_1B_1w, M_1Q_1z), d(L_1P_1w, A_1B_1w), d(M_1Q_1z, S_1T_1z), d(A_1B_1w, S_1T_1z), \\ d(L_1P_1w, S_1T_1z) \end{Bmatrix}\right)$$

$$\leq F(max\{d(L_1P_1w, M_1Q_1z), d(L_1P_1w, M_1Q_1z), d(L_1P_1w, M_1Q_1z), 0,0\})$$

$\tau + F\big(d(w,z)\big) \leq F(d(w,z)$a contradiction, therefore $d(w,z) = 0$ hence w=z

Therefore z is an unique common fixed point of the mappings L_1P_1, M_1Q_1, A_1B_1 and S_1T_1

Lastly we should show that z is a common fixed point of A, B, S, T, L, M, P and Q

Again taking x=z, y= S_1 z in (1) with the assumption that d (L_1P_1z ,$M_1Q_1S_1$z)$\neq 0$,

From condition (iii) we have

F (d (L_1P_1z, $M_1Q_1S_1$z)) = F (d (z, S_1z))

$$\leq F\left(max\begin{Bmatrix} d(L_1P_1z, M_1Q_1S_1z), d(A_1B_1z, M_1Q_1S_1z), d(A_1B_1z, S_1T_1S_1z), \\ d(L_1P_1z, A_1B_1z), d(M_1Q_1S_1z, S_1T_1Sz), \\ d(L_1P_1z, S_1T_1S_1z) \end{Bmatrix}\right) - \tau$$

= F (max$\{d(z, S_1z), d(z, S_1z), 0,0, d(z, S_1z)\}) - \tau$

= F (d (z, S_1 z))- $\tau < F\big(d(z, S_1z)\big)$

That is a contradiction

Hence d (L_1P_1z, $M_1Q_1S_1z$) = d (z, S_1z) = 0, i.e., z=S_1 z

Thus z=S_1S z=S_1T_1z=T_1S_1z=T_1Z

Similarly we also can show that

z= Az =A_1B_1z=B_1A_1 z=B_1z

Since P_1 z=$P_1{}^2$z, Q_1 z=$Q_1{}^2$z and L_1P_1=P_1L_1, M_1Q_1=Q_1M_1

One has z=L_1P_1z=$L_1P_1P_1$ z=pL_1P_1z=P_1z $\Longrightarrow L_1z = z$

z=M_1Q_1z=$M_1Q_1Q_1$ z=$Q_1M_1Q_1$z=Q_1z $\Longrightarrow M_1z = z$

Therefore in the view of above foresaid, we have

z= A_1z = B_1z= L_1z = M_1z = S_1z = T_1z = P_1z = Q_1z

Which shows that A_1, B_1, S_1, T_1, L_1, M_1, P_1 and Q_1has a common fixed point *z* in X

Now we try to show that this common fixed point is unique

Take up that W is another common fixed point of A_1, B_1, S_1, T_1, L_1, M_1, P_1 and $Q_1$with w$\neq z$.It follows that

w= A_1w = B_1w = L_1w = M_1w = S_1w = T_1w = P_1w = Q_1w

taking x=z and y=w in (1) we have

F (d (L_1P_1z, M_1Q_1w))

$$\leq F\left(max\begin{Bmatrix} d(L_1P_1z, M_1Q_1w), d(A_1B_1z, M_1Q_1w), d(A_1B_1z, S_1T_1w), \\ d(L_1P_1z, A_1B_1z), d(M_1Q_1w, S_1T_1w), \\ d(L_1P_1z, S_1T_1w), \end{Bmatrix}\right) - \tau$$

=$F(max\{d(z,w), d(z,w), d(z,z), d(w,w), d(z,w)\}) - \tau$

= F (max{d (z, w),d(z,w),d(z,w),0,0,d(z,w)}) - τ

< F (d (z, w))

That is a contradiction

Therefore d (z, w) =0, i. e z=w

Thus A_1, B_1, S_1, T_1, L_1, M_1, P_1 and Q_1has an unique common fixed point *z* in X

The proof is whole

Assume g=I_X (or f=I_X) in theorem3.1, we can acquire common fixed point results in seven maps and its coincidence

Corolary 3.1. (Y, d) is a cone metric space from which self maps P, Q, R, S, T , U and V are taken let the pairs (TV, PQ) and (U, TL) fulfill ($CLR_{(PQ)(TL)}$) property , Ciric type F_M-contraction, i.e., there is F$\in \mathcal{F}_M$ and $\tau > 0$, for every x, y$\in Y$ *with* d(Tfx, Uy) >0,

$$\tau + F\big(d(Tfx, Uy)\big) \leq F\left(\max\left\{\begin{array}{c} d(Tfx, PQx), d(TLy, Uy), d(PQx, TLy), \\ \dfrac{d(PQx, Uy) + d(Tfx, TLy)}{2} \end{array}\right\}\right) \quad (9)$$

Then a common fixed point exists for (T, PQ) and (U, TL)

And also

(a) (TV, PQ) and (U,TL) are occasionally weakly compatible;

(b)PQ = QP, Tf=fT, TfP=PTf;

(c)TL=LT, UgL=LUg;

(d)Vx =V^2x, for all x$\in Y$;

henceP, Q, R, S, T , U and V will have exclusive common fixed point in Y.

common fixed points for six self maps can be attained by taking V=g=I_x in theorem 3.1

Corolary 3.2.

In a cone metric space (Y,d) we take the self maps as P, Q, R, S, T , U . (T, PQ) and (U, RS) satisfy ($CLR_{(PQ)(RS)}$) as they are Ciric type F_M-contraction, i.e. , we have $\mathcal{F} \in F_M$, $\tau > 0$ so that, for all x, y$\in Y$ with d(Tx, Uy) >0,

$$\tau + F\big(d(Tx, Uy)\big) \leq F(\max\left\{\begin{array}{c} d(Tx, PQx), d(RSy, Uy), d(PQx, RSy), \\ \frac{d(PQx,Uy)+d(Tx,TLy)}{2} \end{array}\right\}) \quad (3.3)$$

Then both pairs (T, PQ) and (U, RS) will have a common fixed point.

and

(a) (T,PQ) , (U,RS) are occasionally weakly compatible;
(b) PQ = QP, TP=PT;
(c) RS=SR,UR=RS;

Hence P, Q, R, S, T , U has a unique common fixed point in Y.

Coincident and common fixed points for five self maps can be achieved if we take T=I_x in corollary 3.2,

Corollary 3.3.

In a cone metric space (Y, d) we take the self maps as P, Q, R, S, T , U. (T, PQ) and (U, RS) satisfy ($CLR_{(PQ)(RS)}$) property and they are Ciric F_M-contraction, i.e , we have $\mathcal{F} \in F_M$ and$\tau > 0$ so that, for all x, y$\in Y$ with d(Tx, Uy) >0,

$$F\big(d(Tx, Uy)\big) \leq F\left(\max\left\{\begin{array}{c} d(Tx, PQx), d(Ry, Uy), d(PQx, Ry), \\ \frac{d(PQx,Uy)+d(Tx,RTy)}{2} \end{array}\right\}\right) - \tau \quad (3.4)$$

Then both pairs (S, PQ) and (U, R) has a common fixed point.

and

(a) (S,PQ) , (U,R) are occasionally weakly compatible;
(b) PQ = QP, SP=PS;
(c) RU=UR;

Hence P, Q, R, S, T , U have a unique common fixed point in Y.

outcomes for four self-maps, can be obtained if B =T=I_X in corollary 3.3which is listed as follows:

Corollary3.4.
In (Y, d) cone metric space the self maps P, Q, R, S, T are taken, (S, P) and (T, R) fulfill (CLR $_{(PR)}$) property, Cirictype F_M-contraction. (S, P) , (T, R) has a common fixed point and also are weakly compatible, SP=PS, TR=RT, so that P, Q, R, S, T has a unique common fixed point in Y.

The following examples are given to suppourt the key results.

Exaample 3.1. Assume Y = [1,∞) and d be a Euclidean metric given by d(x, y) =$|x-y|$, for every

x, y$\in Y$. Define A, B, S, T, L, M: Y → Y by

$$Ay=\begin{cases}2, \text{if } y=1,2\\ 5, \text{if } y\in[1,3)-\{1,2\}\\ 1, \text{if } y\geq 3\end{cases}, By=\begin{cases}2, \text{if } y=1,2\\ 4, \text{if } y\in[1,3)-\{1,2\}\\ 1, if\ y\geq 3\end{cases}$$

$$Sy=\begin{cases}2, if\ y=1,2\\ 8, if\ y\in[1,3)-\{1,2\}\\ 1, if\ y\geq 3\end{cases} Ty=\begin{cases}2, \text{if } y=1,2\\ 3, \text{if } y\in[1,3)-\{1,2\},\\ 1, \text{if } y\geq 3\end{cases}$$

$$Ly=\begin{cases}2, \text{if } y=1,2\\ 5, \text{if } y\in[1,3)-\{1,2\},\\ 1, \text{if } y\geq 3\end{cases} My=\begin{cases}2, \text{if } y=1,2\\ 9, \text{if } y\in[1,3)-\{1,2\},\\ 1, \text{if } y\geq 3\end{cases} gy$$
$$=\begin{cases}2, \text{if } y=1,2\\ 7, \text{if } y\in[1,3)-\{1,2\},\\ 1, \text{if } y\geq 3\end{cases} fy=\begin{cases}2, \text{if } y=1,2\\ 6, \text{if } y\in[1,2)-\{1,2\},\\ 1, \text{if } y\geq 3\end{cases}$$

Hence,

$$Lfy=\begin{cases}2, \text{if } y=1,2 \text{ and } y\geq 3\\ 1, \text{if } y\in[1,3)-\{1,2\}\end{cases} Mgy=\begin{cases}2, \text{if } y=1,2 \text{ and } y\geq 3\\ 1, \text{if } y\in[1,3)-\{1,2\}\end{cases}$$

sequences are$\{x_n\}$ and $\{y_n\}$ in Y and $x_n=3+\frac{1}{n}$,y_n =2 and

$$\lim_{n\to\infty} Lfx_n=\lim_{n\to\infty} Lf(3+\tfrac{1}{n})=2,$$

$$\lim_{n\to\infty} ABx_n=\lim_{n\to\infty} AB(3+\tfrac{1}{n})=2,$$

And

$$\lim_{n\to\infty} Mgy_n=\lim_{n\to\infty} Mg(2)=2,$$

$$\lim_{n\to\infty} STy_n=\lim_{n\to\infty} ST(2)=2,$$

Hence, $\lim_{n\to\infty} Lfx_n = \lim_{n\to\infty} ABx_n = \lim_{n\to\infty} Mgy_n = \lim_{n\to\infty} STy_n = 2,\ 2 \in AB(X) \cap ST(X)$, i. e

(Mg, ST) , (Lf, AB) satisfies the (CLR $_{(AB)\,(ST)}$) property.

hence, Lfy= ABy =2, Mgy=STy=2, where y$\in \{1,2,\}$, y$\geq 3, i.e,$ (Mg, ST) (Lf, AB) , have coincident points in Y.

Also ,LfABy= ABLfy, MgSTy = STMgy, where y$\in \{1,2\}$ and y$\geq 3, i.e$ (Mg, ST) ,(Lf, AB) are occasionally weakily compatible and $\mathrm{AB} = \mathrm{BA}, \mathrm{fL} = \mathrm{Lf}, \mathrm{Mg} = \mathrm{gM}, \mathrm{ST} = \mathrm{TS}, \mathrm{LfA} = \mathrm{ALf}, \mathrm{MgS} = \mathrm{SMg}$,$f^2$=f, g^2=g. also, A, B, S, T, L, M, f , g satisfy ciric type F_M-contraction assumption (5) for $\tau = \ln 3$ and F($\propto$) = ln $\propto$

Therefore y=2 will be an unique common fixed point of A, B, S, T, L, M, f , g

4. Conclusion

Our key results demonstrated fixed points using (CLR $_{(AB)\ (ST)}$) property by implementing eight maps on a cone metric space. Occasionally weakly compatibility condition is more weakly than commutativity of the maps. F- Contraction mapping, an appropriate overview of Banach contraction, our outcomes shorten, outspread and expand results of Wardowski [25], various other authors in the literature.

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