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PROPERTIES AND MATRIX SEQUENCES OF DERIVED K- JACOBSTHAL, DERIVED K- JACOBSTHAL LUCAS

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Abstract. In this paper we delineate Derived k- Jacobsthal and Derived k- Jacobsthal Lucas Matrix Sequences. We analyze some properties of Derived k- Jacobsthal and Derived k- JacobsthalLucas, we show some relationship between them.[1,2,3,4,5].

KEYWORDS: Derived k- Jacobsthal and Derived k- Jacobsthal Lucas Matrix Sequences.

NOTATIONS: $D\hat{j}_{k,n}$, $D\hat{c}_{k,n}$, $DJ_{k,n}$, $DC_{k,n}$

1. Introduction

Determined k-Jacobsthal, Derived k-Jacobsthal Lucas Sequences are talked about by not many of the authors. In this examination we investigate a portion of the properties of Derived k-Jacobsthal, Derived k-Jacobsthal Lucas. [6,7,8,9]Then by utilizing these arrangements we likewise characterize Derived k-Jacobsthal Matrix Sequences, Derived k-Jacobsthal Lucas Matrix Sequences .Integer sequences such as Fibonacci, Lucas ,Jacobsthal, Jacobsthal Lucas ,Pell charm us with their abundant applications in science and arts and very interesting properties[10,11,12]. Many of these properties are deduced from elementary matrix algebra. In this study, we define Derived k-Jacobsthal ,Derived k-Jacobsthal Lucas Sequences[13,14].Then by using these sequences we also define Derived k-Jacobsthal Matrix Sequences , Derived k-Jacobsthal Lucas Matrix Sequences. We discuss some properties of these sequences[15].

2. Definition

For n∈ N, any positive real number k, the ***Derived k- Jacobsthal Sequence***$\{D\hat{j}_{k,n}\}$ is defined by

$$D\hat{j}_{k,n+1} = kD\hat{j}_{k,n} - 2D\hat{j}_{k,n-1} \quad for\ n \geq 1$$

(1)

$$with\ initial\ condition\ D\hat{j}_{k,0} = 0, D\hat{j}_{k,1} = 1$$

First few terms of Derived k-Jacobsthal sequences are given by

$$D\hat{j}_{k,0} = 0, D\hat{j}_{k,1} = 1, D\hat{j}_{k,2} = k, D\hat{j}_{k,3} = k^2 - 2, D\hat{j}_{k,4} = k^3 - 4k, D\hat{j}_{k,5} = k^4 - 6k^2 + 4$$

.

For n∈ N, k>0 any real number then ***Derived k- Jacobsthal Lucas Sequence*** $\{D\hat{c}_{k,n}\}$ is defined

by $D\hat{c}_{k,n+1} = kD\hat{c}_{k,n} - 2D\hat{c}_{k,n-1} \quad for\ n \geq 1$

$with\ initial\ condition\ D\hat{c}_{k,0} = 2, D\hat{c}_{k,1} = k,$ (2)

First few terms of Derived k-Jacobsthal Lucas sequences are

$$D\hat{c}_{k,0} = 2, D\hat{c}_{k,1} = k, D\hat{c}_{k,2} = k^2 - 4, D\hat{c}_{k,3} = k^3 - 6k, D\hat{c}_{k,4} = k^4 - 8k^2 + 8$$

3. BINET FORMULA

For $n \geq 0$ any integer, the Binet formula for n^{th} Derived k- Jacobsthal, Derived k-Jacobsthal Lucas number are given by

$D\hat{j}_{k,n} = \frac{r_1^n - r_2^n}{r_1 - r_2}$; $D\hat{c}_{k,n} = r_1^n + r_2^n$ where $r_1 = \frac{k+\sqrt{k^2-8}}{2}$; $r_2 = \frac{k-\sqrt{k^2-8}}{2}$; (3)

The characteristic Equation associated to (1) is $x^2 = kx - 2$ .We can easily seen $r_1.r_2 = 2$, $r_1 + r_2 = k$,

$r_1 - r_2 = \sqrt{k^2 - 8}$.

4. MAIN RESULTS

1. PROPERTIES OF DERIVED k-JACOBSTHAL AND DERIVED k-JACOBSTHAL LUCAS

THEOREM 1:1 - D'OCAGNE'S PROPERTY FOR DERIVED k-JACOBSTHALANDDERIVED k-JACOBSTHAL LUCAS

a. $D\hat{j}_{k,m} D\hat{j}_{k,n+1} - D\hat{j}_{k,m+1} D\hat{j}_{k,n} = 2^n D\hat{j}_{k,m-n}$ For $m \geq n$ and $n, m \in z^+$

Proof:

By Using (3)

$D\hat{j}_{k,m} D\hat{j}_{k,n+1} - D\hat{j}_{k,m+1} D\hat{j}_{k,n} = \frac{r_1^m - r_2^m}{r_1 - r_2} \cdot \frac{r_1^{n+1} - r_2^{n+1}}{r_1 - r_2} - \frac{r_1^{m+1} - r_2^{m+1}}{r_1 - r_2} \cdot \frac{r_1^n - r_2^n}{r_1 - r_2}$

$= (r_1 r_2)^n \frac{r_1^{m-n} - r_2^{m-n}}{r_1 - r_2}$.

$= 2^n D\hat{j}_{k,m-n}$

b. For $m \geq n$ and $n, m \in z^+$ we have

$D\hat{c}_{k,m+1} D\hat{c}_{k,n} - D\hat{c}_{k,m} D\hat{c}_{k,n+1} = \sqrt{(k^2 - 8)} 2^n D\hat{c}_{k,m-n}$

Proof:

By Using (3)

$D\hat{c}_{k,m+1} D\hat{c}_{k,n} - D\hat{c}_{k,m} D\hat{c}_{k,n+1} = (r_1^{m+1} + r_2^{m+1})(r_1^n + r_2^n) - (r_1^m + r_2^m)(r_1^{n+1} + r_2^{n+1})$

$= (r_1 r_2)(r_1^n\ r_2^n)(r_1^{m-n} + r_2^{m-n})$

$= \sqrt{(k^2 - 8)} 2^n D\hat{c}_{k,\ m-n}$

THEOREM 1:2- CATALAN'S PROPERTY FOR DERIVED k-JACOBSTHAL AND DERIVED

k-JACOBSTHAL LUCAS

a. $D\hat{j}_{k,n+r} D\hat{j}_{k,n-r} - D\hat{j}^2_{k,n} = (-1) 2^{n-r} D\hat{j}^2_{k,r}$ For $n, r \in z^+$

Proof:

$D\hat{j}_{k,n+r} D\hat{j}_{k,n-r} - D\hat{j}^2_{k,n} = \frac{r_1^{n-r} - r_2^{n-r}}{r_1 - r_2} \frac{r_1^{n+r} - r_2^{n+r}}{r_1 - r_2} - \frac{(r_1^n - r_2^n)^2}{(r_1 - r_2)}$

$= \frac{1}{(r_1 - r_2)^2} (-1) 2^n \left(\frac{r_1^{2r} + r_2^{2r}}{r_1^r r_2^r} - 2 \right)$

$= (-1) 2^{n-r} D\hat{j}^2_{k,r}$

b. For $n, r \in z^+$ we have $D\hat{c}_{k,n+r} D\hat{c}_{k,n-r} - D\hat{c}_{k,n}{}^2 = 2^{n-r}(k^2 - 8)$

Proof:

Doing the same procedure as in the above we can prove it

THEOREM 1:3 - CASSINI 'S PROPERTIY OR SIMPSON PROPERTY FOR DERIVED k-JACOBSTHAL AND DERIVED k-JACOBSTHAL LUCAS

Put r = 1 in Catalan's property we get Cassini's property

For n,r$\in z^+$ we have

$D\hat{j}_{k,n+1} D\hat{j}_{k,n-1} - D\hat{j}^2_{k,n} = (-1)\, 2^{n-1}$

$D\hat{c}_{k,n+1} D\hat{c}_{k,n-1} - D\hat{c}_{k,n}{}^2 = 2^{n-1}(k^2-8)$

RELATION BETWEEN DERIVED k-JACOBSTHAL AND DERIVED k-JACOBSTHAL LUCAS AND THE ROOTS α, β

1.$\alpha^n = \alpha D\hat{j}_{k,n} - 2D\hat{j}_{k,n-1}$

2.$\beta^n = \beta D\hat{j}_{k,n} - 2D\hat{j}_{k,n-1}$

3.$\sqrt{k^2-8}\,\alpha^n = \alpha D\hat{c}_{k,n} - 2D\hat{c}_{k,n-1}$

4.-$\sqrt{k^2-8}\beta^n = \beta D\hat{c}_{k,n} - 2D\hat{c}_{k,n-1}$

Proof:

2. $\beta D\hat{j}_{k,n} - 2D\hat{j}_{k,n-1} = \beta\frac{\alpha^n-\beta^n}{\alpha-\beta} - 2\frac{\alpha^{n-1}-\beta^{n-1}}{\alpha-\beta}$

$=\frac{1}{\alpha-\beta}(\beta(\alpha^n-\beta^n) - 2(\alpha^{n-1}-\beta^{n-1}))$

$=\frac{1}{\alpha-\beta}(-\beta^{n+1}+2\beta^{n-1}) = \beta^n$

3. $\alpha D\hat{c}_{k,n} - 2D\hat{c}_{k,n-1} = \alpha(\alpha^n+\beta^n) - 2(\alpha^{n-1}+\beta^{n-1})$

$= \alpha^{n+1} - 2\alpha^{n-1} = \alpha^n(\alpha-\beta) = \sqrt{k^2-8}\,\alpha^n$

Other proofs can be done in a similar way.

THEOREM 1.4

The limit of the quotient of two consecutive terms of Derived k-Jacobsthal and Derived k-Jacobsthal Lucas sequences are

$\lim_{n\to\infty}\frac{D\hat{j}_{k,n+1}}{D\hat{j}_{k,n}} = \alpha$, $\lim_{n\to\infty}\frac{C\hat{j}_{k,n+1}}{C\hat{j}_{k,n}} = \alpha$

Proof:

By Binet formula

$$\lim_{n\to\infty}\frac{D\hat{j}_{k,n+1}}{D\hat{j}_{k,n}} = \lim_{n\to\infty}\frac{\alpha^{n+1}-\beta^{n+1}}{\alpha^n-\beta^n}$$

$=\frac{1-(\frac{\beta}{\alpha})^{n+1}}{\frac{1}{\alpha}-(\frac{\beta}{\alpha})^{n+1}\frac{1}{\beta}} = \alpha$

Taking into the account that $|\beta| < \alpha$, since $\lim_{n\to\infty}(\frac{\beta}{\alpha})^n = 0$.

THEOREM 1:5 GENERATING FUNCTIONS

a. ***DERIVED k- JACOBSTHAL***

$$\sum_{i=0}^{n} D\hat{j}_{k,i}\, x^i = \frac{x}{1-kx+2x^2}$$

Proof:

Let us suppose that the Derived k-Jacobsthal numbers are the coefficient of a power series centred at

the origin and consider the corresponding analytic function$D\hat{j}_k(x)$ the function defined in such a

way is called the generating function of the Derived k-Jacobsthal number.

$$D\hat{j}_k(x) = D\hat{j}_{k,0} + D\hat{j}_{k,1}x + D\hat{j}_{k,2}x^2 + \cdots + D\hat{j}_{k,n}x^n$$

$$kxD\hat{j}_k(x) = kD\hat{j}_{k,0} + k\,D\hat{j}_{k,1}x^2 + D\hat{j}_{k,2}x^3 + \cdots + kD\hat{j}_{k,n}x^{n+1}$$

$$2x^2D\hat{j}_k(x) = 2D\hat{j}_{k,0}x^2 + 2\,D\hat{j}_{k,1}x^3 + \cdots + 2D\hat{j}_{k,n}x^{n+2}$$

$$since\ D\hat{j}_{k,n+1} = kD\hat{j}_{k,n} - 2D\hat{j}_{k,n-1} \quad for\ n \geq 1 \quad with\ initial\ condition\ D\hat{j}_{k,0} = 0, D\hat{j}_{k,1} = 1$$

(1- kx + 2x^2) $D\hat{j}_k(x)$ = x

Hence generating function of Derived k- Jacobsthal is

$$\sum_{i=0}^{n} D\hat{j}_{k,i}\, x^i = \frac{x}{1 - kx + 2x^2}$$

b. ***DERIVED k- JACOBSTHAL LUCAS***

$$\sum_{i=0}^{n} C\hat{j}_{k,i}\, x^i = \frac{2 - kx}{1 - kx + 2x^2}$$

Proof:

Doing the same procedure as in the above we get the result.

THEOREM 1:6 - THE EXPONENTIAL GENERATING FUNCTIONS OF DERIVED k- JACOBSTHAL.

DERIVED k- JACOBSTHAL LUCAS

$\sum_{i=0}^{\infty} D\hat{j}_{k,i}\, \frac{x^i}{i!} = \frac{1}{\sqrt{k^2-8}}\left(e^{\alpha x} - e^{\beta x}\right)$

Proof:

$$\sum_{i=0}^{\infty} D\hat{j}_{k,i}\, \frac{x^i}{i!} = \sum_{i=0}^{\infty} \frac{\alpha^i - \beta^i}{\alpha - \beta} \frac{x^i}{i!}$$

$$= \frac{1}{\sqrt{k^2 - 8}} \sum_{i=0}^{\infty} \frac{(\alpha x)^i - (\beta x)^i}{i!}$$

$= \frac{1}{\sqrt{k^2-8}}\left(e^{\alpha x} - e^{\beta x}\right)$

a. $\sum_{i=0}^{\infty} D\hat{c}_{k,i}\, \frac{x^i}{i!} = e^{\alpha x} + e^{\beta x}$

Proof :

Doing the same procedure as in the above we get the result.

THEOREM 1:7(**Generating function for the Equidistant elements of Derived k- Jacobsthal Sequence and Derived k- Jacobsthallucas Sequence)**

Let n ≥ 0 any integer and $|\alpha^i x| < 1$ and $|\beta^i x| < 1$ then

$$\sum_{n=0}^{\infty} D\hat{j}_{k,in}\, x^n = \frac{D\hat{j}_{k,i}\, x}{1 - D\hat{c}_{k,i}\, x + 2^i x^2}$$

$$\sum_{n=0}^{\infty} \mathrm{D\hat{c}_{k,in}}\, x^n = \frac{2 + x(\alpha^i + \beta^i)\, \mathrm{D\hat{j}_{k,i}}}{1 - \mathrm{D\hat{c}_{k,i}}\, x + 2^i x^2}$$

Proof:

Using (3) and geometric series we obtain

$$\sum_{n=0}^{\infty} \mathrm{D\hat{j}_{k,in}}\, x^n = \sum_{n=0}^{\infty} \frac{(\alpha^{in} - \beta^{in})}{(\alpha - \beta)} x^n$$

$=\frac{1}{(\alpha-\beta)}\left(\frac{1}{1-\alpha^i x} - \frac{1}{1-\beta^i x}\right)$

$=\frac{1}{(\alpha-\beta)}\left(\frac{(\alpha^i-\beta^i)x}{1-x(\alpha^i+\beta^i)+x^2 2^i}\right)$

$= \frac{\mathrm{D\hat{j}_{k,i}}\, x}{1-\mathrm{D\hat{c}_{k,i}}\, x+2^i x^2}$

Using the same procedure as in the above we get the result for Derived k-Jacobsthal Lucas

EXPLICIT EXPRESSION FOR CALCULATING THE GENERAL TERM OF THE DERIVED

K - JACOBSTHAL SEQUENCE

$$\mathrm{D\hat{j}_{k,n}} = \frac{1}{2^{n-1}} \sum_{i=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \binom{n}{2i+1} k^{n-1-2i} (k^2 - 8)^i$$

Where $\lfloor a \rfloor$ is the floor function of a

Proof:

$$\mathrm{D\hat{j}_{k,n}} = \frac{1}{\sqrt{k^2-8}} \left(\left(\frac{k + \sqrt{k^2-8}}{2} \right)^n - \left(\frac{k - \sqrt{k^2-8}}{2} \right)^n \right)$$

$=\frac{1}{\sqrt{k^2-8}} \left\{ \frac{k^n}{2^n} \left(\left(1 + \frac{\sqrt{k^2-8}}{k}\right)^n - \left(1 - \frac{\sqrt{k^2-8}}{k}\right)^n \right) \right\}$

$=\frac{1}{\sqrt{k^2-8}} \left\{ \frac{k^n}{2^n} \left\{ \binom{2n}{1} \frac{\sqrt{k^2-8}}{k} + \binom{2n}{3} \left(\frac{\sqrt{k^2-8}}{k}\right)^3 + \cdots \right\} \right\}$

$=\frac{1}{2^{n-1}} \sum_{i=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \binom{n}{2i+1} k^{n-1-2i} (k^2 - 8)^i$

1. ***DERIVED k- JACOBSTHAL AND DERIVED k- JACOBSTHAL LUCAS MATRIX SEQUENCES***

DEFINITION

By the definition of Derived k- Jacobsthal , we define ***Derived k- Jacobsthal Matrix Sequence***

For n∈ N, k>0 any real number$(\mathrm{DJ_{k,n}})_{n\in N}$ is defined by

$DJ_{k,n+1} = kDJ_{k,n} - 2DJ_{k,n-1} \quad for\ n \geq 1$

(4)

With initial condition $\mathrm{DJ_{k,0}} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $\mathrm{DJ_{k,1}} = \begin{pmatrix} k & -2 \\ 1 & 0 \end{pmatrix}$

DEFINITION

By the definition of Derived k-Jacobsthal Lucaswe define ***Derived k- Jacobsthallucas Matrix***

Sequence For n∈ N, k > 0 any real number,$(\mathrm{DC_{k,n}})_{n\in N}$ is defined by

$DC_{k,n+1} = kDC_{k,n} - 2DC_{k,n-1} \quad for\ n \geq 1$ (5)

With initial condition $DC_{k,0} = \begin{pmatrix} k & 4 \\ -2 & -k \end{pmatrix}$ $DC_{k,1} = \begin{pmatrix} k^2-4 & -2k \\ k & -4 \end{pmatrix}$

Derived k-Jacobsthal$(\mathrm{DJ}_{k,n})_{n\in N}$ Derived k-JacobsthalLucas matrix sequences $(\mathrm{DC}_{k,n})_{n\in N}$ are defined by carrying to matrix theory k-Jacobsthal, k-Jacobsthal Lucas sequences.

The following theorem shows the n^{th} ***general term of the Derived k-Jacobsthal matrix sequences,***

Derived k-Jacobsthal Lucas matrix sequences given in (4),(5)

THEOREM:2.1 For n∈N, k>0 any real number we have $DJ_{k,n} = \begin{pmatrix} D\hat{J}_{k,n+1} & -2D\hat{J}_{k,n} \\ D\hat{J}_{k,n} & -2D\hat{J}_{k,n-1} \end{pmatrix}$ (6)

Proof:

Using Principle of Mathematical Induction We are going to prove this theorem.

Let us consider n=1 in(6)

We know that $D\hat{J}_{k,0} = 0, D\hat{J}_{k,1} = 1, D\hat{J}_{k,2} = k$,

$$DJ_{k,1} = \begin{pmatrix} D\hat{J}_{k,2} & -2D\hat{J}_{k,1} \\ D\hat{J}_{k,1} & -2D\hat{J}_{k,0} \end{pmatrix}$$

$$DJ_{k,1} = \begin{pmatrix} k & -2 \\ 1 & 0 \end{pmatrix}$$

For n=2 we get

$$DJ_{k,2} = \begin{pmatrix} k^2-2 & -2k \\ k & -2 \end{pmatrix}$$

Let us assume that the equality holds for all m ≤ n∈ Z^+

To end up the proof we have to show that the case holds for n+1

$DJ_{k,n+1} = kDJ_{k,n} - 2DJ_{k,n-1}$,

$= k\begin{pmatrix} D\hat{J}_{k,n+1} & -2D\hat{J}_{k,n} \\ D\hat{J}_{k,n} & -2D\hat{J}_{k,n-1} \end{pmatrix} - 2\begin{pmatrix} D\hat{J}_{k,n} & -2D\hat{J}_{k,n-1} \\ D\hat{J}_{k,n-1} & -2D\hat{J}_{k,n-2} \end{pmatrix}$

$= \begin{pmatrix} kD\hat{J}_{k,n+1} - 2D\hat{J}_{k,n} & -2(kD\hat{J}_{k,n} - 2D\hat{J}_{k,n-1}) \\ kD\hat{J}_{k,n} - 2D\hat{J}_{k,n-1} & -2(kD\hat{J}_{k,n-1} - 2D\hat{J}_{k,n-2}) \end{pmatrix}$

$= \begin{pmatrix} D\hat{J}_{k,n+2} & -2D\hat{J}_{k,n+1} \\ D\hat{J}_{k,n+1} & -2D\hat{J}_{k,n} \end{pmatrix}$

COROLLORY (i) For m,n∈ N, Det $DJ_{k,n}{}^{m} = 2^m$

Proof:

For m = 1 it can be easily seen that $(DJ_{k,n}) = 2$

We can write $DJ_{k,n}{}^{m} = DJ_{k,n}. DJ_{k,n}.DJ_{k,n}.DJ_{k,n}.........DJ_{k,n}$

$=2. 2. 2.......2=2^m$

COROLLORY (ii) Let $M^{n} = \begin{pmatrix} D\hat{J}_{k,2n+1} & -2D\hat{J}_{k,2n} \\ D\hat{J}_{k,2n} & -2D\hat{J}_{k,2n-1} \end{pmatrix}$ be defined as in (6) then $\mathrm{Det}(M^{n}) = 2^{2n}$

Proof :

It can be easily done by Principle of Mathematical Induction.

THEOREM:2.2 For n∈N, k>0 any real number we have $DC_{k,n}=\begin{pmatrix} D\hat{c}_{k,n+1} & -2D\hat{c}_{k,n} \\ D\hat{c}_{k,n} & -2D\hat{c}_{k,n-1} \end{pmatrix}$ (7)

Proof:

Applying Principle of Mathematical Induction and Using $DC_{k,n+1} = kDC_{k,n} - 2DC_{k,n-1}$,

doing same procedure as in Theorem :2.1we can prove it.

THEOREM :2.3 For m,n ∈ N, k>0 any real number then $DJ_{k,m+n} = DJ_{k,m}.DJ_{k,n}$,

Proof:

It's proven by Principle of Mathematical Induction

Applying $DJ_{k,0} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $DJ_{k,n+1} = kDJ_{k,n} - 2DJ_{k,n-1}$ and

Executing some mathematical simplifications we can prove this theorem.

THEOREM :2.4 For n∈N, k> 0 any real number then we have $DC_{k,n+1} = DC_{k,1}.DC_{k,n}$

Proof:

This theorem can be proven by Principle of Mathematical Induction.

THEOREM :2.5 For n∈N, k> 0 any real number then we have $DC_{k,n} = kDJ_{k,n} - 4DJ_{k,n-1}$

Proof:

It's proven by Principle of Mathematical Induction.

THEOREM :2.6*(Commutative Property)* For m, n ∈N, k> 0 any real number we have $DJ_{k,m}\, DC_{k,n+1}=DC_{k,n+1}DJ_{k,m}$

Proof:

$DJ_{k,m}\, DC_{k,n+1} = DJ_{k,m} \,.\, DC_{k,1}.\, DJ_{k,,n}$

$= DJ_{k,m} .(kDJ_{k,1} - 4DJ_{k,0})\, DJ_{k,,n}$

$=kDJ_{k,1}.DJ_{k,m+n} - 4DJ_{k,m+n}$

$=DJ_{k,m+n}\left(kDJ_{k,1} - 4DJ_{k,0}\right)$

$= DC_{k,1}DJ_{k,n}.DJ_{k,m}$

$= DC_{k,n+1}DJ_{k,m}$

THEOREM :2.7 For any integer n ≥ 1 we get $DJ_{k,n} = DJ_{k,1}^{n}$

Proof:

This theorem can be prove by Principle of Mathematical Induction

THEOREM :2.8 For n ≥ 0 any integer we have

a. $DC^2_{k,n+1}=DC^2_{k,1}.DJ_{k,2n}$

b. $DC^2_{k,n+1}=DC_{k,2n+1}$

c. $DC^2_{k,2n+1}=DJ_{k,n}\, DC_{k,n+1}$

Proof:

a. $DC^2_{k,n+1}= DC_{k,n+1}\, DC_{k,n+1}= DC_{k,1}DJ_{k,n}.\, DC_{k,1}DJ_{k,n}= DC^2_{k\,,\,1}\, DJ_{k,2n}$

b. $DC^2_{k,n+1}= DC^2_{k,1}.\, DJ_{k,2n}= DC_{k,1}DC_{k,1}\, DJ_{k,2n}= DC_{k,1}DC_{k\,,2n+1}$

c. $DC_{k,2n+1}= DC_{k,1}\, DJ_{k,2n}= DJ_{k,n}\, DC_{k,n+1}$

THEOREM :2.9 For $n \geq 0$ $DJ_{k,n} = \left(\frac{DJ_{k,1} - r_2 DJ_{k,0}}{r_1 - r_2}\right) r_1^n - \left(\frac{DJ_{k,1} - r_1 DJ_{k,0}}{r_1 - r_2}\right) r_2^n$

Proof:

$$\left(\frac{DJ_{k,1} - r_2 DJ_{k,0}}{r_1 - r_2}\right) r_1^n - \left(\frac{DJ_{k,1} - r_1 DJ_{k,0}}{r_1 - r_2}\right) r_2^n$$

$$= \frac{r_1^n}{(r_1 - r_2)} \begin{pmatrix} k - r_2 & -2 \\ 1 & -r_2 \end{pmatrix} - \frac{r_2^n}{(r_1 - r_2)} \begin{pmatrix} k - r_1 & -2 \\ 1 & -r_1 \end{pmatrix}$$

$$= \frac{1}{(r_1 - r_2)} \begin{pmatrix} k(r_1^n - r_2^n) - r_1 r_2 (r_1^{n-1} - r_2^{n-1}) & 2(r_1^n - r_2^n) \\ (r_1^n - r_2^n) & -r_1 r_2 ((r_1^{n-1} - r_2^{n-1}) \end{pmatrix}$$

$$\begin{pmatrix} D\hat{J}_{k,n+1} & -2D\hat{J}_{k,n} \\ D\hat{J}_{k,n} & -2D\hat{J}_{k,n-1} \end{pmatrix} = DJ_{k,n}$$

9. Conclusion

In this paper we discussed some properties of Derived k –Jacobsthal , Derived k Jacobsthal Lucas .We have derived amazing relationship between Derived k-Jacobsthal, Derived k Jacobsthal Lucas, and its Matrices.

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