

# Meta-heuristic algorithms with immigrant techniques for nurse duty roster in public hospitals in Sindh, Pakistan.

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## Abstract

The integration of immigrant approaches to the meta-heuristics is a trending research topic to increase evolutionary algorithm's efficiency for a variety of real-world optimization problem. Several researchers have focused on optimization problems with soft and hard constraints for example University timetabling, nurse rostering, job scheduling, vehicle routing and many more. Three proposed immigrant schemes have been studied in this paper; these techniques are integrated with meta-heuristic algorithms to generate better population within reasonable time to improve the individual's quality. Experiments are conducted to compare the proposed immigrant approaches with canonical meta-heuristic on different wards(1-to-5) for nurse rostering problem of public hospitals in Sindh, Pakistan. The experimental result validates those two out of three proposed techniques get better performance on nurse rostering and overall better in all wards on NSP.

## 1 Introduction

Genetic algorithms [1] are population-based search approach having a genetic inheritance and natural evolution. It has shown that these are powerful techniques for solving many real-world combinatorial optimization problems for example job scheduling, vehicle routing, DNA sequence alignment and traveling salesman problems with static environment due to their properties of easy-to-use and robustness for finding good solutions to difficult problems [1, 10, 21, 19].

Optimization is the process to find the optimal solution to the objective function under the constraints. Hard and soft constraints are compulsory that must be satisfied. Optimization is useful in several fields. The basic concept of optimization may be different in different fields.

In today's world, one of the problems faced by health care globally is Nurse Rostering Problem (NRP). This problem is a highly constrained combinatorial optimization problem. Several NRPs have been explained in the literature [2, 3, 4], which deal with the allocation of shifts and days-off to nurses of public hospitals in Sindh, Pakistan. The goal is to generate roster that would cover all time round the clock while

considering the personal preferences of nurses. If the nurses are not satisfied with the roster, it has a direct effect on the services given to patients by them.

During the research, it has been observed that the public hospitals of Sindh in Pakistan are still applying the manual nurse duty roster approaches. The survey of manual system of staff nurse roster has been conducted in different hospitals of Sukkur, Shaheed Benazirabad and Hyderabad divisions. Manual staff scheduling is a time-consuming task and unbalanced approach of assigning duties of the nurses in different wards in the public hospitals of Sindh. The proposed meta-heuristic algorithms (including GA, DE and PSO) are used for creating automatic nurse/staff scheduling at different wards of the public hospitals. Each algorithm utilizes three different immigrant schemes to increase the effectiveness of algorithm [5,6].

## 2 Literature Review

### 2.1 Nurse Scheduling Problem NSP

The different variants of Nurse Scheduling problem (NSP) were found in the literature that showed that some researchers have solved that problem by using different meta-heuristics. NSP is a specific form of the combinatorial optimization problem, which belongs to NP-hard problem. Hard and soft constraints are basic components of the NSP.

In [7], BCO algorithm has been applied for an NRP comprising three hard and 14 soft constraints. If any soft constraint is violated that must be penalized. The conclusion was to minimize the penalties and to give the comparison with different algorithm with proposed BCO.

An NRP having joint normalized shift and day off preference satisfaction, containing manpower, day off and shift requirements were studied by Lin et al [2]. To calculate the preference of each nurse for his/her shift, work shift weight and day-off weight was used by them. Moreover, a genetic algorithm with immigrant scheme (GAIS) was developed by them and the results were compared with recovery scheme based on 20 to 100 nurses.

Wu et al. [8] developed a PSO based NRP, which start with a mathematical model and then proposed a technique

for creating work stretch patterns that could be used as templates in searches for initial individuals.

Hybrid meta-heuristic algorithms were developed by Jin et al [9], these approaches are combined with the harmony search(HS) and artificial immune systems(AIS)to make a hybrid schemes. The experimental results can be seen, that the performance of hybrid is better than all presented algorithms in this paper.

In [8], a two-stage heuristic technique was proposed for NRP in the emergency department of a hospital and mathematical programming model was mentioned for it. In the first step, randomly generated solution was proposed by using schedule assignment algorithm that satisfied all of the hard constraints. In second, A local search method was used to locate for the neighborhood solutions to reduce the penalties incurred from soft constraint violations. The outcome of this research proposed a two-stage heuristic approach to solve NRPs.

Recently, two heuristic algorithms were developed for NRP by researchers [10], which are the decision tree method and the greedy search algorithm, these approaches are integrated with meta-heuristics algorithms in order to generate better initial solutions in less time and to improve solutions' quality.

## 2.2 Genetic Algorithms (GAs)

Genetic Algorithm (GA), is an evolutionary algorithm developed by John Holland and his coworkers in 1960s and 1970 [1]. GAs was inspired by Darwin's theory of survival. This algorithm belongs to class of numerical and combinatorial optimization solving problems and represent one area of research studies called evolutionary computations. The basic idea of GA is to evolve and optimize (i.e., minimize or maximize) the problem's overall cost. It imitates the biological process of reproduction and natural selections in which it takes randomly generated solutions, technically called individuals or initial population, and process these all individuals under genetic operators called selection, crossover, and mutation operators. The cost (fitness values) of processed population is then evaluated and individuals with optimum cost value is passed for next round as new population. This process continues to a fix number called number of generations.

## 2.3 Particle swarm Optimization (PSO)

Particle Swarm Optimizations (PSO), is an iterative type, computational science algorithm that tries to optimize the solution concerning given measures of desired qualities. It was introduced with intention to simulate the social behaviors of birds flock and fish school, by Kennedy and Eberhard in 1995 [11, 12]. It then become most popular techniques applied in various global optimization problems, because of its ease and ability to search local optimum solution using current dimensions. PSO is also based on two main models, exploration (global best) and exploitation (local best), and addition of few parameters, these parameters are given in the following section.

After implementation of many real world and academic problem in PSO, different modified version of PSO has been introduced. In simple PSO, each particle is represented by  $i$  having a position vector  $\vec{x}_i$  and velocity vector  $\vec{v}_i$  both of these vector quantities will be updated using equation 1 and equation 2, respectively.

$$v_i'^d = \omega v_i^d + n_1 r_1 (x_{pbest_i}^d - x_i^d) + n_2 r_2 (x_{gbest_i}^d - x_i^d) \quad (1)$$

$$x_i'^d = x_i^d + v_i'^d \quad (2)$$

In equation 1, and 2,  $v_i'^d$  and  $x_i^d$  are current and previous position of particle  $i$ , respectively in the dimension.  $v_i'$  and  $v_i$  are the current and previous velocities of particle  $i$  respectively.  $\omega \in (0,1)$  is an inertia weight, which determines the velocity change factor,  $x_{pbest_i}^d$  and  $x_{gbest_i}^d$  is the  $i$  particle so far best-known position and best position found by the whole population,  $n_1$  and  $n_2$  both are the acceleration constant, and  $r_1$  and  $r_2$  are randomly generated real values in between 0 and 1.

## 2.4 Differential Evolution

In the 1997 [13], DE was introduced by Rainer Storn and Kenneth Price for solving optimization problem over the continuous domain it was considered as the search heuristic technique by them which is simple,ease of use and speed and robustness. DE is one of the best evolutionary algorithms for solving problems with the real valued variables and it has been successfully applied in the field of Science, engineering, numerical and combinatorial optimization problems [14, 15, 16].

DE is a population-based heuristic optimization strategy in which a randomly created population is utilized to choose all of the solutions from the population to be the  $N$  parents using a general selection scheme. The  $N$  parents are mutated to produce new children. These new children are evaluated, and  $N$  survivors are selected from the current population to make the next generation. The main motivation behind DE is a new approach for creating tail parameter vectors.

## 3 Research Methodology

### 3.1 Proposed Meta-heuristic Scheme for NRP

Evolutionary computing is a quickly developing region. It assumes a significant part inside the field of Computer Science. Evolutionary computing utilizes computational models of evolutionary cycles as key components for planning and carrying out PC based critical thinking frameworks. A few developmental calculation models have been proposed and considered in the literature, which is known as evolutionary algorithms [1, 12, 13]. They utilize normal properties of simulating the development of individual constructions through the cycle of determination and recombination.

According to the literature, the NRPs are NP-hard optimization problem, and it may never find polynomial time

solution and different heuristics or meta-heuristics have been applied to solve these problems efficiently to "search for the sub-optimal solution". Meta-heuristic is a very generic approach in nature and It can often be applied for different problems along with slight modification. The three meta-heuristics presented here are Genetic algorithms (GAs), Differential evolution (DE ), and Particle swarm optimization (PSO). In addition, three immigrants schemes are also combined with GAs, DE, and PSO respectively. These algorithms are applied to solve the nurse duty roster.

**Problem Description** In this section, we describe the multi ward nursing scheduling problem along with the mathematical model, its constraints, and solution representation. This is a minimization type optimization problem, so the solution with the minimum cost will be considered best over one with maximum cost. Cost calculation or objective function or fitness function is such that on violation of any soft and hard constraints (discussed in section 3.2), solution will be penalized with 1 and 10 numbers, respectively, and sum of all the penalties will be the cost of that particular instance. Designed the schedule for multi ward nurse rostering problem for one week. In each day of week there are three shifts, i.e. Morning, Evening, and Night with specific number of nurses required in each shift. For each ward there are distinct number of nurses working-in. Each nurse has its own working and off shift preferences.

**Constraints** In this problem, there are two sets of constraints for a valid solution to avoid violations which are hard and soft constraint, respectively, shown in Table 1.

Table 1: Basic constraints

Type	Constraint
Hard Constraints	I. Consecutive shift violations, $C_1$ , II. Shift per week violations, $C_2$ , III. Nurses per shift violations, $C_3$ .
Soft Constraints	I. Shift preferences violations, $C_4$

I. Consecutive shift violations, represented by  $C_1$  penalized to solution when any nurse in the solution is assigned more than one consecutive shift, calculated as shown in the equation 3.

$$C_1 = \sum_{i=0}^z x_i, \text{ where } x = \begin{cases} 1, & \text{if } s_j = s_{j+1} = 1 \\ 0, & \text{if } s_j = 0 \mid s_{j+1} = 0 \end{cases} \quad (3)$$

II. Shifts per week violations, denoted by  $C_2$  is penalized when any nurse is assigned shifts more allowed shifts per week, as given in equation 4.

$$C_2 = \sum_{i=0}^z x_i, \text{ where } x = \begin{cases} 1, & \text{if } n_{assign} > n_{max} \\ 0, & \text{if } n_{assign} \leq n_{max} \end{cases} \quad (4)$$

Where  $z$  is number of shifts in solution,  $n_{assign}$  and  $n_{max}$  are number of assigned shifts, and maximum allowed shifts per nurse in a week, respectively.

III. Nurse per shift violations, represented by  $C_3$  is penalized over solution when in any shift, number of assigned nurses is greater than number of maximum allowed nurses, as shown in equation 5.

$$C_3 = \sum_{i=0}^z x_i, \text{ where } x = \begin{cases} 1, & \text{if } y_{assign} > y_{max} \\ 0, & \text{if } y_{assign} \leq y_{max} \end{cases} \quad (5)$$

Where  $z$  represents the number of shifts in solution,  $y_{assign}$  and  $y_{max}$  are number of assigned and maximum allowed nurses per shift, respectively.

IV. Shift preference violations, represented by  $C_4$  is penalized when sum of shifts assigned to nurses not as per one's preferences, as shown in equation 6.

$$C_4 = \sum_{i=0}^z x_i, \text{ where } x = \begin{cases} 1, & \text{if } p_{shift} \neq 1 \\ 0, & \text{if } p_{shift} = 1 \end{cases} \quad (6)$$

### 3.2 Objective Function

Objective function or Fitness function  $f$  schedule  $s$  can be calculated using the following equation 7.

$$f(s) = hCP * hCV + sCV \quad (7)$$

where  $hCP$  is hard Constraint Penalty and can be any positive integer,  $hCV$  is hard Constraints Violations is total sum of  $C_1$ ,  $C_2$ , and  $C_3$ , and  $sCV$  is soft Constraints Violations is sum of  $C_4$ .

### 3.3 Solution Representation

In table2, there is given a schedule by the public government hospitals from three different divisions (Sukkur, Nawabshah, and Hyderabad) for one week. There are three shifts in each day, i.e. Morning (M), Evening (E), Night(N), and Day off (O). The generalized weekly schedule of nurses in public hospital in above three division is given in Table 3.

Table 2: Sample duty roster of Nurses in Government hospitals in Sindh.

	$W_{1,1}$	$W_{1,2}$	$W_{1,3}$	$W_{1,4}$	$W_{1,5}$	$W_{1,6}$	$W_{1,7}$	...	$W_{w,[1...7]}$
$X^{1,1}$	M	O	M	E	E	M	O	...	$W_{w,[1...7]}$
$X^{1,2}$	M	E	O	O	E	O	O	...	$W_{w,[1...7]}$
$X^{1,3}$	O	O	O	E	O	E	O	...	$W_{w,[1...7]}$
$X^{1,4}$	E	E	E	M	O	O	E	...	$W_{w,[1...7]}$
$X^{1,5}$	N	O	N	O	M	N	N	...	$W_{w,[1...7]}$
$X^{1,6}$	O	M	M	M	O	E	M	...	$W_{w,[1...7]}$
$X^{1,7}$	E	M	O	N	N	O	M	...	$W_{w,[1...7]}$
$X^{1,8}$	O	N	E	O	M	M	E	...	$W_{w,[1...7]}$
$X^{2,1}$	O	M	M	E	E	O	M	...	$W_{w,[1...7]}$
$X^{2,2}$	M	M	E	O	M	O	M	...	$W_{w,[1...7]}$
$X^{2,3}$	N	O	O	N	O	M	N	...	$W_{w,[1...7]}$
$X^{2,4}$	E	O	E	E	M	E	O	...	$W_{w,[1...7]}$
$X^{2,5}$	O	N	E	O	N	O	N	...	$W_{w,[1...7]}$
$X^{2,6}$	O	M	M	M	O	M	E	...	$W_{w,[1...7]}$
$X^{2,7}$	E	E	N	O	E	O	E	...	$W_{w,[1...7]}$
$X^{2,8}$	M	E	O	M	O	E	E	...	$W_{w,[1...7]}$
...	...	...	...	...	...	...	...	...	...
$X^{M,n}$	$x_{w,1}^{M,n}$	$x_{w,2}^{M,n}$	$x_{w,3}^{M,n}$	$x_{w,4}^{M,n}$	$x_{w,5}^{M,n}$	$x_{w,6}^{M,n}$	$x_{w,7}^{M,n}$	...	$W_{w,[1...7]}$

Table 3 shows the generalized duty roster of nurses of Government hospitals in Sindh.

	$W_{1,1}$	$W_{1,2}$	$W_{1,3}$	$W_{1,4}$	$W_{1,5}$	$W_{1,6}$	$W_{1,7}$	...	$W_{w,[1...7]}$
$X^{1,1}$	$x_{1,1}^{1,1}$	$x_{1,1}^{1,2}$	$x_{1,3}^{1,1}$	$x_{1,4}^{1,1}$	$x_{1,5}^{1,1}$	$x_{1,6}^{1,1}$	$x_{1,7}^{1,1}$	...	$W_{w,[1...7]}$
$X^{1,2}$	$x_{1,1}^{1,2}$	$x_{1,2}^{1,2}$	$x_{1,3}^{1,2}$	$x_{1,4}^{1,2}$	$x_{1,5}^{1,2}$	$x_{1,6}^{1,2}$	$x_{1,7}^{1,2}$	...	$W_{w,[1...7]}$
$X^{1,3}$	$x_{1,1}^{1,3}$	$x_{1,3}^{1,2}$	$x_{1,2}^{1,3}$	$x_{1,4}^{1,3}$	$x_{1,5}^{1,3}$	$x_{1,6}^{1,3}$	$x_{1,7}^{1,3}$	...	$W_{w,[1...7]}$
...	...	...	...	...	...	...	...	...	...
$X^{2,1}$	$x_{1,1}^{2,1}$	$x_{1,2}^{2,1}$	$x_{1,3}^{2,1}$	$x_{1,4}^{2,1}$	$x_{1,5}^{2,1}$	$x_{1,6}^{2,1}$	$x_{1,7}^{2,1}$	...	$W_{w,[1...7]}$
$X^{2,2}$	$x_{1,1}^{2,2}$	$x_{1,2}^{2,2}$	$x_{1,3}^{2,2}$	$x_{1,4}^{2,2}$	$x_{1,5}^{2,2}$	$x_{1,6}^{2,2}$	$x_{1,7}^{2,2}$	...	$W_{w,[1...7]}$
$X^{2,3}$	$x_{1,1}^{2,3}$	$x_{1,2}^{2,3}$	$x_{1,3}^{2,3}$	$x_{1,4}^{2,3}$	$x_{1,5}^{2,3}$	$x_{1,6}^{2,3}$	$x_{1,7}^{2,3}$	...	$W_{w,[1...7]}$
...	...	...	...	...	...	...	...	...	...
$X^{M,n}$	$x_{w,1}^{M,n}$	$x_{w,2}^{M,n}$	$x_{w,3}^{M,n}$	$x_{w,4}^{M,n}$	$x_{w,5}^{M,n}$	$x_{w,6}^{M,n}$	$x_{w,7}^{M,n}$	...	$W_{w,[1...7]}$

3.4 Genetic algorithms with immigrants schemes for NRP  
 The simple GA is the process of evolving a population of candidate solution via selection and variation. A set of solution of population is first initialized and then evolved from

generation to generation by an iterative process of evolution, selection, recombination, and mutation. This process continues until some termination condition becomes true, for example, the maximum allowable number of generations is reached.

The property of immigrant approaches to maintaining diversity in the population and jump-out from local optima during the evolutionary process. These immigrant schemes are combined with conventional GAs to maintain diversity and to enhance the performance of GA towards getting optimum

solution within reasonable time. By using this hybrid approach to solve the NRP problem. The Pseudo-code of the GA with one of the immigrant techniques in this report, is also shown in algorithm 1. Random with random immigrants denoted by RND\\_RNDI, however only one of them shall execute at one time.

**3.5 Particle swarm optimization with immigrants for NRP**  
To find the settings or parameters required to maximize a specific objective, to explore the search space for the particular problem a mechanism is used for this purpose, which is called PSO which is an inspiration from the social behavior of organisms because of its bird flocking and fish schooling attributes. It has been observed that many particles "fly" in the search space in this mechanism. Tracking of best positions achieved by each article is maintained along with the best positions of the surrounding neighbors and this

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#### Algorithm 1 GA with an immigrant scheme

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1. Randomly generate an initial population pop
  2. Evaluate the fitness of each individual of pop
  3.  $t := 0$
  4. while  $t < \text{max\_gen do}$
  5.   for each individual  $i$  in pop do
  6.     Select individual  $j$  by the roulette wheel method
  7.     Crossover individual  $i$  with individual  $j$  using the two point crossover method
  8.     Mutate individual  $i$  by using the bit flip method
  9.   end for
  10. Apply immigrant scheme(from RND\\_RNDI or Elitism or RND\\_WRST) with GA
  11. Apply repair method
  12. end while
- 

information is also shared with the counterparts. To increase the ratio of success for finding the global optima, researchers have suggested a few models in which gbest and lbest models are specified for the original PSO. This meta\\_heuristic approach to solve the NRP problem.

The Pseudocode of the PSO with one of the immigrant techniques in this report, is also shown in algorithm 2. Random with random immigrants denoted by RND\\_RNDI and Elitism and RND\\_WRST.

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#### Algorithm 2 PSO with an immigrant scheme

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1. Create the initial swarm by randomly generating the positions and velocities for each particle.
2. Evaluate the fitness of each particle
3.  $t := 0$
4. repeat
5.   for each particle  $i$  do

6.   update particle  $i$  according to equation 1 and 2
  7.   if  $f(\vec{x}_i) < f(\vec{x}_{pbest_i})$  then
  8.      $(\vec{x}_{pbest_i}) := \vec{x}_i$
  9.   if  $f(\vec{x}_i) < f(\vec{x}_{gbest})$  then
  10.      $(\vec{x}_{gbest}) := \vec{x}_i$
  11.   end if
  12. end if
  13. end for
  14. Apply immigrant scheme(from RND\\_RNDI or Elitism or RND\\_WRST) with PSO
  15. Apply repair method
  16. until The stop condition is satisfied.
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**3.6 Differential evolution with immigrants for NRP**  
Differential evolution (DE) is an effective tool, which is the class of stochastic search method for solving optimization problems. Nowadays, DE has been successfully applied for solving many optimization problems such as Wind energy, numerical optimization, Science, and constraint problems [15, 16, 17] due to the easy-to-use and robustness. In DE, number of trail vector generation strategies have been used out of which some may be suitable for solving a specific problem. The effectiveness of DE depends on several parameters such as initial population size  $NP$ , Scaling factor  $FF$  and crossover rate (CR) may significantly impact on the optimization performance of the DE.

Several evolutionary algorithms have been developed to deal with NSP. The general approaches that are applied are linear programming, meta\\_heuristics, and artificial intelligence schemes. Recently, many strategies have been suggested and integrated with DE to maintain a high-quality solution with reasonable time. Immigrant scheme has been integrated to DE, to find the optimized result during the evolutionary process. The detailed description of DE with immigrant schemes has been shown in algorithm 3.

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#### Algorithm 3 DE with an immigrant scheme

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1. Start
2. Randomly generate an initial population of size pop
3. Evaluate the fitness of each individual of pop
4. repeat
5.   for each individual  $j$  in pop do
6.     Generate three random individual  $r1, r2, r3 \in (0, \text{pop})$ , with  $r1 \neq r2 \neq r3 \neq j$
7.     Generate a random integer  $ir$  and  $ir \in (1, n)$
8.     for each parameter  $i$  do
9.        $x'_{i,j} = \begin{cases} x_{i,j} + F * (x_{i,r1} - x_{i,r2}) & \text{if } \text{rand}(0,1) < CR \text{ or } i = i_{\text{rand}} \\ x_{i,j} & \text{otherwise} \end{cases}$
10.    end for
11.    Replace  $x_i$  with the child  $x'_j$ , if  $x'_j$  is better
12. end for
13. Apply immigrant scheme(from RND\\_RNDI or Elitism or RND\\_WRST) with DE
14. Apply repair method

15. until The stop condition is satisfied

### 3.7 Computational power for experiments

The experiments were run using Intel(R) Xeon(R), @ 2.30GHz, 2 cores, Haswell family CPU, and 12GB of RAM with 25GB of disk Space on CPU only VMs provided by google co-laboratory. The python language was applied to implement the suggested methods. The detailed description of the parameters and operators of meta heuristics are given in the table 1 and 2.

The idea of immigrant techniques was taken from [6,18], which is successfully implemented in the several evolutionary algorithms. The population size(150) and total number of generations(300) are same for the proposed approaches for nurse scheduling problem, the total number of wards was set to 1 to 5. All these algorithms with different immigrant schemes were run 10 times independently on particular NSP.

Table4: Parameters of GA, PSO, and DE techniques

No.	Attributes	Values
1.	Population size	110
2.	Maximum generations	300
3.	Hard constraint penalty	10
4.	Soft constraint penalty	30
5.	Hall of fame	30
6.	Wards	$w \in [1,2,3,4,5]$

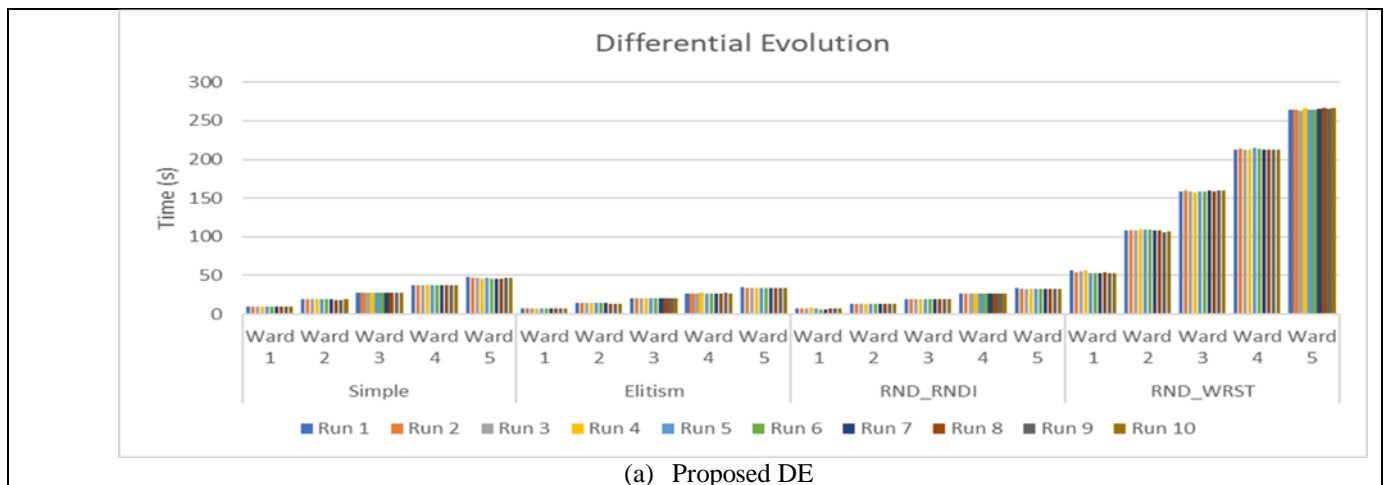
Figure 1: Execution Time (s) over ten independent runs.

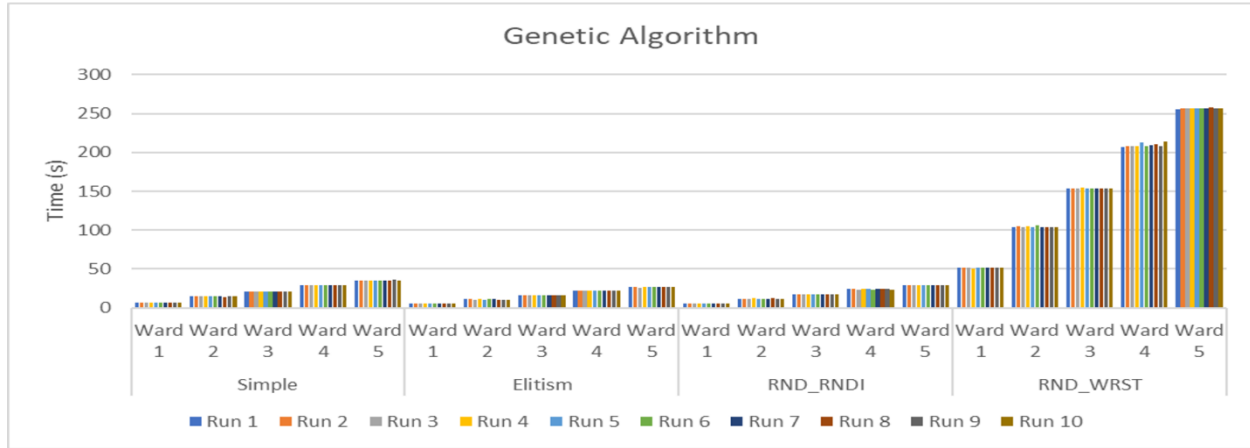
7.	Nurses in a single ward		8
8.	Shift in a day		3
9.	Nurses allowed for each shift	Minimum	Maximum
	I. SM	2	3
	II. SE	2	4
	III. SN	1	2

## 4 Result and Discussion

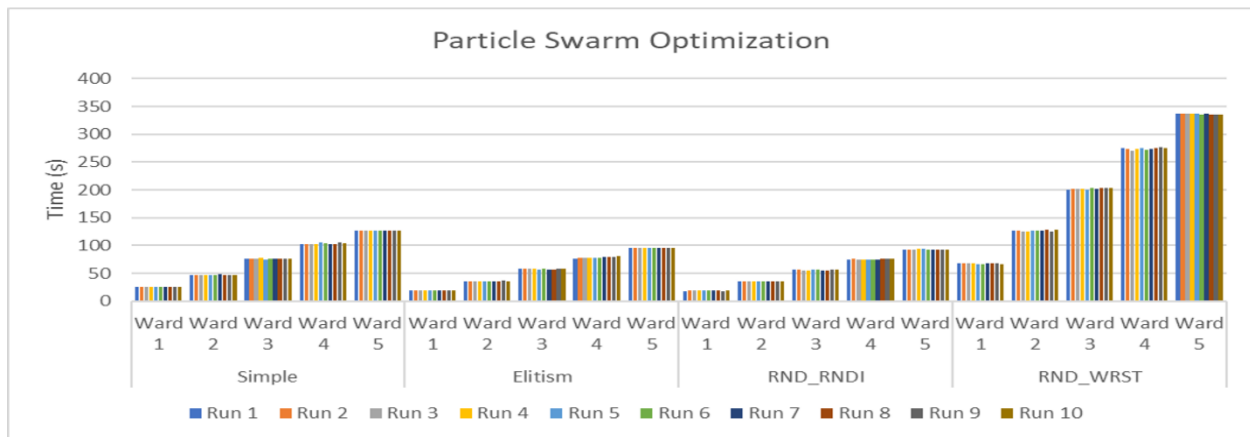
In this section the results of the experiments are presented for each algorithms with immigrant schemes and the experiments were divided into six phases; in first phase the execution time is plotted against independent runs, simple and immigrant approach wise for metaheuristic algorithm, solving 1,2,3,4, and 5 wards NSP, in figure 1 for GA, PSO, DE respectively.

In Figure 1, Execution time in seconds, for each independent run for 1-to-5 wards is plotted for three proposed approaches(such as Elitism, RND-RNDI, and RND-WRST) are given for GA, DE, and PSO. It can be observed in figure 1(a), 1(b), 1(c), from left to right as number of wards increases it causes uniform linear increase in time taken by each metaheuristic to generate the solutions, in each scheme. The result of Elitism and RND-RNDI of each algorithm is better than Simple and RND-WRST of metaheuristics on different wards of NSP during the evolutionary process in term of execution time(s) over the independent runs. RND-WRST immigrant approach is expensive in term of time over independent runs for GA,DE, and PSO respectively.





(b) Proposed GA

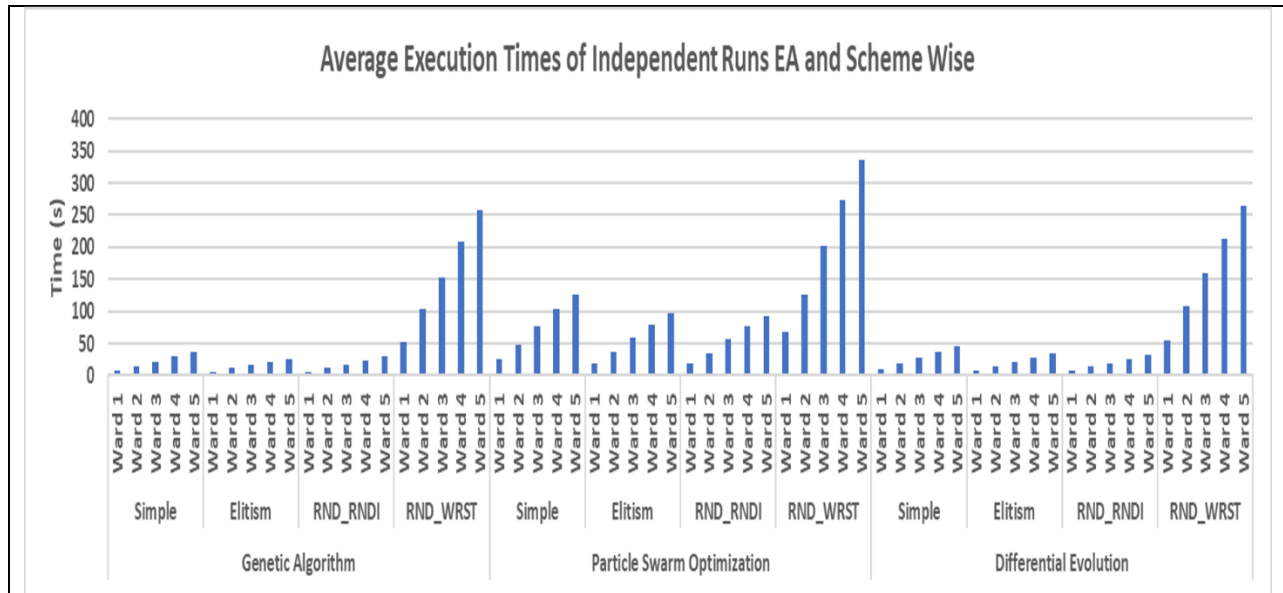


(c) Proposed PSO

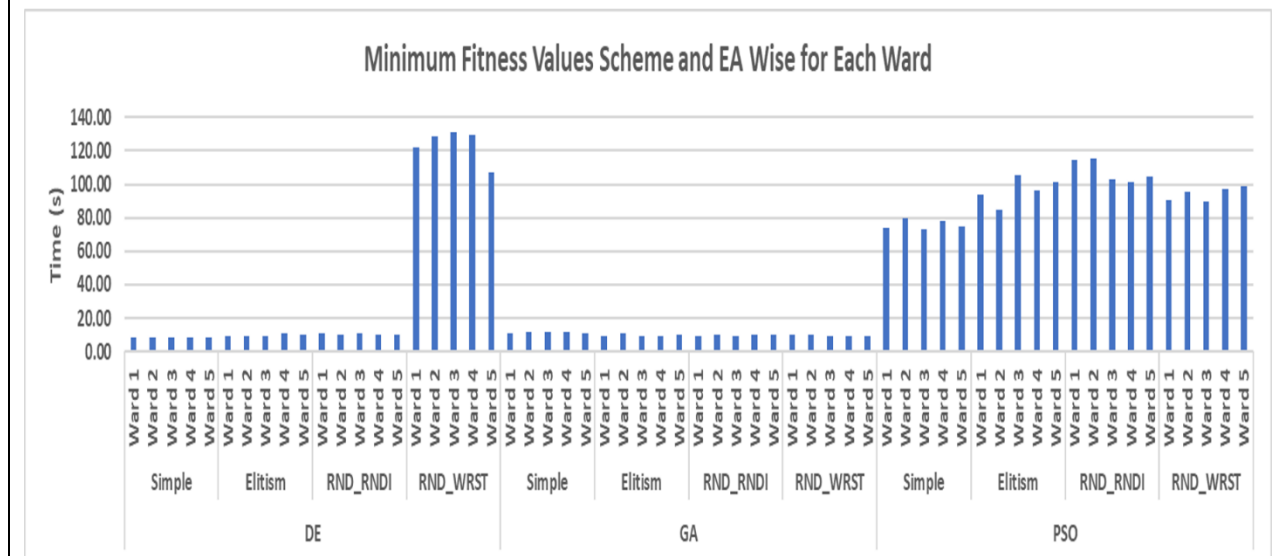
In Figure 2, each variants of proposed scheme are applied for different wards upto 5 wards for NRP. Average time spent in seconds for each of variants of suggested technique in plotted as shown in figure 2 (a). In Figure 2(b) present the evolutionary process for modified of suggested approaches of GA, DE, and

PSO on Ward 1 to Ward 5 respectively, where the result is presented in minimum fitness value over the independent runs. Figure 2(b), it can be seen that comparatively Elitism and RND RNDI takes less time during the process of each EA.

Figure 2: Average Execution Time and Mini: Fitness value over ten independent runs.



(a) Avg: Ext(T) for each ward



(b) Mini Fit(T) for each ward

Ward and technique wise minimum and mean fitness values are given in table 5. Two conclusions can be analyzed from table 5. First, Elitism and RND\\_RNDI is better performance among the different variants of Meta-heuristics on five wards except

simple PSO. Second, the number of ward is the important factor regarding the complexity of the NRP, increasing the number of wards the complexity of problem would be very high.

Table 5: Comparison minimum and mean fitness result of between GA, DE, and PSO with immigrant schemes on different wards of public hospital.

Algorithm	Wards	Simple		Elitism		RND_RNDI		RND_WRST	
		Minimum	Mean	Minimum	Mean	Minimum	Mean	Minimum	Mean
GA	1	11.30	12.40	9.50	10.93	9.60	12.99	10.40	12.07
	2	11.60	12.83	11.20	13.14	10.25	15.06	9.90	11.61
	3	12.03	13.50	8.90	10.07	9.53	14.04	9.53	10.76
	4	12.08	13.39	8.98	10.32	9.98	14.84	9.03	10.22
	5	11.04	12.18	9.90	11.22	9.98	13.93	9.40	11.27



DE	1	8.40	8.42	9.20	9.52	11.10	11.10	121.60	123.43
	2	8.20	8.23	8.90	9.16	9.90	9.92	128.60	130.98
	3	8.67	8.79	9.63	9.94	11.23	11.27	130.90	132.86
	4	8.55	8.61	10.70	11.00	10.28	10.34	129.50	131.43
	5	8.40	8.48	10.36	10.85	10.28	10.31	107.48	109.62
PSO	1	9.85	10.41	9.35	10.23	10.35	12.05	66.00	67.75
	2	9.90	10.53	10.05	11.15	10.08	12.49	69.25	71.29
	3	10.35	11.15	9.27	10.01	10.38	12.66	70.22	71.81
	4	10.31	11.00	9.84	10.66	10.13	12.59	69.26	70.82
	5	9.72	10.33	10.13	11.04	10.13	12.12	58.44	60.45

From figure 3, it can be observed that first column(left side of figure 3) and second column (right side of figure 3)represents the minimum fitness results of different meta-heuristic with immigrant schemes on ward-5, and ward-1 respectively, where the results of ward-5 and ward-1 are shown in a log scale. Minimum fitness convergence rate of Elitism and RND-WRST is better among four approaches in figure 3(a) on ward-5, somewhere in the middle of evolutionary process of GA with immigrants schemes shown better performance of Simple,RND-RNDI and RND-WRST on ward-1 and ward-5(see figure 3(a) and (b)). Over all the performance of RND-WRST is better on ward-5 and RND-RNDI is better on ward-1.

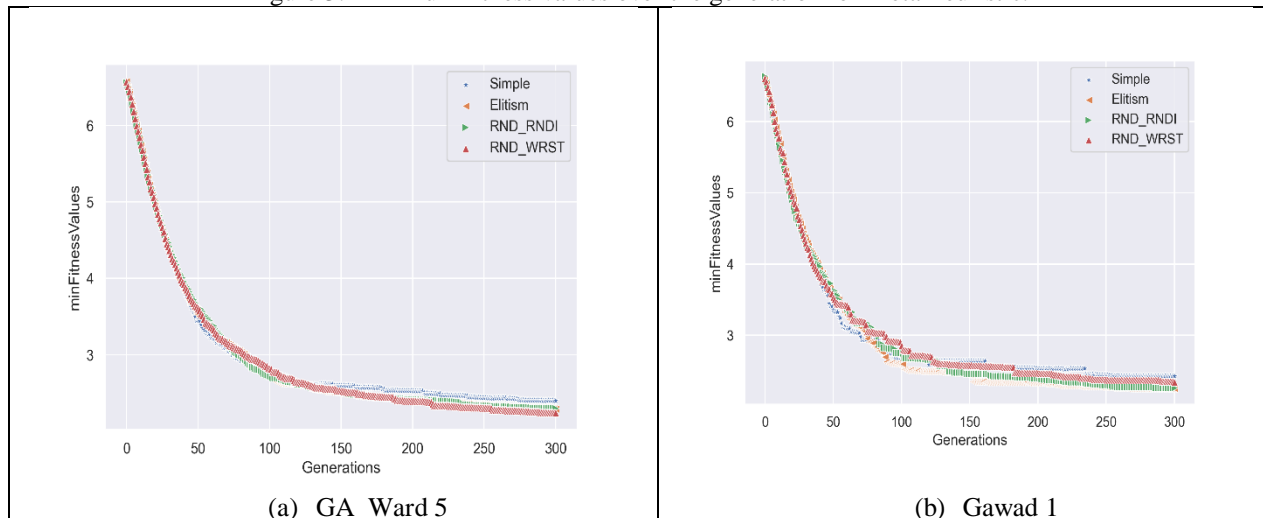
The convergence rate of simple DE is better on ward-5 during the process of iterations. RND-RNDI and Elitism approaches also show that they are close to the simple scheme during the evolutionary process of DE on ward-5 and elitism is better than RND-RNDI technique on ward-1.

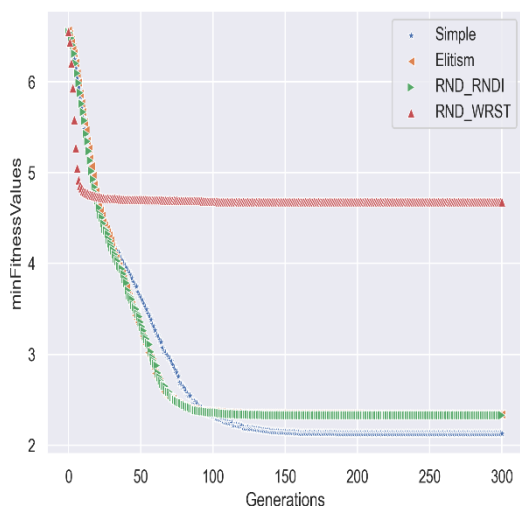
The result of minimum fitness curve of PSO using immigrant approaches plotted similar curves in graph except RND-WRST(see figure 3(e)). Figure 3(f) presents the evolutionary process for Simple, Elitism, RND-RNDI, and RND-WRST approaches on ward-1. It can be seen that the convergence curve of Elitism is faster than other three schemes on ward-1, and simple and RND-RNDI are close to the Elitism. RND-WRST scheme stuck into local optimum on different wards due to fast convergence curve which can be observed in figure 3(c,d,e,and f).

## 5 Conclusions

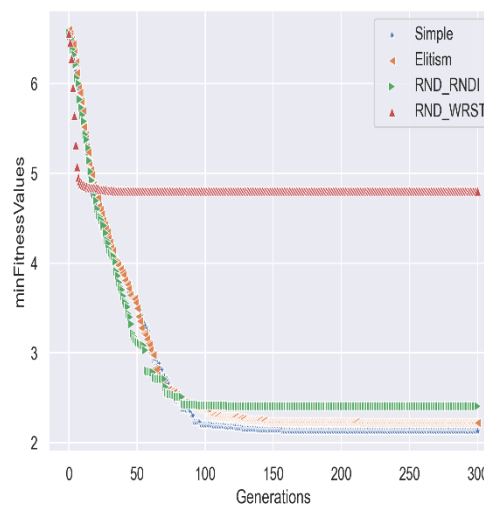
Various immigrants' approaches have been combined to different heuristics to cope the different problems [19, 5, 20, 21]. In this work, we integrate immigrants techniques with Meta heuristics such as GA, DE, and PSO to solve the nurse roster problem of public hospitals in Sindh, Paksitan. The name of these algorithms are RND \_RNDI, Elitism, RND WRST, where random immigrant replaced with random solutions, some percentage of elitism based immigrants are generated respectively, and replace

Figure 3: Minimum fitness values over the generation for Meta-heuristic.

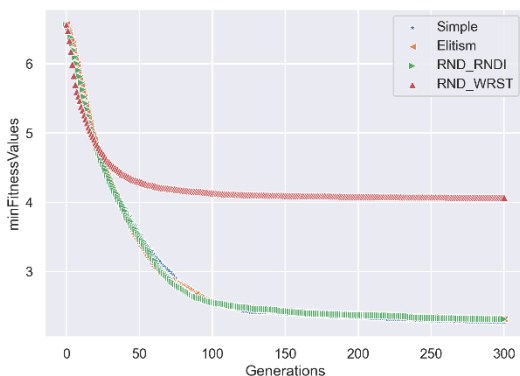




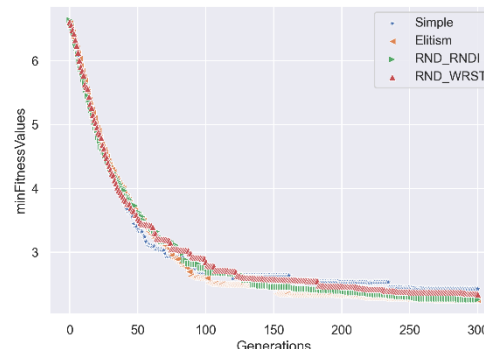
(c) DE\_Ward 5



(d) DE\_Ward 1



(e) PSO\_Ward 5



(f) PSO\_Ward 1

the worst individuals in the current population in order to add more information in the population and maintain the diversity of the population, which is important to find the optimum solution for NSP.

To analyze the experimental result, immigrants schemes enhance the performance of conventional GA, DE, PSO. the ability of random immigrant approach to jump out from local optimum due to the property of generating more diversity in the current population. The elitism and RND\_WRST immigrant approaches can able to get optimum solution quickly. Due to this reason it may create a chance to trap into local optimum.

After doing some experiments, it has been learned that Performance of Random and Random Elitism both is better than simple Meta heuristics and RND WRST Meta heuristics on Nurse Rostering and overall better in all wards on NSP.

#### Acknowledgment

This work was supported and funded by the Higher Education Commission of Pakistan under National Research Program for Universities (NRPU Project No 9907\sinh\NRPU\R&D\HEC\2017).

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