

Characterizations of Supra Generalized Preregular Closed Sets

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Abstract: In this paper we extend the study of gpr^μ - closed sets in a supra topological space. In particular we study the relation of gpr^μ - closed sets with various sets in supra topological space.

Keywords and Phrases: supra preclosed set, gpr^μ - closed set, supra extremally disconnected space.

1.Introduction: The concept of topological space grew out of the study of the real line and euclidean space and the study of continuous functions on these spaces. In topology the fundamental notion of generalized closed sets was introduced by N. Levine in 1970 [5]. This contribution made the topologists Bhattacharya et al [3], Balachandran et al [2], Maximilian Ganster [7] and Gnanambal et al [6] to investigate new concepts and results in general topology. In 1983, A. S. Mashour et al [9] introduced the supra topological spaces and studied S - continuous maps and S^* - continuous maps. The study on supra topological space was explored by several researchers. Devi et al [4], Sayed et al [11], Sayed [10], Ravi et al [8], Arockiarani et al [1] and Vidhya Menon [12] introduced supra α - open sets and S_α - continuous functions, supra b - open sets and supra b - continuity, supra preopen sets and supra pre - continuity on topological spaces, supra sg - closed sets and supra gs - closed sets, supra generalized b - regular closed sets and supra generalized preregular closed sets respectively. In this paper we study the characteristics of gpr^μ - closed sets.

Throughout this paper (X, τ) , (Y, σ) and (Z, η) (or simply X, Y and Z) denote topological spaces on which no separation axioms are assumed unless explicitly stated. Let (X, τ) be a topological space and μ be a supra topology on X. We call μ a supra topology associated with τ if $\tau \subset \mu$. (X, μ) is called a supra topological space. A subset A of (X, μ) is said to be supra preclosed [10] if $\text{cl}^\mu(\text{int}^\mu(A)) \subseteq A$. The complement of supra preclosed set is called supra preopen set [10]. The supra pre-closure of A, denoted by $\text{pcl}^\mu(A)$ is the intersection of the supra preclosed sets containing A. The supra pre-interior of A, denoted by $\text{pint}^\mu(A)$ is the union of the supra pre - open sets contained in A.

2. Preliminaries :

Definition 2.1: A subset A of a supra topological space (X, μ) is called

- supra generalized preclosed (briefly gp^μ - closed) [12] if $\text{pcl}^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is supra open in (X, μ) .

- supra generalized semi - closed (briefly gs^μ - closed) [1] if $scl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is supra open in (X, μ) .
- supra semi generalized closed (briefly sg^μ - closed) [1] if $scl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is supra semi open in (X, μ) .
- supra generalized - α closed (briefly $g\alpha^\mu$ - closed) [1] if $\alpha cl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is supra α - open in (X, μ) .
- supra generalized semi - preclosed (briefly gsp^μ) - closed) [12] if $spcl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is supra open in (X, μ) .
- supra generalized preregular closed (briefly gpr^μ) - closed) [12] if $pcl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is supra regular open in (X, μ) .
- supra generalized b - closed (briefly $g^\mu b$ - closed) [1] if $bcl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is supra open in (X, μ) .
- supra generalized b - regular closed (briefly $g^\mu br$ - closed) [1] if $bcl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is supra regular open in (X, μ) .

3. gpr^μ - closed sets

Theorem 3.1: For a supra topological space (X, μ) the following properties hold:

- Every $g\alpha^\mu$ - closed set is gpr^μ - closed.
- Every gpr^μ - closed set is $g^\mu br$ - closed.

Proof: Obvious.

However the converse of the above said theorems are not true.

Example 3.2:

- Let (X, μ) be a supra topological space where $X = \{a, b, c, d\}$ and $\mu = \{\phi, X, \{a, c, d\}, \{a, b\}\}$. Here $\{a, b\}$ is gpr^μ - closed but not $g\alpha^\mu$ - closed.
- Let (X, μ) be a supra topological space where $X = \{a, b, c, d\}$ and $\mu = \{\phi, X, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}, \{a\}\}$. Here $\{b, c\}$ is $g^\mu br$ - closed but not gpr^μ - closed.

Remark 3.3: If (X, μ) is supra topological space then

- gpr^μ - closed set and $g^\mu b$ - closed set are independent of each other.
- gpr^μ - closed set and gs^μ - closed set are independent of each other.
- gpr^μ - closed set and sg^μ - closed set are independent of each other.

These results are proved by the following examples.

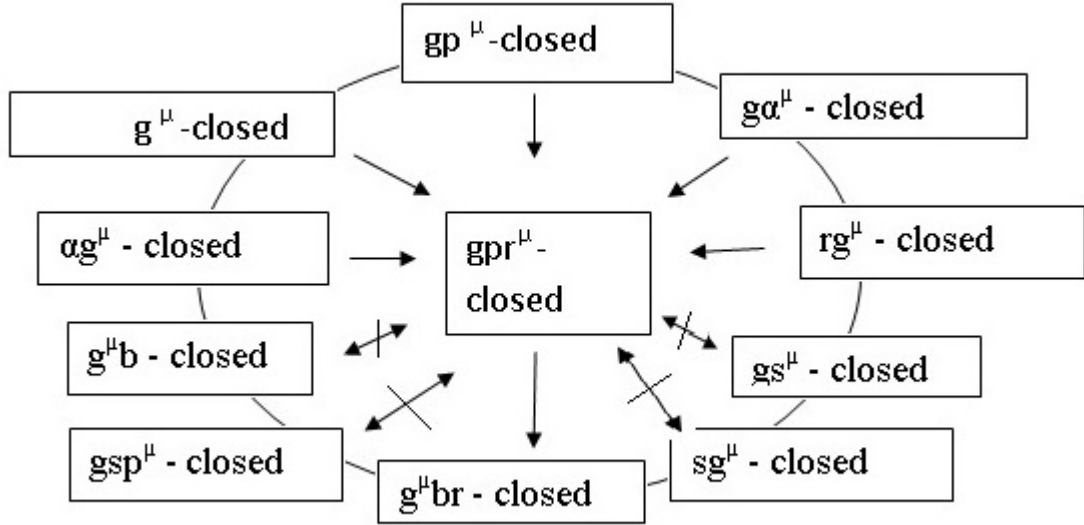
Example 3.4: (i) In example 3.2 (ii) $A = \{b, c\}$ is $g^\mu b$ - closed but not gpr^μ - closed. Let (X, μ) be a supra topological space where $X = \{a, b, c\}$ and $\mu = \{\phi, X, \{a, b\}, \{a, c\}\}$. Consider $A = \{a, b\}$. A is gpr^μ - closed but not $g^\mu b$ - closed.

(ii) Let (X, μ) be a supra topological space where $X = \{a, b, c, d\}$ and $\mu = \{\phi, X, \{a,$

$c, d\}, \{a, b\}\}$. Consider $A = \{a, b\}$. A is gpr^μ - closed but not gs^μ - closed and sg^μ - closed.

(iii) In example 3.2 (ii) $A = \{b, c\}$. A is gs^μ - closed and sg^μ - closed but not gpr^μ - closed.

3.5 From the above results we have the following diagram



Fig(3.5.1)

Theorem 3.6: Let (X, μ) be any supra topological space, then the following are equivalent

- (i) Every $g^\mu br$ - closed set is gpr^μ - closed.
- (ii) Every supra b - closed set is gpr^μ - closed.

proof: (i) \rightarrow (ii) is obvious as every supra b - closed set is $g^\mu br$ - closed.

(ii) \rightarrow (i) Let A be a $g^\mu br$ - closed and $A \subseteq U$ where U is supra regular open. Then $bcl^\mu(A)$ is supra b - closed and $bcl^\mu(A) \subseteq U$. Then by (ii), $bcl^\mu(A)$ is gpr^μ - closed. So $pcl^\mu(A) \subseteq pcl^\mu(bcl^\mu(A)) \subseteq U$. Hence A is gpr^μ - closed.

Theorem 3.7: Let (X, μ) be a supra topological space. If every supra semiclosed set is supra preclosed in (X, μ) , then

- (i) Every gs^μ - closed set is gpr^μ - closed.
- (ii) Every sg^μ - closed set is gpr^μ - closed.

Proof: (i) Let A be gs^μ - closed in (X, μ) and $A \subseteq U$ where U is a supra regular open. Then $scl^\mu(A) \subseteq U$ whenever $A \subseteq U$. Since every supra semiclosed set is supra preclosed (by assumption), $pcl^\mu(A) \subseteq scl^\mu(A) \subseteq U$. This implies $pcl^\mu(A) \subseteq U$. Thus A is gpr^μ - closed.

(ii) Since every sg^μ -closed set is gs^μ -closed, every sg^μ - closed set is gpr^μ - closed by(i).

Theorem 3.8: If every supra open set U is supra regular open in a supra topological space (X, μ) then

(a) Every gpr^μ - closed set is gsp^μ - closed.

(b) The following statements are equivalent:

(i) Every gpr^μ - closed set is αg^μ - closed.

(ii) Every supra preclosed set is αg^μ - closed.

Proof: (a) Let A be gpr^μ - closed in (X, μ) and $A \subseteq U$ where U is supra open. By assumption every supra open set is supra regular open, then $pcl^\mu(A) \subseteq U$. Since every supra preclosed set is supra semi - preclosed, $spcl^\mu(A) \subseteq pcl^\mu(A) \subseteq U$. Hence A is gsp^μ - closed set.

(b) (i) \rightarrow (ii). Every supra preclosed is gpr^μ - closed. This implies every supra preclosed set is αg^μ - closed.

(ii) \rightarrow (i). Let A be gpr^μ - closed in (X, μ) and $A \subseteq U$ where U is supra open. If $B = pcl^\mu(A)$ then $B \subseteq U$. By assumption B is αg^μ - closed. Thus $\alpha - cl^\mu(A) \subseteq \alpha - cl^\mu(B) \subseteq U$. Hence A is αg^μ - closed.

Remark 3.9: $pcl^\mu(A) = A \cup cl^\mu(int^\mu(A))$

Proposition 3.10: Let A be a subset of a supra topological space (X, μ) . If $A \in SO^\mu(X, \mu)$ then $pcl^\mu(A) = cl^\mu(A)$.

proof: We know that every supra closed set is supra preclosed $pcl^\mu(A) \subseteq cl^\mu(A)$. Now given $A \in SO^\mu(X, \mu)$, that is $A \subseteq cl^\mu(int^\mu(A))$. Thus $cl^\mu(A) \subseteq cl^\mu(cl^\mu(int^\mu(A))) \Rightarrow cl^\mu(A) \subseteq (cl^\mu(int^\mu(A))) \Rightarrow cl^\mu(A) \cup A \subseteq cl^\mu(int^\mu(A)) \cup A \Rightarrow cl^\mu(A) \subseteq cl^\mu(int^\mu(A)) \cup A \Rightarrow cl^\mu(A) \subseteq pcl^\mu(A)$. Hence $pcl^\mu(A) = cl^\mu(A)$.

Definition 3.11: A space (X, μ) is supra extremally disconnected if the supra closure of every supra open subset of X in (X, μ) is supra open or equivalently if every supra regular closed subset of X in (X, μ) is supra open.

Theorem 3.12: For a space (X, μ) the following are equivalent:

(i) Every gsp^μ - closed subset of X is gpr^μ - closed

(ii) Every supra semi - preclosed subset of X is gpr^μ - closed

(iii) The space (X, μ) is supra extremally disconnected.

Proof:(i) \rightarrow (ii) is obvious as every supra semi - preclosed set is gsp^μ - closed, every supra semi - preclosed subset of X is gpr^μ - closed.

(ii) \rightarrow (iii) Let A be a supra regular open subset of X . Then $A = int^\mu(cl^\mu(A))$. This implies $int^\mu(cl^\mu(int^\mu(A))) \subseteq int^\mu(cl^\mu(A)) = A$. Thus A is supra semi - preclosed. By hypothesis A is gpr^μ - closed and so $pcl^\mu(A) \subseteq A$. Hence $cl^\mu(int^\mu(A)) = A$. Therefore A is supra regular closed, i.e A is supra closed. Hence the space (X, μ) is supra extremally disconnected.

(iii) \rightarrow (i) Let A be a gsp^μ - closed subset of X and let $U \subseteq X$ be supra regular open with $A \subseteq U$. If $B = \text{spcl}^\mu(A)$, then by assumption $A \subseteq B \subseteq U$. Since B is supra semi - preclosed, $B = \text{spcl}^\mu(B) = B \cup \text{int}^\mu(\text{cl}^\mu(\text{int}^\mu(B)))$. By assumption $\text{cl}^\mu(\text{int}^\mu(B)) = \text{int}^\mu(\text{cl}^\mu(\text{int}^\mu(B)))$. Thus $B = \text{pcl}^\mu(B)$, i.e B is supra preclosed. Thus $\text{pcl}^\mu(A) \subseteq \text{pcl}^\mu(B) = B \subseteq U$, i.e A is gpr^μ - closed.

Theorem 3.13: For a space (X, μ) the following are equivalent:

- (i) Every gs^μ - closed subset of X is gpr^μ - closed.
- (ii) Every sg^μ - closed subset of X is gpr^μ - closed.
- (iii) The space (X, μ) is supra extremally disconnected.

Proof: (i) \rightarrow (ii) is obvious.

(ii) \rightarrow (iii) Let A be a supra regular open subset of X . Then $A = \text{int}^\mu(\text{cl}^\mu(A))$. That is $\text{int}^\mu(\text{cl}^\mu(A)) \subseteq A$, A is supra semi - closed and hence sg^μ - closed. By assumption A is gpr^μ - closed, which implies $\text{pcl}^\mu(A) \subseteq A$. Thus $A = \text{pcl}^\mu(A)$, so A is supra preclosed. Hence $\text{cl}^\mu(\text{int}^\mu(A)) \subseteq A \Rightarrow \text{cl}^\mu(A) \subseteq A$ as A is supra regular open. Therefore $\text{pcl}^\mu(A) \subseteq \text{cl}^\mu(A) \subseteq A$, i.e A is supra closed and thus the space (X, μ) is supra extremally disconnected.

(iii) \rightarrow (i) Let U be supra regular open and A be gs^μ - closed with $A \subseteq U$. Then $\text{scl}^\mu(A) = A \cup \text{int}^\mu(\text{cl}^\mu(A)) \subseteq U$. By assumption $\text{int}^\mu(\text{cl}^\mu(A))$ is supra closed and so $\text{pcl}^\mu(A) = A \cup \text{cl}^\mu(\text{int}^\mu(A)) \subseteq U$, i.e A is gpr^μ - closed.

Theorem 3.14: A space is supra extremally disconnected iff every g^μbr - closed subset of (X, μ) is gpr^μ - closed.

proof: Suppose that the space (X, μ) is supra extremally disconnected. Let A be g^μbr - closed and let U be supra regular open containing A . Then $\text{bcl}^\mu(A) = A \cup [\text{int}^\mu(\text{cl}^\mu(A)) \cap \text{cl}^\mu(\text{int}^\mu(A))] \subseteq U$. Since $\text{int}^\mu(\text{cl}^\mu(A))$ is supra closed, we have $\text{cl}^\mu(\text{int}^\mu(A)) \subseteq \text{cl}^\mu[\text{int}^\mu(\text{cl}^\mu(A)) \cap \text{int}^\mu(A)] \subseteq [\text{cl}^\mu(\text{int}^\mu(\text{cl}^\mu(A))) \cap \text{cl}^\mu(\text{int}^\mu(A))] \subseteq U$. Hence $\text{pcl}^\mu(A) = A \cup \text{cl}^\mu(\text{int}^\mu(A)) \subseteq U$. This implies A is gpr^μ - closed. Conversely, let every g^μbr - closed subset of (X, μ) be gpr^μ - closed. Let A be a supra regular open subset of X in (X, μ) . Then $\text{bcl}^\mu(A) = A \cup [\text{int}^\mu(\text{cl}^\mu(A)) \cap \text{cl}^\mu(\text{int}^\mu(A))] = A \cup [A \cap \text{cl}^\mu(\text{int}^\mu(A))] \subseteq A$. By hypothesis A is g^μbr - closed and so A is gpr^μ - closed. Since every supra regular open set is supra semi open set, by the proposition 3.10 and A is gpr^μ - closed we have $\text{pcl}^\mu(A) = \text{cl}^\mu \subseteq A$. Therefore A is supra closed and (X, μ) is supra extremally disconnected.

Theorem 3.15: Let F be a family of supra regular closed sets in (X, μ) . Then the following are equivalent:

- (i) $\mu = F$
- (ii) Every subset of (X, μ) is gpr^μ - closed set.

proof: (i) \rightarrow (ii). Let $\mu = F$ and let $A \subseteq G$ and G be supra regular open set in $(X,$

μ). Then $\text{pcl}^\mu(A) \subseteq \text{pcl}^\mu(G) = G$, since G is supra regular closed. This implies that $\text{pcl}^\mu(A) \subseteq G$. That is A is gpr^μ - closed.

(ii) \rightarrow (i) Suppose every subset of (X, μ) is gpr^μ - closed set. Let G be supra regular open. Since $G \subseteq G$ and G is gpr^μ - closed, it follows that $\text{pcl}^\mu(G) \subseteq G$ and so $G \in F$. Thus $\mu \subseteq F$. On the other hand if $H \in F$, then $X-H \in \mu \subseteq F$ and so $H \in \mu$. Thus $\mu = F$.

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