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AN APPROACH TO CONTROL MIMO TIME–DELAY PROCESSES

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Abstract. This work develops an approach to control MIMO (Multiple Inputs Multiple Outputs) time–delay processes that can be represented by stable time–delay transfer matrices. A method to transform a time–delay transfer matrix into a delay–free one is also developed. Such a delay–free transfer matrix is used to obtain its state space representation in order to design a linear quadratic regulator controller. This controller is applied to the time–delay transfer matrix with the purpose of obtaining controlled outputs with the ability to follow arbitrary reference signals. Such outputs are also employed for an observer to estimate the state vector of the delay–free transfer matrix. The estimated vector is then employed to compute the controller. Two time–delay processes representing industrial distillation columns were used to demonstrate the validity of the design approach.

1. Introduction

This work develops an approach to control time–delay processes of the form

$$\mathbf{G}_d(s) = \begin{bmatrix} G_{d11}(s)e^{-\tau_{d11}} & \cdots & G_{d1p}(s)e^{-\tau_{d1p}} \\ \vdots & \ddots & \vdots \\ G_{dm1}(s)e^{-\tau_{dm1}} & \cdots & G_{dmp}(s)e^{-\tau_{dmp}} \end{bmatrix} \qquad p \geq m \tag{1.1}$$

In (1.1), each τ_{dij} is a time delay, also known as a transport delay, and every $G_{dij}(s)$ is a stable transfer function.

Time delays are intrinsic in various engineering processes such as pneumatic and hydraulic processes, networked control systems, nuclear reactors, rolling mills, to mention a few. The presence of time delays in a process is a source of oscillations, instability, or poor control performance. A time delay increases the phase lag limiting the required amount of control action. Such a difficulty is greater in MIMO processes due to the existence of several time delays in different control loops, where interactions between inputs and outputs are also unavoidable.

Over the last decades, even in the present, the control of industrial time–delay processes, as described in equation (1.1), has been of great concern. A MIMO time–delay process is controlled if all elements of the vector output $\mathbf{y}$ in Figure 1, follow all elements of the reference vector $\mathbf{r}$, correspondingly, meeting design specifications previously established, despite the presence of disturbance signals

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and interactions between elements of the vector control $\mathbf{u}$ and elements of the vector output $\mathbf{y}$.

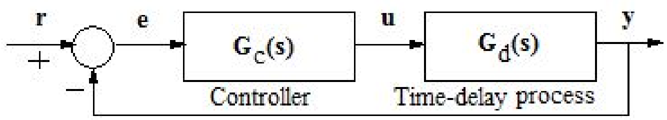

FIGURE 1. MIMO feedback control system.

Various methods to control time–delay processes of the form (1.1) have been published. To cite a few, in [1], a decoupled Smith predictor is designed to control a MIMO non–square time–delay process. A multi–model Smith predictor based control for MIMO processes with uncertain bounded delays is presented in [2], while in [3] a self–tuning predictive control for time–delay processes is designed. In this work, the time–delay transfer matrix $\mathbf{G}_d(s)$ is transformed into a delay–free transfer matrix $\mathbf{G}(s)$, preserving the dynamic characteristics of the original matrix, as explained in the next section. A delay–free transfer matrix $\mathbf{G}(s)$ makes easier the task of designing a MIMO controller $\mathbf{G}_c(s)$.

This paper is organized as follows. Section 2 develops the method to obtain a delay–free transfer matrix. The MIMO controller $\mathbf{G}_c(s)$ is designed in Section 3. Some conclusions derived from this work are presented in Section 4.

2. Obtaining the Delay–Free Transfer Matrix

There are several methods to determine a rational approximation of the exponential term $e^{-s\tau_d}$ as cited in [4]. The Padé approximation is expressed as

$$e^{-s\tau_d} \approx \frac{P(s)}{P(-s)}; \quad P(s) = \sum_{k=0}^{n} \frac{(2n-k)!}{k!(n-k)!} (-s\tau_d)^k \tag{2.1}$$

In (2.1), n is the order of the approximation. The Laguerre shift operator form of $e^{-s\tau_d}$ is given by

$$e^{-s\tau_d} \approx \lim_{n\to\inf} \left[\frac{1-\frac{s\tau_d}{2n}}{1+\frac{s\tau_d}{2n}}\right]^n \tag{2.2}$$

On the other hand, the approximation based in the Fourier analysis has the form

$$\begin{aligned} e^{-s\tau_d} &= \frac{W(s)}{1-W(s)} \\ W(s) &= \frac{1}{2} - \frac{1}{4}\tau_d s + \frac{2}{\pi^2\tau_d^3 s^2} \sum_{k=0}^{n} \frac{1}{(2n+1)^2} \frac{1}{s^2\tau_d^2 + (2n-1)\pi} \end{aligned} \tag{2.3}$$

A variation of the Padé approximation given by (2.1) is used in [5] to control a SISO (Single Input Single output) time–delay process. A first order Padé approximation together with the polynomial method are used in [6] to control SISO unstable processes.

The method developed in this paper consists in finding a delay–free transfer function $G_{ij}(s)$ that substitutes the corresponding time–delay transfer function expressed as $G_{dij}(s)e^{-s\tau_{dij}}$. The latter transfer function is an element of the transfer matrix given by (1.1). As a starting point, let us use Table 1 to extract parameters n and T_{nij} in order to establish the following approximation

$$G_{dij}(s)\, e^{-s\tau_{dij}} = \frac{K_{pij}}{1+T_{dij}s}\, e^{-s\tau_{dij}} \approx \frac{K_{pij}}{(1+T_{nij}s)^n} \tag{2.4}$$

In (2.4), K_{pij} is the process gain.

TABLE 1. Determination of parameters n and T_{nij} of (2.4).

n	2	3	4	5	6	7	8	9	10
τ_{dij}/T_{dij}	0.104	0.218	0.320	0.410	0.493	0.591	0.641	0.709	0.775
T_{nij}/T_{dij}	0.368	0.270	0.224	0.195	0.175	0.151	0.148	0.140	0.132

For the special case $0 < \tau_{dij} < 0.104$, we may use the following approximation

$$\frac{K_{pij}}{1+T_{dij}}\, e^{-s\tau_{dij}} \approx \frac{K_{pij}}{(1+T_{1ij}s)(1+T_{2ij}s)} \tag{2.5}$$

In (2.5), parameters T_{1ij} and $T_{2ij} = kT_{1ij}$ can be obtained using Table 2.

TABLE 2. Determination of parameters T_{1ij} and $T_{2ij} = kT_{1ij}$, $k > 0$, of (2.5).

k	1	2	3	4	5	6	7	8	9
τ_{dij}/T_{dij}	0.094	0.090	0.085	0.080	0.075	0.069	0.064	0.058	0.053
T_{1ij}/T_{dij}	0.238	0.175	0.140	0.120	0.107	0.097	0.088	0.081	0.074

As a first example, consider the following time–delay transfer matrix corresponding to the distillation column of Wood and Berry found in [7]

$$\begin{aligned} \mathbf{G}_d(s) &= \begin{bmatrix} \frac{12.8}{1+16.7s}e^{-s} & \frac{-18.9}{1+21s}e^{-3s} \\ \frac{6.6}{1+10.9s}e^{-7s} & \frac{-19}{1+14s}e^{-3s} \end{bmatrix} \\ &= \begin{bmatrix} G_{d11}(s)e^{-T_{d11}s} & G_{d12}(s)e^{-T_{d12}s} \\ G_{d21}(s)e^{-T_{d21}s} & G_{d22}(s)e^{-T_{d22}s} \end{bmatrix} \end{aligned} \tag{2.6}$$

For the transfer function located in position $(i,j) = (1,1)$ of (2.6), we have 1/16.7 = 0.06 < 0.104. Hence, using Table 2, we obtain $k = 8$, $T_{111} = 0.081$, and $T_{211} = 0.648$. Then, the delay–free transfer function is expressed as

$$\frac{12.8}{1+16.7s}\, e^{-s} \approx \frac{12.8}{(1+0.081s)(1+0.648s)}$$

However, a better approximation using the trial–and–error method to find new values of the required parameters, is obtained with the following transfer function

$$\frac{12.8}{1+16.7s}\,e^{-s} \approx \frac{12.8}{(1+1.35s)(1+16s)} \tag{2.7}$$

Figure 2 compares step responses and Bode diagrams of both transfer functions of (2.7). From its Bode diagram, we observe that the approximated transfer function (dashed line) performs good for a bandwidth of 2 rad/s.

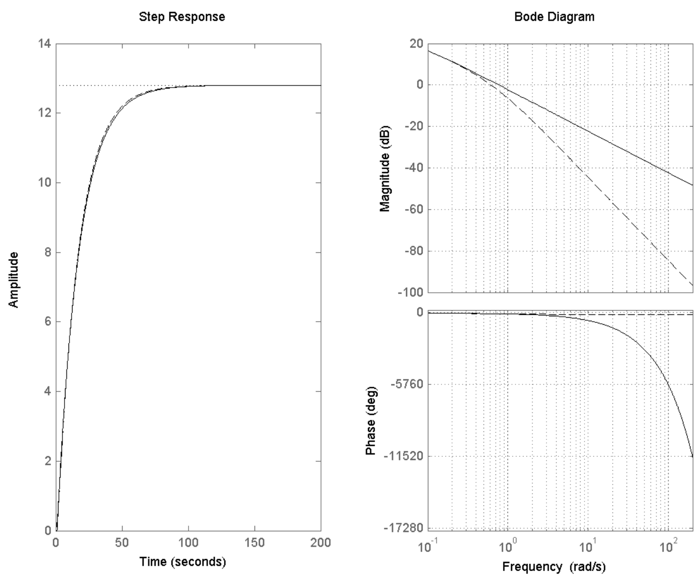

FIGURE 2. Step responses and Bode diagrams of transfer functions $\frac{12.8}{1+16.7s}\,e^{-s}$ (solid line) and $\frac{12.8}{(1+1.35s)(1+16s)}$ (dashed line) of equation (2.7).

Following a similar procedure, we obtain the delay–free transfer matrix of (2.6)

$$\mathbf{G}(s) = \begin{bmatrix} \frac{12.8}{(1+1.35s)(1+16s)} & \frac{-18.9}{(1+13s)^2} \\ \frac{6.6}{(1+5s)^4} & \frac{-19.4}{(1+8.8s)^2} \end{bmatrix} \tag{2.8}$$

As a second example, let us consider the Oggunaike and Ray distillation column found in [7], represented as a 3× 3 transfer matrix

$$\mathbf{G}_d(s) = \begin{bmatrix} \frac{0.66e^{-2.6s}}{1+6.7s} & \frac{-0.61e^{3.6s}}{1+8.64s} & \frac{-0.0049e^{-s}}{1+9.06s} \\ \frac{1.11e^{-6.5s}}{1+3.25s} & \frac{-2.36e^{-3s}}{1+5s} & \frac{-0.01e^{-1.2s}}{1+7.09s} \\ \frac{-34.68e^{-9.2s}}{1+8.15s} & \frac{46.2e^{-9.4s}}{1+10.9s} & \frac{(0.87)(1+11.61s)e^{-s}}{(1+3.89s)(1+18.8s)} \end{bmatrix} \tag{2.9}$$

The corresponding delay–free transfer matrix of (2.9) is

$$\mathbf{G}(s) = \begin{bmatrix} \frac{0.66}{(1+4.5s)^2} & \frac{-0.61}{(1+6s)^2} & \frac{-0.0049}{(1+3.33s)^2} \\ \frac{1.11}{(1+1.3s)^7} & \frac{-2.36}{(1+2.7s)^3} & \frac{-0.01}{(1+4.6s)^2} \\ \frac{-34.68}{(1+4s)^4} & \frac{46.2}{(1+6.5s)^3} & \frac{(0.87)(1+11.61s)}{(1+3.89s)(1+1.4s)(1+12.6s)} \end{bmatrix} \tag{2.10}$$

Note that the term $\frac{e^{-s}}{(1+18.8s)}$ of the transfer function $G_{d33}(s)$ (see (2.9)) has been replaced by the term $\frac{1}{(1+1.4s)(1+12.6s)}$.

3. MIMO Controller Design

A Linear Quadratic Regulator (LQR) controller will be used to control the MIMO time–delay process formulated in (1.1). It follows the procedure design of the LQR–controller.

1. Find the delay–free transfer matrix $\mathbf{G}(s)$ from the time–delay transfer matrix $\mathbf{G}_d(s)$ given by (1.1).
2. Obtain the state space representation $\frac{d\mathbf{x}}{dt} = \mathbf{Ax} + \mathbf{Bu}$, $\mathbf{y} = \mathbf{Cx}$, where $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{u}$ are the state, output, and control vectors, respectively, and, $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ are the state, control and output matrices, correspondingly.
3. Use the state space representation to obtain the controller gain $\mathbf{K}$, and the observer gain $\mathbf{K}_e$.
4. Calculate the estimated state vector $\mathbf{x}_e$ using the following state observer $\frac{d\mathbf{x}_e}{dt} = \mathbf{Ax}_e + \mathbf{Bu} + \mathbf{K}_e(\mathbf{y} - \mathbf{y}_e)$, $\mathbf{y}_e = \mathbf{Cx}_e$, where $\mathbf{y}_e$ is the estimated output vector.
5. Compute the control law $\mathbf{u} = -\mathbf{Kx}_e$ using the separation principle.
6. Apply $\mathbf{u}$ to the MIMO time–delay process $\mathbf{G}_d(s)$ in order to obtain the controlled vector output from $\mathbf{y} = \mathbf{G}_d(s)\mathbf{u}$. Tune weighted matrices $\mathbf{Q}$ and $\mathbf{R}$ to obtain the controller gain $\mathbf{K}$, as well as weighted matrices $\mathbf{Q}_e$ and $\mathbf{R}_e$ to compute the observer gain $\mathbf{K}_e$, so that the elements of the vector output $\mathbf{y}$ follow the elements of an arbitrary reference vector $\mathbf{r}$, correspondingly.

According the procedure described above, Figure 3 depicts the controlled outputs $y_1(t)$ and $y_2(t)$ following arbitrary references $r_1(t) = 0.005t/T + sin(0.05t/T)$ and $r_2(t) = -0.005t/T + cos(0.05t/T)$, where T is a proper sampling time.

4. Conclusions

This work developed a procedure described in Section 3 to control MIMO time–delay processes represented by stable time–delay transfer matrices.

This procedure was applied to a MIMO time–delay process representing a distillation column (Section 3) to demonstrate the validity of the design approach.

More research should be performed to employ the developed approach to control unstable MIMO time–delay transfer matrices.

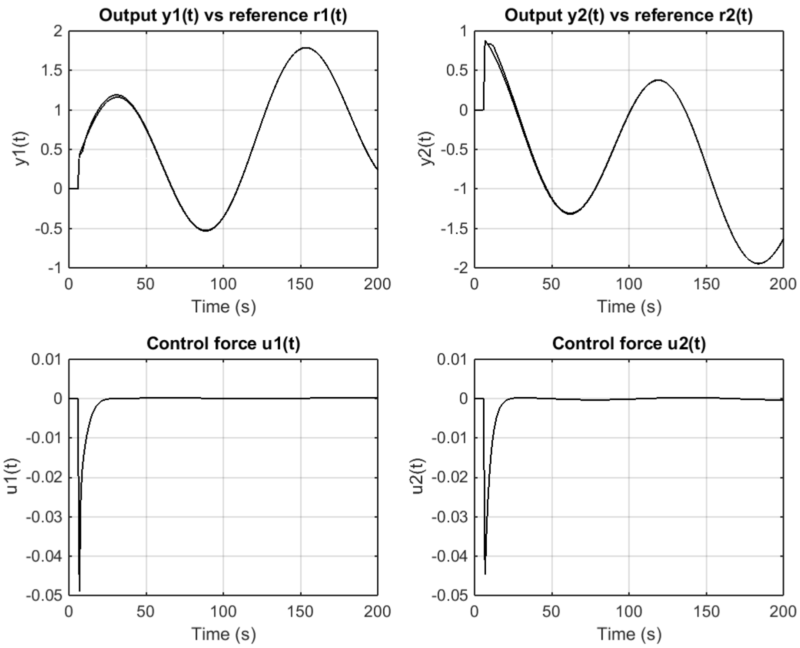

FIGURE 3. Controlled outputs $y_1(t)$ and $y_2(t)$ and the corresponding control signals $u_1(t)$ and $u_2(t)$ using a LQR–controller.

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