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MODIFIED ARTIFICIAL BEE COLONY ALGORITHM FOR ENGINEERING DESIGN PROBLEMS

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ABSTRACT: It is very necessary and applicable to optimize all disciplines. In practical engineering problems the optimization has been a significant component. This article presents the implementation of modified approach combined with two novel techniques during the early bees of employed bees and the determination of a new location for scout bees. Modified Artificial Bee Colony (MABC) Algorithm was responsible for resolving well-known unconstrained engineering design problems like Three-Bar Truss Design Problem, Tension / Compression Spring Design Problem and Belleville Spring Design Problem. Statistical results of MABC Algorithm were compared with those of previous ABCs and many methods from previous studies. Comparisons have shown that MABC Algorithm has improved on its competing with accuracy, consistency and speed of reliability.

Keywords: Optimization, Artificial bee colony algorithm, Constraint optimization problems, engineering designs problem.

INTRODUCTION

In many programs, in general, the problem of optimization is defined as

Min $f_r(\underline{x})$; $\underline{x} \in R^d$ and $1 \leq r \leq M$

Such that

$h_l(\underline{x}) = 0,\ 1 \leq l \leq p$

$g_m(\underline{x}) \leq 0,\ 1 \leq m \leq q$

where f_r, h_l and g_m are real-valued functions defined on R^d. In this problem $x_1, x_2, x_3, \ldots, x_d$ are called the design variables, the functions f_r is considered as the cost function or objective function, g_m and h_l are known as the problem constraints. And the space it covers is recognized as the search space or design space [1]. Objective function may also be constructed as a problem of maximization and inequality can be stated as a greater or equal form [2]. Meanwhile the introduction of algorithms such as the annealing (SA) algorithm and genetic algorithm (GA), adaptation has been an effective and efficient part of the development of practical and efficient strategies [3, 4]. Particle swarm optimization (PSO) algorithm is constructed on the concept of swarm behavior [5]. Other examples of PSO type algorithms are bats algorithm (BA), Firefly algorithm (FA), Artificial bee colony (ABC), and krill swarm (KH) [6-9]. Comparisons show that ABC offers better performance than PSO, DE, EA and can be successfully hired to solve engineering problems [10, 11]. Similarly, the Differential Evolution (DE) algorithm is based on the idea of improving the quality of a member of society with social and other differences [12]. The Mine Blast Algorithm (MBA) and the Grenade Explosion Method (GEM) promote the explosion of grenades and mines, respectively [13, 14]. Bee Colony Optimization Algorithm (BCOA) is proposed to solve numerical problems such as mobile retailer, traffic, and transportation [15]. Encouraging results have been reported in complex engineering problems using the BCOA [16]. Yang proposed a virtual bee algorithm (VBA) and proved its effectiveness in solving problems with two-dimensional numbers [17]. The first BA [18] suffers from a serious lack of proper programming of many parameters.

One of the serious problems of the ABC is its lack of search-related content, which is sufficiently accurate during the search space exploration but is limited compared to the exploitation process [19]. Some useful modifications from previous studies are available to address such shortcomings of ABC. Wei-Feng et al. have proposed an improved version of ABC based on the orthogonal learning (OL) program [19]. A global best artificial bee colony (GABC) was introduced [20] to improve ABC's level of convergence. Promoted by DE, modified ABC used to conduct local searches [21]. The concepts of best information, inertial weight and acceleration coefficient were used as in Improved Artificial Bee Colony (IABC) [22]. The idea of Rosenbrock's rotational method is hybridized with that of ABC [23]. In the onlooker bee phase, a memory board-based mechanism for the selection of neighboring solutions has been proposed [24].

Some recent studies like [25] also investigate these kinds of developments. Most of the methods like traditional analytical and finite difference schemes does not handle the properties of the model like boundedness, positivity, feasibility. There is the dire need of developing such a method that may be capable of handling these properties and give true insights into model dynamics.

In recent years a lot of sophisticated meta-heuristics have been introduced to solve the most complex problems by transforming them into problems of optimization [26-28]. Improvisations to differential equations of these suggested metaheuristics can be seen in [29] as well, but the application of these meta-heuristics [30-33] to widely distributed and disease models are difficult to see. The study conducted by Farhan et al. for the treatment of the HIV/AIDS epidemic model with vertical transmission, Hepatitis-B Model [34] and

Smoking Model by using evolutionary Pad´e-approximation, extend this work to solve under line measles dynamical model. Unfortunately, previous modifications are not very specific to improve on two very important components of ABC, namely the initialization phase and the phase of scout-bees. The concepts of reflection and effective radius, respectively, have been incorporated to improve the initialization process and the scout-bees phase. The resulting method is called as Radial Artificial Bee Colony Algorithm (RABC). The main objective of the current work is to improve the improved and more efficient version of ABC by improving the two mentioned components.

MARTIALS AND METHODS

Artificial bee colony algorithm: The Artificial Bee Colony (ABC) categories are structured as follows [7].

Initialization phase: This category should start with a user-defined size that can vary from problem to problem. Half of the population consisted of employed bees and the rest were treated as onlooker bees. Each randomly generated area describes a food source provided by the employed bee and is constructed using the following equation.

$x_{u,v} = {x_v}^{min} + \lambda({x_v}^{max} - {x_v}^{min})$

u= 1,2, 3…, P and v = 1,2, 3…, Q

here $x_{u,v}$ represented the v^{th} limit of the u^{th} food source or an employed bee,

${x_v}^{max}$ and ${x_v}^{min}$ were the bound on the v^{th} constraints, respectively, λ was a randomly selected number between 0 and 1, P represented the number of employed bees and Q was the magnitude of the problem to be optimized. In addition, in this section, the reset counter (AC) counter for each food source also occurred. After that, the following formula was used to calculate the suitability of each food source.

$$fit_u = \{ \begin{matrix} \frac{1}{1+f_u} & if(f_u \geq 0 \\ 1 + abs(f_u) & otherwise \end{matrix} \quad (1)$$

Where fit_u denoted the fitness of u^{th} employed bee at its relevant food source and f_u was the objective function value of u^{th} food source.

Employed bee phase: In this stage, each food source was improved by waggle dance of corresponding employed bee by using following equation:

$y_{u,v} = x_{u,v} + \emptyset(x_{u,v} - x_{w,v})$ $u, w \in 1, 2,\ldots,P$, $v \in 1,2,\ldots,Q$ and $u \neq w$

Here $y_{u,v}$ was v^{th} part of the u^{th} solution vector, $x_{u,v}$ was v^{th} part of the u^{th} food source, $x_{w.v}$ was v^{th} component of w^{th} source and $\emptyset$ act as a randomly selected number between − 1 and +1. In addition, the component (j) and the neighboring candidate (k) solution were randomly selected from the remaining members.

Using the figure (1) the suitability of a new location $y_{u,v}$was obtained and assigned as $x_{u,v}$, as long as the suitability of the new resource was high. The AC value is reset to zero if it is successful and rises by 1 in case of a failure.

Onlooker bee phase: At this stage, the waggle dance of the employed bees helped to make the better position. The onlooker bees were then selected by food sources according to the selection probability (p_u) calculated by the following equation:

$$p_u = \frac{fut_u}{\sum_{v=1}^{N} fut_v}$$

Later, onlooker bees aiming for improvement in shared food sources shared with employed bees using the waggle dance equation. If the solution obtained by the observer was better than the employed bees, the newly found solution for the onlooker category was returned to the used bee and the AC was reset.

Scout bee phase: In this step, the discarded counter with maximum content matched the predefined limit value. When the amount of high AC content was greater than the original value, the corresponding bee was converted into a scout bee and a new food source was produced randomly. AC reset. Having found a new solution, the scout bee is back to normal as a employed bee.

PROPOSED MODIFICATIONS

In the original ABC format, random searches are performed in the search field to select food sources. However, this startup method did not guarantee the correct availability of search space. In response to this challenge algorithm change was introduced according to review # 1 of the introduction of N food sources in the search area.

Modification # 1:

Step-1 u=1

Step-2 for v= 1, 2, 3…, P/2 , x_v = L + rand (0, 1) (U-L) , $x_{u+1} = x_u$ + 2((L+U)/2-x_u)

Step-3 u = u+ 2, go to step-4

Step-4 if u≤P go to step-2 else stop

The third line of step 2 has done the job of identifying the location of the search space that does not allow the solution and its indicated point to occur in the same search area.

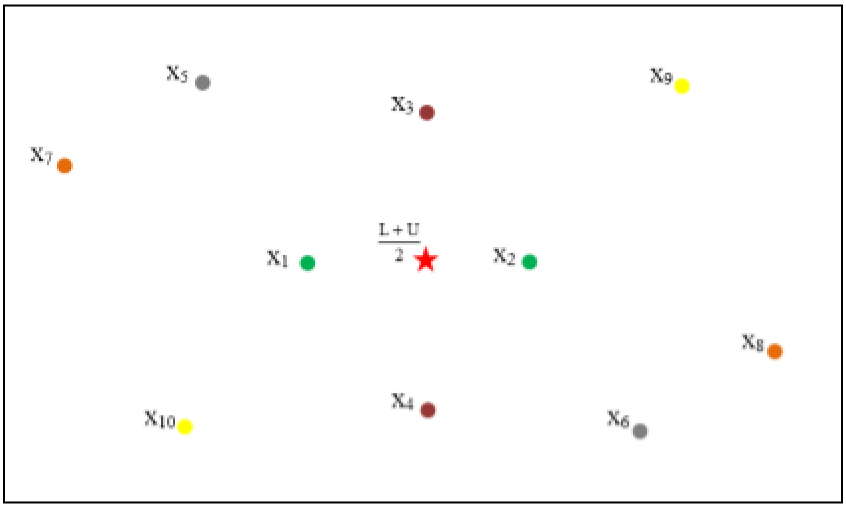

Fig.1: The initial locations N are randomly distributed in 2-dimensional search space

Modification 2:
The second modification was to determine the proper position of the scout bee and its waggle dance budget has expired. With the use of employed bee, the scout bee explores the search space while taking on a new location listed in equation 2.

$$x_{new} = x^{d} + Rad \times rand\,(-1, 1) \quad (2)$$

Here $Rad = max\,\{|x_u{}^{d} - l_u|\,, |x_u{}^{d} - h_u|: 1 \leq u \leq n\}$ (3)

Continuous rand (-1, 1) indicates the vector of random numbers generated by the interval (-1, 1).
Due to the effect of eqn. (6), each scout bee is introduced into the space listed in equation 3 next to the current solution in a random way. The RABC algorithm emerged embedded in modification 1 and 2 in the original ABC. The full RABC algorithm was stated below.

MABC Algorithm
Step 1: Selected parameters.
Step 2: The initialization population is created using modification # 1.
Step 3: Employed Bees Phase was activated.
Step 4: Onlooker bee stage is created.
Step 5: The AC parameter is tested and the scout bees are introduced using modification # 2.

RESULTS AND DISCUSSION
Engineering benchmark constrained problems: The success of the proposed new and dynamic algorithm MABC is shown by the resolution of various problems in optimization, which are commonly used to validate optimization approaches and are known as benchmarks in the literature. These test cases consist of three practical benchmarks for optimization of design problems [13].

Three-Bar Truss Design Problem
This problem has been taken from, which has two types of design variables, x_1 and x_2 for cross-sectional areas, and has one linear objective function of minimization with three nonlinear inequality constraints [13]. The graphical representation of the three-bar truss design problem is shown in Figure 2.

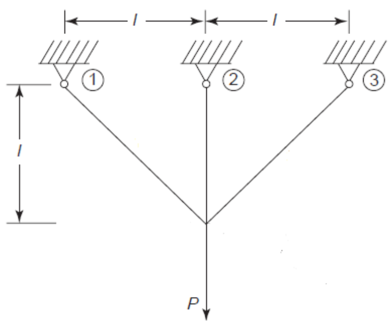

Figure2. The three-bar truss design problem

$$min f(x) = (2\sqrt{2}x_1 + x_2) \times l$$

$$subject\ to \begin{cases} g_1(x) = \dfrac{\sqrt{2}x_1 + x_2}{\sqrt{2}x_1{}^2 + 2x_1x_2} P - \sigma \leq 0 \\ g_2(x) = \dfrac{x_2}{\sqrt{2}x_1{}^2 + 2x_1x_2} P - \sigma \leq 0 \\ g_3(x) = \dfrac{1}{\sqrt{2}x_2 + x_1} P - \sigma \leq 0 \end{cases}$$

$$0 \leq x_1, x_2 \leq 1$$

$$\boldsymbol{Data}:$$
$$length = \ l \ = 100cm, \qquad applied\ load = P = 2\frac{KN}{cm^2}, \qquad stress = \sigma = 2\frac{KN}{cm^2}$$

Abbreviations use in this problem are as follows: MBA [13] Mine Blast Algorithm, DEDS [12] Differential Evolution with Dynamic Stochastic PSO-DE [5], Particle Swarm Optimization with Differential Evolution, Modified Artificial Bee Colony.

Table 1 shows the comparison of the best solution between different optimizers and the variability of the corresponding design. The results obtained by MABC are compared to 3 advanced algorithms. Differential Evolution with Dynamic Stochastic, Particle Swarm Optimization with Differential Evolution and Mine Blast Algorithm which is the first constraint to the final solution violated but MABC satisfies all the barriers to the final solution.

Table1.Reported results for three-bar truss design problem from different optimizers

$\boldsymbol{Methods}$	$\boldsymbol{Design\ variables}$		$\boldsymbol{f(x)}$	$\boldsymbol{Constraints}$		
	$\boldsymbol{x_1}$	$\boldsymbol{x_2}$		$\boldsymbol{h_1(x)}$	$\boldsymbol{h_2(x)}$	$\boldsymbol{h_3(x)}$
$DEDS$	0.78867513	0.40824828	263.895841	$1.77797E-08$	-1.464101618	-0.535898364
$PSO-DE$	0.7886751	0.4082482	263.8958245	$1.42718E-07$	-1.464101647	-0.535898211
MBA	0.788565	0.4085597	263.8958336	$1.41887E-07$	-1.463747582	-0.536252276
$\boldsymbol{MABC}$	**0.788675135**	**0.40824829**	**263.895843**	**0**	$\boldsymbol{-1.464101616}$	$\boldsymbol{-0.535898384}$

It is clear from Table 1 that the proposed MABC algorithm works best and is superior to all high-quality methods without breaking the law. The convergence curve shows the performing values compared to the number of generations of the three bar-truss design problem. 30 trials of the best solution found in the MABC algorithm are given in Figure 3.

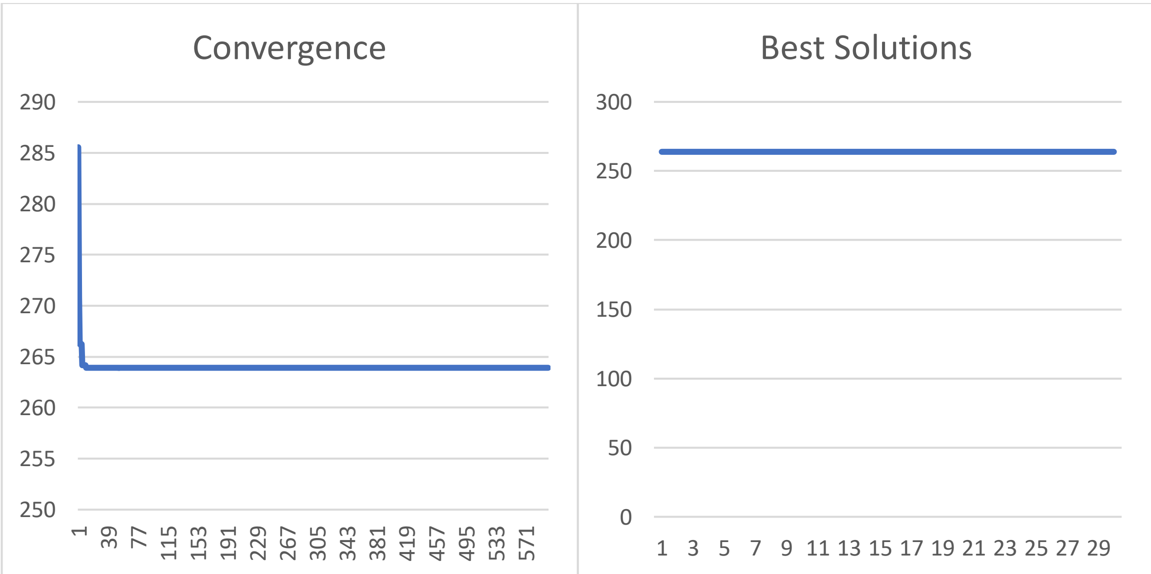

Figure 3. Convergence curve and 30 best solutions for the three-bar truss design problem

Tension / Compression Spring Design Problem

This problem is derived from [13] which has three types of design variables and has one linear objective function of minimization with four nonlinear inequality constraints. The detailed view of the tension/compression spring design problem is shown in Figure 4. The four design variables are:

$d: wire\ diameter = x_1,$

$D: the\ mean\ coil\ diameter = x_2,$

$P: the\ number\ of\ active\ coils = x_3.$

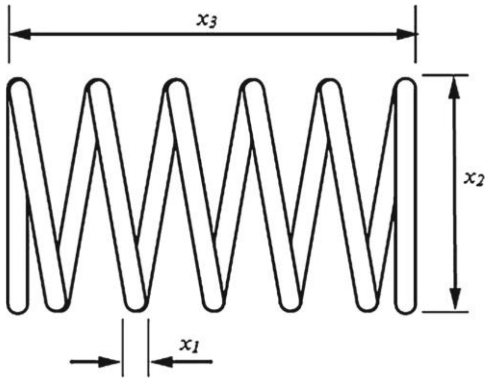

Figure 4. Tension/ compression spring design problem

$$f(x) = (x_3 + 2)x_2 x_1^2$$

$$\text{subject to} \begin{cases} g_1(x) = 1 - \left(\dfrac{x_2^3 x_3}{71{,}785 x_1^4}\right) \le 0 \\ g_2(x) = \left(4x_2^2 - \dfrac{x_1 x_2}{12{,}566(x_2 x_1^3 - x_1^4)}\right) + \left(\dfrac{1}{5108 x_1^2}\right) - 1 \le 0 \\ g_3(x) = 1 - \left(\dfrac{140.45 x_1}{x_2^3 x_3}\right) \le 0 \\ g_4(x) = \dfrac{x_2 + x_1}{1.5} - 1 \le 0 \end{cases}$$

$$0.05 \le x_1 \le 2.00, 0.25 \le x_2 \le 1.30, 2 \le x_3 \le 15$$

Abbreviations use in this problem are as follows: DELC [12] Differential Evolution with Level Comparison, CPSO [5] Chaotic Particle Swarm Optimization, HPSO [5] Hybrid Particle Swarm Optimization, NM-PSO [5] Nelder - Mead Particle Swarm Optimization, MBA [13] Mine Blast Algorithm, HEAA [35] Hybrid Evolutionary Algorithm and Adaptive technique, GQPSO [5] Genetic Quantum particle Swarm Optimization, DEDS [12] Differential Evolution with Dynamic Stochastic.

Table 2 shows the comparison of a better solution than several optimizers and the variability of the corresponding design. Results obtained by *MABC* comparing 8 state-of-the-art algorithms. *DELC*, *DEDS*, *NM-PSO* include two constraints with *HPSO*, *G-QPSO*, *HEAA*, one *MBA* violet facing the final the final solution but *MABC* satisfies all aspects of final solution.

Table2.*Reported results for tension/compression spring design problem from different optimizers*.

Methods	***Design* Variables**			$f(x)$	***Constraints***			
	x_1	x_2	x_3		$h_1(x)$	$h_2(x)$	$h_3(x)$	$h_4(x)$
$DELC$	0.051689	0.356717	11.28897	0.01266517	$1.56136E-06$	$1.6457E-06$	−4.053800957	−0.727729333
$DEDS$	0.051689	0.356717	11.28897	0.01266517	$1.56136E-06$	$1.6457E-06$	−4.053800957	−0.727729333
$CPSO$	0.051728	0.357644	11.24454	0.01267474	−0.00082509	$-2.52741E-05$	−4.051306652	−0.727085333
$HPSO$	0.051706	0.357126	11.26508	0.01266523	$-3.06754E-06$	$1.39164E-06$	−4.054583211	−0.727445333
$NM-PSO$	0.05162	0.355498	11.33327	0.01263019	0.001010113	0.00099487	−4.061859753	−0.728588
$G-QPSO$	0.051515	0.352529	11.53886	0.01266614	$4.83412E-05$	$-3.57742E-05$	−4.045483267	−0.730637333
$HEAA$	0.051689	0.356729	11.28829	0.01266496	$-3.98291E-05$	$2.87099E-05$	−4.053761789	−0.727721333
MBA	0.051656	0.35594	11.34467	0.01267435	−0.00093359	$3.86419E-05$	−4.047743157	−0.728269333
$\boldsymbol{RABC}$	**0.0516881**	**0.3566964**	**11.290215**	**0.0126652**	$\mathbf{-2.22E-15}$	$\mathbf{-1.54E-14}$	**−4.0537435**	**−0.7277435**

It is clear from Table 2 that the proposed *MABC* algorithm has performed better and superior to all other high-level methods without violation the rules others are violated the constraints than 30 trial of the best solution found in the MABC algorithm are given in Figure 5.

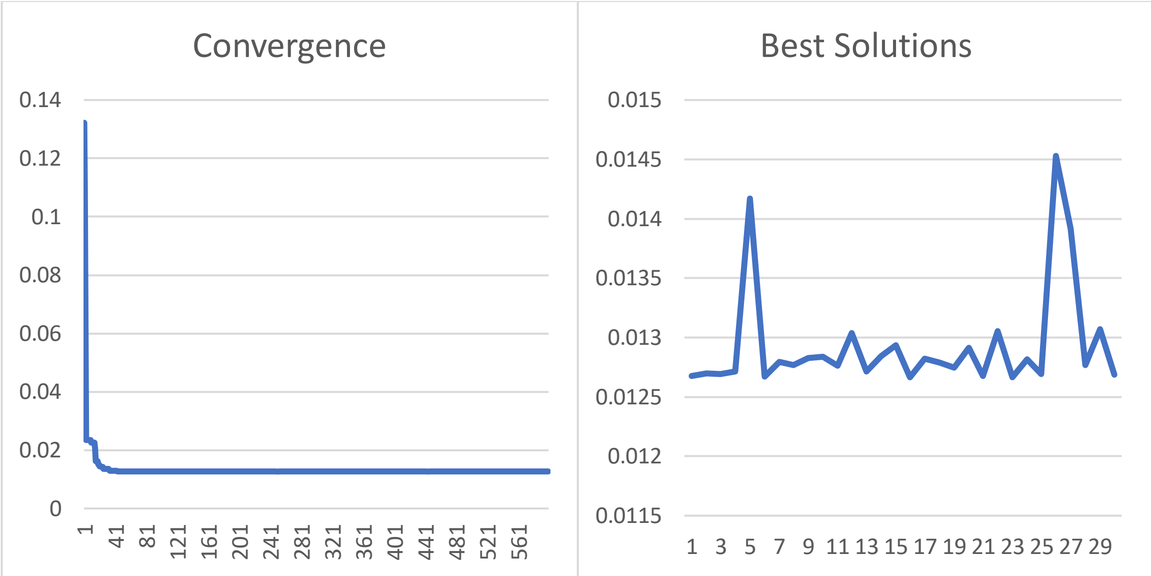

Figure 5. Convergence curve and 30 best solutions for tension/compression spring design problem

Belleville Spring Design Problem

This problem has been taken from which has four types of design variables and has a linear objective function of minimization with seven non-linear inequality constraints [13]. The graphic view of the Belleville spring design problem is shown in Figure 6. This problem has the following four design variables:

D_e: *external diameter of the spring* $= x_1$,
D_i: *internal diameter of the spring* $= x_2$,
t: *thickness of the spring* $= x_3$, and
h: *the height of the spring* $= x_4$.

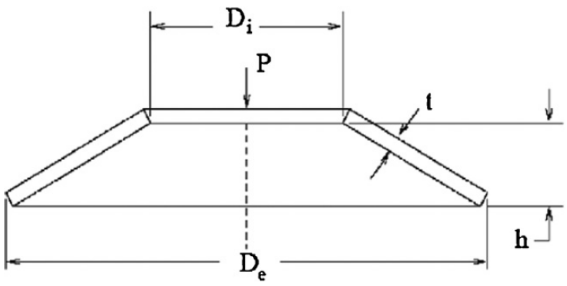

Figure 6. Graphic view of the Belleville spring design problem

$$\min f(x) = 0.07075\pi\left({D_e}^2 - {D_i}^2\right)t$$

$$\textit{subject to} \begin{cases} g_1(x) = S - \dfrac{4E\delta_{max}}{(1-\mu^2)\alpha {D_e}^2}\left[\beta\left(h - \dfrac{\delta_{max}}{2}\right) + \gamma t\right] \geq 0 \\ g_2(x) = \left(\dfrac{4E\delta_{max}}{(1-\mu^2)\alpha {D_e}^2}\left[\left(h - \dfrac{\delta}{2}\right)(h-\delta)t + t^3\right]\right)_{\delta=\delta_{max}} - P_{max} \geq 0 \\ g_3(x) = \delta_1 - \delta_{max} \geq 0 \\ g_4(x) = H - h - t \geq 0 \\ g_5(x) = D_{max} - D_e \geq 0 \\ g_6(x) = D_e - D_i \geq 0 \\ g_7(x) = 0.3 - \dfrac{h}{D_e - D_i} \geq 0 \end{cases}$$

Data:

$$\alpha = \frac{6}{\pi lnK}\left(\frac{K-1}{K}\right)^2, \quad \beta = \frac{6}{\pi lnK}\left(\frac{K-1}{lnK} - 1\right), \quad \gamma = \frac{6}{\pi lnK}\left(\frac{K-1}{2}\right)$$

$$P_{max} = 5400lb.\ E = 30e6Psi,\ \delta_{max} = 0.2in,\ \mu = 0.3, \quad S = 200KPsi, \quad H = 2in,$$

$$D_{max} = 12.01in, \qquad K = \frac{D_e}{D_{i,}}, \quad \delta_1 = f(a)a, \quad a = \frac{h}{t}$$

Abbreviations use in this problem are as follows: Coello [36] Single-Objective Evolutionary Optimization, Gene AS [36] Combined Genetic Search Technique, Siddal [37] Optimal Engineering Design, ABC [38] Artificial Bee Colony, NDE [12] Novel Differential Evolution, Rank- iMDDE [39] Ranking-based Improved Dynamic Diversity Mechanism.

Table 3 shows the best solution between the various optimizers and the variations of the corresponding design. The results obtained by *MABC* are compared to 6 state-of-art algorithms. Gene AS I and Siddal violet are the one constraint to find a final solution but the *SABC* satisfies all aspects of the final solution.

Table 3*. Described results for Belleville spring design problem from distinct optimizers*

Methods	**Design Variables**				$f(x)$	**Constraints**						
	x_1	x_2	x_3	x_4		$h_1(x)$	$h_2(x)$	$h_3(x)$	$h_4(x)$	$h_5(x)$	$h_6(x)$	$h_7(x)$
Coello	0.208	0.2	8.751	11.067	2.121964	2145.4109	39.75018	0	1.592	0.943	2.316	0.21364
Gene AS I	0.205	0.201	9.534	11.627	2.01807	−10.3396	2.8062	0.001	1.594	0.383	2.093	0.20397
Siddal	0.204	0.2	10.03	12.01	1.979715	134.0816	−12.5328	0	1.596	0	1.98	0.19899
Gene AS II	0.21	0.204	9.268	11.499	2.16256	2127.2624	194.2225	0.004	1.586	0.511	2.231	0.20856
MBA	0.204143	0.2	10.030473	12.01	1.9796747	$4.58E-04$	$3.04E-07$	$9.24E-10$	1.595856	0	1.979526	0.198965
RABC	**0.204**	**0.2**	**10.0304**	**12.01**	1.979674	**0**	**0**	**0.7797**	**1.59585**	$\mathbf{1.24E-14}$	**1.97952**	**0.19896**

The results obtained by the *MABC* are also compared with 5 state-of-the-art algorithms, comparison of the statistical results of the Belleville spring construction problem is shown in Table 4.

Table 4. Statistical comparison of results for the Belleville spring design problem of several algorithms

Method	***Worst***	***Mean***	***Best***	***SD***
ABC	2.104297	1.995475	1.979675	0.07
MBA	2.005431	1.984698	1.9796747	$7.78E-03$
Rank − iMDDE	1.979683	1.979675	1.979675	*N.A*
NDE	1.97969110	1.97967661	1.97967477	$4.82E-06$
MABC	2.047196	2.75	**1.979674**	1.642

It is clear from Tables 3 & 4 that the proposed *MABC* algorithm has done better and superior to all other forms of art without destruction. The convergence curve shows the performance rates relative to the number of generations of the Belleville spring problem. 30 trials of the best solution found in the *MABC* algorithm are shown in Figure 7.

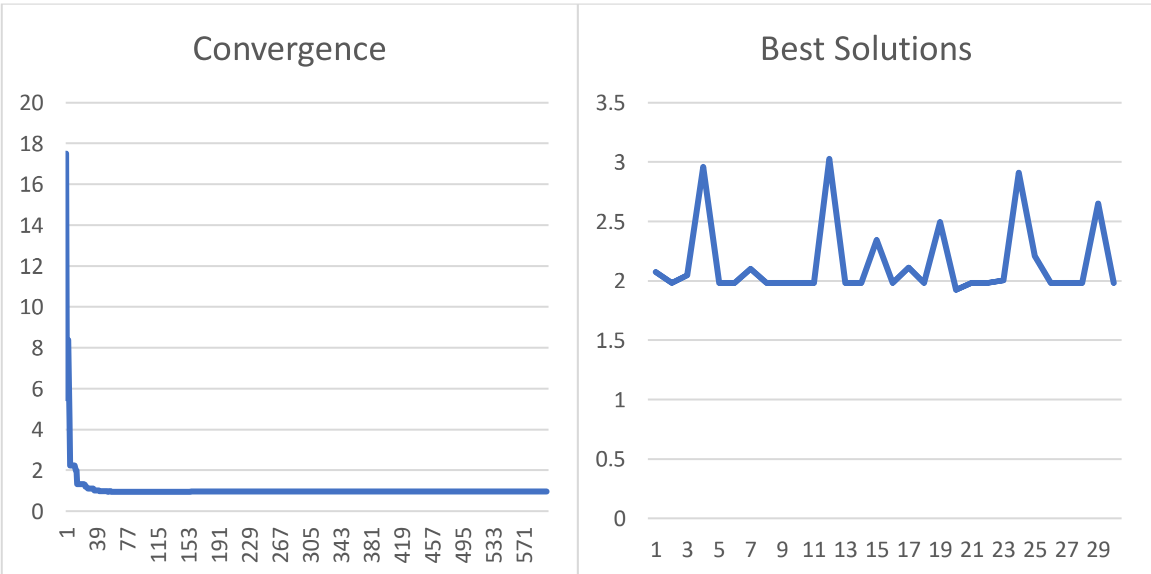

Figure 7*.* Convergence curve and 30 best solutions for Belleville spring design problem

CONCLUSION

Numerical comparisons of the study have witnessed that the M*ABC* found better solution to the non-linear inequality constrains optimization problems in comparison with other meta-heuristic optimizers. The effectiveness and quality of these solutions depends on the nature and complexity of the problem.

REFERENCES

[1] Yang, X. S., Gandomi, A. H., “Bat algorithm: a novel approach for global engineering optimization”, Engineering Computations, Vol. 29, No. 5, 2012, pp.1-18.

[2] Tabassum, M.F., M. Saeed, N. A. Chaudhry, J. Ali and A. Sana, “Modeling and optimizing oxygen production system using differential evolution”. Science International, Vol. 28, No. 1, 2015, pp. 1-6.

[3] Goldberg, D. E., “Genetic algorithms in search, optimization, and machine learning”. Studies in Computational Intelligence, Addison-Wesley Longman Publishing, 1989, pp. 146.

[4] Kirkpatrick, S., C. D. Gellat and M. P. Vecchi, “Optimization by simulated annealing”. Science. Comp. Int, Vol. 220, 1983, pp. 671-680.

[5] Kennedy, J. and R. Eberhart, “Particle swarm optimization”. Proc. of the IEEE Int. Conf. on Neural Networks, Piscataway, 1995, pp. 1942.

[6] Gandomi, A. H and A. H. Alavi, “Krill herd: A new bio-inspired optimization algorithm”. Commun. Nonlinear Sci. Numer. Simulate, Vol. 17, 2012, pp. 4831-4845.

[7] Karaboga D. and B. Akay, “A modified artificial bee colony (ABC) algorithm for constrained optimization problems”. Appl. Soft Compute, Vol. 11, 2011, pp. 3021-3031.

[8] Yang, X. S., “A new meta-heuristic bat-inspired algorithm”, NICSO, Springer, Vol. 284, 2010, pp. 65-74.

[9] Yang, X. S., “Firefly algorithm, stochastic test functions and design optimization”. INT J BIOINSPIR COM, Vol. 2, 2010, pp. 78-84.

[10] Singh, A., “An Artificial Bee Colony algorithm for the leaf-constrained minimum spanning tree problem”. Appl. Soft Compute, Vol. 9, No. 2, 2009, pp. 625–631.

[11] Karaboga, D. and B. Basturk, “On the performance of artificial bee colony”. Appl. Soft Compute, Vol. 8, No. 1, 2008, pp. 687–697.

[12] Storn, R. and K. Price, “Differential evolution-a simple and efficient heuristic for global optimization over continuous spaces”. J. GLOBAL OPTIM, Vol. 11, 1997, pp. 341-359.

[13] Sadollah, A., A. Bahreininejad, H. Eskandar and M. Hamdi, “Mine blast algorithm: A new population-based algorithm for solving constrained engineering optimization problems”. Appl. Soft Compute. Vol. 13, 2013, pp. 2592–2612.

[14] Ahrari, A. and A. A. Atai, “Grenade explosion method - A novel tool for optimization of multimodal functions”. Appl. Soft Compute, Vol. 10, 2010, pp. 1132-1140.

[15] Teodorovic, D. and M. D. Orco., “Bee colony optimization a cooperative learning approach to complex transportation problems”, PONZAN, 10th EWGT Meeting, 2005, pp. 3-16.

[16] Teodorovic, D., “Transport modeling by multi-agent systems: a swarm intelligence approach”. TRANSPORT. PLAN. TECHN, Vol. 26, No. 4, 2003, 99. 289–312.

[17] Yang, X., “Engineering optimization via nature inspired virtual bee algorithms”. J. Mira, J.R. Alvarez. IWINAC, 2005, pp. 317.

[18] Pham, D. T., A. Granbarzadeh, E. Koc, S. Otri, S. Rahim and M. Zaidi.” The bee’s algorithm – a novel tool for complex optimization problems”. Innovation Production Machines and System Virtual Conference, 2006, http://conference.iproms.org.

[19] Wei, F. G., S. Liu, and L. L. Huang. “A novel artificial bee colony algorithm based on, modified search equation and orthogonal learning”, IEEE TRANS CYBERN, Vol. 43, 2013, pp. 1011–1024.

[20] Zhu, G. and S. Kwong. “Gbest-guided artificial bee colony algorithm for numerical function optimization”. APPL MATH COMPUT, Vol. 217, No. 7, 2010, pp. 3166–3173.

[21] Gao, W. F. and S. Y. Liu. “A modified artificial bee colony algorithm”. COPMUT OPER RES, Vol. 39, No. 3, 2012, 687–697.

[22] Li, G., P. Niu, and X. Xiao. “Development and investigation of efficient artificial bee colony algorithm for numerical function optimization”. APPL. Soft COMPUT, Vol. 12, No. 1, 2012, pp. 320–332.

[23] Kang, F., J. Li and Z. Ma. “Rosen brock artificial bee colony algorithm for accurate global optimization of numerical functions”. INF SCI, Vol. 181, No. 16, 2011, pp. 3508–3531.

[24] Mustafa, S. K., A. Babalik. “Improved artificial bee colony algorithm for continuous optimization problems”. J COMPUT COMMUN, Vol. 2, 2014, pp. 108-116.

[25] M. F. Tabassum, M. Saeed, N.A. Chaudhry, J. Ali and M. Farman, S. Akram. “Evolutionary simplex adaptive Hooke-Jeeves algorithm for economic load dispatch problem considering valve point loading effects”, Ain Shams Eng. J. Vol. 12, No. 1, 2021, pp. 1001–1015.

[26] M. F. Tabassum, S. Akram, S. Mahmood-ul-Hassan, R. Karim, P.A. Naik, M. Farman and M. H. Ahmad, “Differential gradient evolution plus algorithm for constraint optimization problems: A hybrid approach”, Int. J. Optim. Control Theory Appl. Vol. 11, No. 2, 2021, pp. 158–177.

[27] M. F. Tabassum, M. Farman, P.A. Naik, A. Ahmad, A.S. Ahmad and H. Saadia, “Modeling and simulation of glucose insulin glucagon algorithm for artificial pancreas to control the diabetes mellitus”, Network Model. Anal. Health Inf. Bioinf. Vol. 10, No. 1, 2021, pp. 1-8.

[28] M. F. Tabassum, M. Saeed, A. Akg¨ul, M. Farman and S. Akram, “Solution of chemical dynamic optimization systems using novel differential gradient evolution algorithm”, Phys. Scr. Vol. 96, No. 3, 2021, pp. 035212.

[29] M. F. Tabassum, M. Saeed, N. Ahmad, A. Sana and N. Riaz, “Solution of war planning problem using derivative free methods”, Sci. Int. Vol. 27, No. 1, 2015, 395–398.

[30] M. F. Tabassum, M. Saeed, A. Sana and N. Ahmad, “Solution of 7 bar tress model using derivative free methods”. Proc. Pakistan Acad. Sci. Vol. 52, No. 3, 2015, 265–271.

[31] M. F. Tabassum, M. Saeed, N. Chaudhry, A. Sana and Z. Zafar, “Optimal design of oxygen production system by

pattern search methods", Pak. J. Sci. Vol. 67, No. 4, 2015, pp. 371–376.

[32] M. F. Tabassum, M. Saeed, N.A. Chaudary, J. Ali and A. Sana, "Integrated approach of set theory and pattern search methods for solving aircraft attacking problem", Pak. J. Sci. Vol. 69, No. 1, 2017, pp. 136–143.

[33] M. F. Tabassum, M. Saeed, A. Akg¨ul, M. Farman and N.A. Chaudhry, "Treatment of HIV/AIDS epidemic model with vertical transmission by using evolutionary Pad´e-approximation", Chaos Solitons Fractals, Vol. 134, 2020, pp. 109686.

[34] M. F. Tabassum, M. Saeed, N.A. Chaudhry and S. Akram, "Treatment of non-linear epidemiological smoking model using evolutionary Pad´e-approximation", Proc. Pakistan Acad. Sci. Vol. 57, No. 2, 2020, pp. 11–19.

[35] Dog˘an, B. and T. Ölmez. "A new meta-heuristic for numerical function optimization: vortex search algorithm", INF SCI, Vol. 293, 2015, pp. 125–145.

[36] Coello, C. A. C. and E. M. Montes. "Constraint handling in genetic algorithms through the use of dominance-based tournament selection". ADV ENG INFORM. Vol. 16, 2002, pp. 193–203.

[37] Coello, C. A. C. "Use of a self-adaptive penalty approach for engineering optimization problems". Compute. Ind. Vol. 41, 2000, pp. 113–127.

[38] Akay, B. and D. Karaboga. "Artificial bee colony algorithm for large-scale problems and engineering design optimization". J. Intell. Manuf. Vol. 45, 2010, pp. 45-54. http://dx.doi.org/10.1007/s10845-010-0393-4.

[39] Kannan, B. K. and S. N. Kramer. "An augmented lagrange multiplier-based method for mixed integer discrete continuous optimization and its applications to mechanical design". J MECH. DESIGN. Vol. 116, 1994, pp. 405–411.