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On Nano β-open sets

A.Revathy[1] and Gnanambal Ilango[2]
[1]Department of Mathematics, Sri Eshwar College of Engineering, Coimbatore, Tamilnadu, India.
[2]Department of Mathematics, Government Arts College, Coimbatore, Tamilnadu, India.
e-mail : [1]revsabs@gmail.com

ABSTRACT

In this paper we introduce a new class of sets called nano β-open sets in nano topological spaces. Various forms of nano β-open set corresponding to different cases of approximations are also derived.

KEYWORDS

Nano topology, nano open sets, nano closed sets, nano interior, nano closure, nano β-open sets.

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1 INTRODUCTION

Abd El Monsef et al. [1] introduced the notion of β-open set in topology, and the equivalent notion of semi-pre open set was given independently by Andrijevic in [2], and further investigated by Ganster and Andrijevic [3]. The notion of nano topology was introduced by Lellis Thivagar [4], which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it. The subset generated by lower approximations is characterized by certain objects that will definitely form part of an interest subset, whereas the upper approximation is characterized by uncertain objects that will possibly form part of an interest subset. The elements of a nano topological space are called the nano open sets. He has also defined nano closed sets, nano interior and nano closure. He has introduced the weak forms of nano open sets namely nano α-open sets, nano semi-open sets, nano pre open sets and nano regular open sets [5]. In this paper we introduce nano β-open sets. Also various forms of nano β-open set are studied for various cases of approximations.

2 PRELIMINARIES

Definition 2.1. [5]
Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

(i) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is,$L_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.

(ii) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is,$U_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \cap X \neq \emptyset\}$

(iii) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Proposition 2.1. [4]
If (U, R) is an approximation space and $X, Y \subseteq U$, then

(i) $L_R(X) \subseteq X \subseteq U_R(X)$;

(ii) $L_R(\emptyset) = U_R(\emptyset) = \emptyset$ and $L_R(U) = U_R(U) = U$;

(iii) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$;

(iv) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$;

(v) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$;

(vi) $L_R(X \cap Y) \subseteq L_R(X) \cap L_R(Y)$;

(vii) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$;

(viii) $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$;

(ix) $U_R U_R(X) = L_R U_R(X) = U_R(X)$;

(x) $L_R L_R(X) = U_R L_R(X) = L_R(X)$.

Definition 2.2. [4]
Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \emptyset, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by the Property 2.2, $\tau_R(X)$ satisfies the following axioms:

(i) U and $\emptyset \in \tau_R(X)$,

(ii) The union of the elements of any subcollection of $\tau_R(X)$ is in $\tau_R(X)$.,

(iii) The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$..

That is, $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called as nano open sets and $[\tau_R(X)]^c$ is called as the dual nano topology of $[\tau_R(X)]$.

Remark 2.1. [4]
If $[\tau_R(X)]$ is the nano topology on U with respect to X, then the set $B = \{U, \emptyset, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.

Definition 2.3. [4]
If $(U, \tau_R(X))$ is a nano topological space with respect to X and if $A \subseteq U$, then the nano interior of A is defined as the union of all nano open subsets of A and it is denoted by $Nint(A)$. That is, $Nint(A)$ is the largest nano open subset of A. The nano closure of A is defined as the intersection of all nano closed sets containing A and it is denoted by $Ncl(A)$. That is, $Ncl(A)$ is the smallest nano closed set containing A.

Definition 2.4. [4]
Let $(U, \tau_R(X))$ be a nano topological space and $A \subseteq U$. Then A is said to be

(i) nano semi-open if $A \subseteq Ncl(Nint(A))$

(ii) nano pre open if $A \subseteq Nint(Ncl(A))$

(iii) nano α-open if $A \subseteq Nint(Ncl(Nint(A)))$

(iv) nano regular open if $A = Nint(Ncl(A))$

$NSO(U, X)$, $NPO(U, X)$ and $N\alpha O(U, X)$ respectively denote the families of all nano semi-open, nano pre open and nano α-open subsets of U.

Definition 2.5. [4]
$(U, \tau_R(X))$ be a nano topological space and $A \subseteq U$. A is said to be nano semi-closed (resp. nano pre closed, nano α-closed), if its complement is nano semi-open (nano pre open, nano α-open).

3 Nano β-open sets

Throughout this paper $(U, \tau_R(X))$ is a nano topological space with respect to X where $X \subseteq U$, R is an equivalence relation on U, U/R denotes the family of equivalence classes of U by R.

Definition 3.1.
A Subset A of a nano toplogical space $(U, \tau_R(X))$ is nano β-open in U if $A \subseteq Ncl(Nint(Ncl(A)))$. The set of all nano β-open sets of U is denoted by $N\beta O(U, X)$.

Proposition 3.1.
If $U_R(X) = U$ in a nano topological space then $N\beta O(U, X)$ is $P(U)$.
Proof
Let $U_R(X) = U$.

(i) If $L_R(X) = \emptyset$, then $B_R(X) = U$. And then $\tau_R(X) = \{U, \emptyset\}$. Thus for any set A, $Ncl(Nint(Ncl(A))) = U$. Hence $A \subseteq Ncl(Nint(Ncl(A)))$. Therefore A is nano β-open in U. Hence $N\beta O(U, X)$ is $P(U)$.

(ii) If $L_R(X) \neq \emptyset$. Each element of U is either in $L_R(X)$ or in $B_R(X)$, since $B_R(X) = U_R(X) - L_R(X)$.

If $A \subseteq L_R(X)$ then $Ncl(Nint(Ncl(A))) = L_R(X)$ and if $A \subseteq B_R(X)$ then $Ncl(Nint(Ncl(A))) = B_R(X)$ because $[L_R(X)]^c = B_R(X)$, $[B_R(X)]^c = L_R(X)$ and $U_R(X) = U$.. If A intersects both $L_R(X)$ and $B_R(X)$, $Ncl(A) = U$. Then $Ncl(Nint(Ncl(A))) = U$. Hence $A \subseteq Ncl(Nint(Ncl(A)))$. Therefore A is nano β-open in U. Hence $N\beta O(U, X)$ is $P(U)$.

Proposition 3.2.
If $U_R(X) \neq U$ then $U, \emptyset$ and any set which intersects $U_R(X)$ are nano β-open set in U.
Proof

Case(i) $L_R(X) = \emptyset$ or $L_R(X) = U_R(X)$. In both situations $\tau_R(X) = \{U, \emptyset, U_R(X)\}$. If A intersects $U_R(X)$ then $Ncl(A) = U$. And hence $Ncl(Nint(Ncl(A))) = U$. Therefore A is nano β-open set in U. If $A \subseteq [U_R(X)]^c$, then $Ncl(A) = [U_R(X)]^c$. And hence $Ncl(Nint(Ncl(A))) = \emptyset$. Therefore A is not nano β-open in U.

Case(ii) If $L_R(X) \neq \emptyset$ then $\tau_R(X) = \{U, \emptyset, L_R(X), U_R(X), B_R(X)\}$.

1. If $A \subseteq U_R(X)$
   Suppose $A \subseteq L_R(X)$, then $Ncl(A) = [B_R(X)]^c$ and $Nint(Ncl(A)) = L_R(X)$ and hence $Ncl(Nint(Ncl(A))) = [B_R(X)]^c \supseteq L_R(X) \supseteq A$. Therefore A is nano β-open in U. Suppose $A \subseteq B_R(X)$, then $Ncl(A) = [L_R(X)]^c$ and $Nint(Ncl(A)) = B_R(X)$ and hence $Ncl(Nint(Ncl(A))) = [L_R(X)]^c \supseteq B_R(X) \supseteq A$. Therefore A is nano β-open in U. Suppose A intersects both $L_R(X)$ and $B_R(X)$, $Ncl(A) = U$ and hence $Ncl(Nint(Ncl(A))) = U$. Therefore A is nano β-open in U.

2. If $A \subseteq [U_R(X)]^c$, $Ncl(A) = [U_R(X)]^c$. And hence $Ncl(Nint(Ncl(A))) = \emptyset$. Therefore A is not nano β-open in U.

3. If A intersects both $U_R(X)$ and $[U_R(X)]^c$.

Suppose A contains elements of $L_R(X)$ and $[U_R(X)]^c$, $Ncl(Nint(Ncl(A))) = [B_R(X)]^c = [U_R(X)]^c \cup L_R(X) \supseteq A$. Therefore A is nano β-open in U. Suppose A contains elements of $B_R(X)$ and $[U_R(X)]^c$, $Ncl(Nint(Ncl(A))) = [L_R(X)]^c = [U_R(X)]^c \cup B_R(X) \supseteq A$. Therefore A is nano β-open in U. Suppose A intersects $L_R(X), B_R(X)$ and $[U_R(X)]^c$ then $Ncl(A) = U$ and hence $Ncl(Nint(Ncl(A))) = U$. Therefore A is nano β-open in U

Proposition 3.3.

If A is nano open in $(U, \tau_R(X))$, then it is nano β-open in U.

Proof

Since A is nano open in U, $Nint(A) = A$. And $A \subseteq Ncl(A)$ always. Then $A \subseteq Ncl(Nint(A)) \subseteq Ncl(Nint(Ncl(A)))$. Therefore $A \subseteq Ncl(Nint(Ncl(A)))$ Hence A is nano β-open in U.

Proposition 3.4.

Every nano semi-open set is nano β-open in $(U, \tau_R(X))$.

Proof

If A is nano semi-open, then $A \subseteq Ncl(Nint(A)) \subseteq Ncl(Nint(Ncl(A)))$. Hence A is nano β-open.

Proposition 3.5.

Every nano pre open set is nano β-open in $(U, \tau_R(X))$.

Proof

If A is nano pre open, then $A \subseteq Nint(Ncl(A)) \subseteq Ncl(Nint(Ncl(A)))$. Hence A is nano β-open.

Proposition 3.6.

Every nano regular open set is nano βopen in $(U, \tau_R(X))$.

Proof

If A is nano regular open, then $A = Nint(Ncl(A)) \subseteq Ncl(Nint(Ncl(A)))$. Hence A is nano β-open in U.

Proposition 3.7.

Every nano α-open set is nano β-open in $(U, \tau_R(X))$.

Proof

Since $Nint(A) \subseteq A$, $Ncl(Nint(A)) \subseteq Ncl(A)$. Therefore $Nint(Ncl(Nint(A))) \subseteq Nint(Ncl(A)) \subseteq Ncl(Nint(Ncl(A)))$. Hence A is nano β-open in U.

Remark 3.1.

The converse of the above proposition 3.3, 3.4, 3.5, 3.6 and 3.7 need not be true always as seen from the following example.

Example 3.1.

Let $U = \{a, b, c, d\}$, U/R=$\{\{a, b\}, \{c\}, \{d\}\}$ and $X = \{a, c\}$ Then $L_R(X) = \{c\}$, $U_R(X) = \{a, b, c\}$, $B_R(X) = \{a, b\}$. Therefore $\tau_R(X) = \{U, \emptyset, \{c\}, \{a, b, c\}, \{a, b\}\}$. Now $A = \{a, d\}$ is nano β-open in U but not nano open, nano semi-open, nano pre open, nano regular open and nano α-open in U.

Proposition 3.8.

If A and B are nano β-open sets in a space $(U, \tau_R(X))$ then $A \cup B$ is also nano β-open in U.

Proof

Since A and B are nano β-open sets, $A \subseteq Ncl(Nint(Ncl(A)))$ and $B \subseteq Ncl(Nint(Ncl(B)))$.

Then $A \cup B \subseteq Ncl(Nint(Ncl(A))) \cup Ncl(Nint(Ncl(B))) = Ncl[Nint(Ncl(A)) \cup Nint(Ncl(B))] \subseteq Ncl[Nint[Ncl(A) \cup Ncl(B)]]$. Therefore $A \cup B \subseteq Ncl(Nint(Ncl(A \cup B)))$. Hence $A \cup B$ is also nano β-open in U.

Remark 3.2.

The intersection of two nano β-open sets need not be nano β-open set. In Example 3.1, the sets $\{a, d\}$ and $\{b, d\}$ are nano β-open sets in U but their intersection $\{d\}$ is not nano β-open in U.

Remark 3.3.

$N\beta O(U, X)$ is supra topology on U.

Proposition 3.9.

$NSO(U, X) \cup NPO(U, X) \subset N\beta O(U, X)$

Proof

Proof follows from proposition 3.4, 3.5 and 3.8.

Remark 3.4.

The equality in the above theorem does not hold in general. In Example 3.1, the set $A = \{a, d\}$ is nano β-open but not in $NSO(U, X) \cup NPO(U, X)$.

Proposition 3.10.

If V is nano open and A is nano β-open then $V \cap A$ is nano β-open.

Proof

$V \cap A \subseteq V \cap Ncl(Nint(Ncl(A))) \subseteq Ncl(V \cap Nint(Ncl(A))) \subseteq Ncl(Nint(Ncl(V \cap A)))$. Therefore $V \cap A$ is nano β-open.

Proposition 3.11.

If B is nano subset of U and A is nano pre open in U such that $A \subseteq B \subseteq Ncl(Nint(A))$. Then B is nano β-open in U.

Proof

Since A is nano preopen in U, $A \subseteq Nint(Ncl(A))$. Now $B \subseteq Ncl(Nint(A)) \subseteq Ncl(Nint(Nint(Ncl(A)))) = Ncl(Nint(Ncl(A))) \subseteq Ncl(Nint(Ncl(B)))$. Hence $B \subseteq Ncl(Nint(Ncl(B)))$. Therefore B is nano β-open in U.

Proposition 3.12.

Each nano β-open set which is nano semi-closed is nano semi-open.

Proof

Let $A \in N\beta O(U, X) and \in NSC(U, X)$. Then $A \subseteq Ncl(Nint(Ncl(A)))$ and $Nint(Ncl(A)) \subseteq A$. Hence $Nint(Ncl(A)) \subseteq A \subseteq Ncl(Nint(Ncl(A)))$. Since $Nint(Ncl(A)) = G$ is a nano open set in $(U, \tau_R(X))$, i.e., there exists a nano open set such that $G \subseteq A \subseteq Ncl(G)$. Therefore A is a nano semi-open set.

Proposition 3.13.

Each nano β-open set which is nano α-closed is nano closed.

Proof

Obvious.

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