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GREEN INTEGRATED MODEL FOR IMPERFECT PRODUCTION PROCESS UNDER RELIABILITY

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Abstract. This paper presents the green integrated inventory model for single producer and single supplier with imperfect production process for deteriorating items under the effect of inflation.Nowadays, many policies have been implemented in many countries for the reduction of carbon emissions. The industry implements these policies for the goal of its maximum profit along with ecology protection. This papers also considers carbon emission reduction technology under the reliability effect. The objective of this study is to develop a supply chain model that minimizes the total cost of the system with carbon tax. The proposed model is illustrated with numerical example along with the sensitivity analysis.

1. Introduction

Supply chain management is known to be a popular method of coordination between industries, companies, suppliers and customers. Since the relationship between all the components involved in a business is very important in a competitive environment, many companies have kept striving to improve the relationship between parties. It seeks to further improve the performance of the business. As a result, supply chain management reduces the chances of delays or spoilage of goods to the customer, while increasing revenue, reducing the cost of transportation. Easy access to customers through a well-organized supply chain that builds trust between the manufacturer, supplier and customer and maintains demand for the product. The infrastructure of the supply chain generally includes processing units and retail units. In the infrastructure of the supply chain, a product must go through several stages, from the process of supply and conversion of raw materials to storage of finished goods. This study presents a multi-echelon supply chain model for deteriorating items, with reduction of carbon emissions in the process of imperfect production. Along with this, the model also discusses the deteriorating effect of the items. In real life many inventory items tend to deteriorate over time or due to loss of quality during the production. Also, the proposed model is formulated in the inflationary environment. Supply chain management is a topic of discussion along with carbon emissions and carbon footprint management. Efforts to reduce carbon emissions have been discussed in several

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current studies, including carbon tax, carbon emission limits and carbon offsets. To reduce carbon emissions, companies can invest in various components of supply chain management such as production units, transportation, design, inventory, etc. Also, carbon emissions can be reduced by using less polluting energy sources and investing in green technology. Thus, this work is more suitable from the point of view of supply chain management because if all the members of the supply chain management make common investments for the related facilities, then along with reduction in carbon emissions, large profits can be obtained at the minimum cost. Due to the pressure of law, green lover and various organizations, business and industry are trying to make themselves eco-friendly. From the production, transportation, distribution and consumption of a product, carbon is emitted in some way or the other at all stages. Therefore, the reduction of carbon emissions in the supply chain provides large-scale interest in the study of researchers, whose aim is to create an environmentally friendly inventory model by reducing carbon emissions through the supply chain. Rest part of paper is organized as follows: Section2 represents the literature regarding the supply chain, carbon reduction technology, deteriorating items, imperfect production and inflation. Assumptions and Notations are explained in section.3. Mathematical development of this paper shown in section.4. Numerical example along with sensitivity analysis with the help of observation table is shown in section .5. Finally, conclusion and scope of future research is provided in section. 6.

2. Literature Review

2.1. Inventory model based on supply chain: The supply chain model involves the process of raw material being manufactured in the factory to the finished goods to the end customer.The supply chain includes the producer as well as the buyer, distributor, transporter and customer. The action of one member in the supply chain affects the profit or loss of other members of the chain. The success of any business depends on its efficient supply chain management. According to the demand of the market, it works to carry the information to the end customer, its demand from the lowest level to the top, so that the effect of that product or service in the market remains and all the members of the chain get maximum profit by fulfilling the wishes of the customers. The supply chain ensures the safe passage of goods from their origin to the end. In recent years, many research papers have been published by researchers taking the supply chain. Manna et al. [1] presented a two-layer green supply chain inventory model for imperfect production in which three tier credit period is taken. Ullah et al.[2] established a two-echelon supply chain model under the effect of PT on waste production. The investment in the PT used by the researchers to control the amount of deterioration is assumed to be variable taken in this paper.Bhuniya et al.[3] presented a smart production inventory system in which production process of items under supply chain management for maximum energy consumption is considered. Rout et al.[4] developed an inventory model for deteriorating and imperfect items keeping in mind the carbon emission norms under sustainable supply chain. Through this paper, they developed a model to reduce the emission of carbon caused by various sources of SCM like transportation, warehousing by implementing various carbon reduction policies. Daryanto and Wee[5]discussed a green supply chain inventory model having deteriorated imperfect quality items.

2.2. Inventory model based on carbon reduction technology: At present, along with the progress of human beings, its adverse effects on the environment are also visible, of which the most important concern is global warming, the main reason for many reasons is carbon emissions. This is a matter of very important concern for environmental lovers and people who keep the earth healthy and clean. It is true that industries are the backbone of any country's development. Without industrial development, the economic development of any country cannot be imagined, but these industries are becoming the main cause of global warming by emitting carbon. In addition, components of the supply chain, such as the transport involved in freight transport which causes carbon emissions. In this context, many policies have been made by the governments of countries to create a pollution-free environment so that emissions can be reduced. In this direction, companies are emphasizing on developing such inventory models that can minimize carbon emissions. In order to protect the environment, many works have been done by researchers, industry organizations and other green organizations in the last few years to control carbon emissions. Huising et al.[6] presented a research paper for evaluation and modelling of policies, technologies to know the progress of carbon emission reduction in recent years. The transportation sector is one of the major causes of carbon emissions. Carbon emissions from transportation are mainly due to burning of fossil fuels in our cars, trucks, airplanes and other means of transport. Ahmed and Sarkar[7] presented a supply chain inventory model for biofuel under the effect of carbon emission. The deterioration of goods is their natural property. Researchers have published lots of research papers regarding the deterioration during transportation, storage of goods.Daryanto et al.[8] presented an inventory model for imperfect quality items by considering the carbon supply chain coordination under deterioration. Mashud et al.[9] worked on a supply chain inventory model for imperfect items under the inflation and carbon emission. A growing item inventory is a set of items whose level is increased over the period of stocking. This type of business is seen in the poultry industry and the livestock business, where the new born is fed until its weight reaches a marketable norm, then it is sent to slaughterhouses throughout the process. There is also the emission of carbon which is harmful to the environment. De-la-Cruz-Márquez et al.[10] developed carbon emission inventory model for growing items under shortage.

2.3. Inventory model based on deteriorating items: Deterioration refers to decay or damage of a substance. Deterioration refers to decay or damage of a substance. Many substances such as food items, medicines, chemicals etc. can get spoiled during the storage period. Deterioration of any substance after a certain period is its natural property. The deterioration rate of stock during storage has attracted the attention of researchers. The concept of deterioration of items plays a very important role in developing the inventory model, so deterioration cannot be ignored while developing the inventory model. In recent years, many researchers have developed lots of research papers by using the concept of deterioration of items in their research. Tiwari et al. [11] presented a sustainable inventory model for deteriorating and imperfect quality items. Khurana et al. [12]proposed a production inventory model for deteriorating item, where demand is taken as a variable under the shortage of items. Mahapatra et al.[13] discussed the fuzzy EOQ model for deteriorating items under learning effect having promotional effect where time interval is finite. Tayal et al.[14] presented a limited storage problem inventory model for deteriorating items having different demand under shortage, where both the warehouse as a rented warehouse. In recent years lots of research papers-based discount policy has been published. Saren et al.[15] presented a price discount policy in advanced inventory model for deteriorating item having delay in payment.

2.4. Inventory model based on imperfect production process: Many Inventory Models were developed on the basis that all the products produced during production are of 100% perfect quality, but in real situation it is not possible, as sometimes due to machine failure or other reasons, perfect as well as imperfect quality items may also be produced. It is important to properly monitor and list the imperfect products produced in the production system. Usually, these products are sold at low prices without making them recyclable, because their quality is less than the quality of the original product. Various research was done to solve the problems faced in the model of EPQ with imperfect quality, lots of research papers have been published by many researchers regarding the problem of production of imperfect material during the production process. The performance of a person engaged in any business, or an organization can be improved with time. This effect of learning reduces the cost of the product and increases the quality of the product and possibility of getting the profit is high. Saha and Chakrabarti[16] proposed an integrated inventory model for imperfect production having different demands for deteriorating items under trade credit. A flexible manufacturing system is a system in which during the production, with the help of computerized system the type and quantity of the product being manufactured can be changed. Dem et al.[17]presented an inventory model for imperfect items having different demands based on flexible manufacturing system. Kumar et al.[18] presented a fuzzified inventory model for imperfect production process having different demands.

2.5. Inventory model based on inflation: Inflation is one of the most popular terms in economics. Inflation refers to the rate of increase in the price of a commodity over a given period. Inflation reflects an increase in the cost of living in a country along with an increase in prices. Singh and Singh[19] presented a production inventory models for imperfect production with exponential demand and inflation. Uncertainty is defined as the ambiguity that is characteristic of fuzzy set theory. Singh et al.[20] derived an imperfect production model for storage problem under the effect of inflation and having shortage of items in fuzzy environment. They also calculated fuzzy total cost of the inventory model with the help of triangular fuzzy number. Volume flexibility is a flexible system which allows production to be adjusted up or down during the time of manufacturing. Pal et al.[21] discussed a production inventory model for deteriorating items under shortage under the effect of inflation. They also assumed linear demand with time value of money and constant deterioration rate. Kumar and Rajput[22] established an inventory model under probabilistic deterioration and inflation with shortage having ramp type demand. Singh et al.[23]proposed an inventory model for deteriorating items with trade credit and shortage under replenishment policy. Barman et al.[24] gives a cloudy fuzzy inventory model for deteriorating items with shortage under the effect of inflation where the total cost is defuzzified by cloudy fuzzy number as well as ranking index method of fuzzy. Padiyar et al.[25] established an inventory model for deteriorating items with price dependent consumption and shortages under fuzzy environment. Kuraie et al.[26] worked on an imperfect production process in a supply chain system with multivariate demand and limited storage capacity. Padiyar et al.[27] investigated an integrated supply chain subjected to imperfect production with probabilistic demand and variable production rate. Padiyar et al.[28] developed a production inventory model considering limited storage for perishable commodities with learning and inspection using fuzzy parameters.

3. Assumption and Notation

The following assumptions are mainly made to develop the inventory model:

(1) Demand rate and Production rate are constants.
(2) Shortage is not allowed.
(3) System involves single supplier and single producer.
(4) Multi deliveries per order are considered.
(5) The investment made in technology to rate carbon emissions is represented by $m(L)$ where $0 < m(L) < 1$, and $m(L)$ is increasing the work of L.
(6) The investment ratio between supplier and producer to reduce carbon emissions is the capital investment ratio $1-\gamma$ and γ.

4. Problem Description

4.1. Model Development for Producer: In the producer's inventory system, the planning horizon T is divided into two intervals, namely, $[0, T_1]$ and $[T_1, T]$, where $[0, T_1]$ is production period and $[T_1, T]$ is the non-production period. There are the following differential equations for the inventory level of producer:

$$\frac{dI_{P_1}}{dt} = AP - D - \theta_1 I_{P_1}(t), \qquad 0 \le t \le T_1, \tag{4.1}$$

$$\frac{dI_{P_2}}{dt} = -D - \theta_1 I_{P_2}(t), \qquad T_1 \le t \le T. \tag{4.2}$$

With boundary condition $I_{P_1}(0) = 0$ and $I_{P_2}(T) = 0$, the solution of the differential equation (4.1) and (4.2) are follows,

$$I_{P_1}(t) = \frac{(AP-D)}{\theta_1}[1 - e^{-\theta_1 t}] \tag{4.3}$$

$$I_{P_2}(t) = \frac{D}{\theta_1}[e^{(T-t)\theta_1} - 1] \tag{4.4}$$

By the continuity

$$I_{P_1}(T_1) = I_{P_2}(T_2),$$

TABLE 1. Notations

T	Length of the inventory cycle
d_i	Rework cost for imperfect item
n	Number of deliveries from producer to supplier
h_s	Holding cost for supplier
d_s	Deterioration cost for supplier
A_s	Ordering cost for supplier
r	Constant rate of inflation per unit time
h_p	Holding cost for producer
d_p	Deterioration cost for producer
c_p	Production cost for producer
A_p	Setup cost for producer
θ_1	Deterioration rate for producer
θ_2	Deterioration rate for supplier
λ	Percentage of the defective items
A	Reliability rate to produced good items
P	Production rate per unit time
D	Demand rate per unit time
L	Technology investment for carbon emission reduction
$m(L)$	Function of L, which proportion of reduced the carbon
$\overline{\overline{A_p}}$	Order cost for producer under carbon emission
$\overline{\overline{h_p}}$	Holding cost per unit for producer under carbon emission
$\overline{\overline{d_p}}$	Deteriorating cost per unit for producer under carbon emission
$\overline{\overline{d_l}}$	Amount of rework cost in carbon emission per unit
$\overline{\overline{h_s}}$	Holding cost for supplier under carbon emission
$\overline{\overline{d_s}}$	Deteriorating cost per unit for supplier under carbon emission
$\overline{\overline{A_s}}$	Ordering cost for supplier under carbon emission

Then

$$T_1 = \frac{1}{\theta_1} Log\Big[\frac{De^{T\theta_1} + AP - D}{AP}\Big] \tag{4.5}$$

Producer's total cost depends on the following factor;

(a) Set-up Cost: The cost of setting up the machines prior to production or purchase from an external supplier. These costs include loading, unloading, telephone bills, transportation costs, etc.

$$SC_p = A_p \tag{4.6}$$

(b) Holding Cost: The cost incurred as a result of moving the inventory. It includes the cost of storing and managing the inventory such as warehousing, transportation, cost of staff to maintain stores, insurance, deterioration and obsolescence cost etc.

$$HC_p = h_p\Big[\int_0^{T_1} I_{P_1}(t)e^{-rt}dt + \int_{T_1}^{T} I_{P_2}(t)e^{-rt}dt\Big]$$

$$\begin{aligned} HC_p &= h_p\Big[\frac{(AP-D)}{\theta_1}\Big[\Big(\frac{1-e^{-rT_1}}{r}\Big)+\Big(\frac{e^{-(\theta_1+r)T_1}-1}{\theta_1+r}\Big)\Big] \\ &\quad +\frac{D}{\theta_1}\Big[\Big(\frac{e^{-rT}-e^{-rT_1}}{r}\Big)+e^{\theta_1 T}\Big(\frac{e^{-(\theta_1+r)T_1}-e^{(\theta_1+r)T}}{r+\theta_1}\Big)\Big]\Big] \end{aligned} \tag{4.7}$$

(c) Deterioration Cost: Deterioration of goods is a common phenomenon which is found in many substances such as food items, medicines, fruits, blood etc. The biggest problem facing any supply manager is inventory and maintenance of deteriorating items. The cost of deterioration is also affected by inflation in economy.

$$DC_p = d_p\theta_1\Big[\int_0^{T_1} I_{P_1}(t)e^{-rt}dt+\int_{T_1}^{T} I_{P_2}(t)e^{-rt}dt\Big]$$

$$\begin{aligned} DC_p &= d_p\theta_1\Big[\frac{(AP-D)}{\theta_1}\Big[\Big(\frac{1-e^{-rT_1}}{r}\Big)+\Big(\frac{e^{-(\theta_1+r)T_1}-1}{\theta_1+r}\Big)\Big] \\ &\quad +\frac{D}{\theta_1}\Big[\Big(\frac{e^{-rT}-e^{-rT_1}}{r}\Big)+e^{\theta_1 T}\Big(\frac{e^{-(\theta_1+r)T_1}-e^{(\theta_1+r)T}}{r+\theta_1}\Big)\Big]\Big] \end{aligned} \tag{4.8}$$

(d) Number of defective items: There are two cases: First, if the machine turn to out of control state in the time interval $[T_1, T]$, then there will be no defective items, but if the machine is in out of control state in the time interval $[0, T_1]$,then there will be defective items as given below

$$\begin{aligned} N &= \begin{cases} 0 & \text{if } X \geq T_1 \\ \lambda\int_X^{T_1} P dt & \text{if } X < T_1 \end{cases} \\ &= \begin{cases} 0 & \text{if } X \geq T_1 \\ \lambda P(T_1-X) & \text{if } X < T_1 \end{cases} \end{aligned} \tag{4.9}$$

Therefore, the expected number of defective items in a production cycle is

$$E(N) = \int_0^{T_1} \lambda P(T_1-X)f(X)dx \tag{4.10}$$

After each inspection for defective items process through laser and coating operations, rework parts are returned to workstation for processing, which includes the costs viz. processing cost, labour cost, material cost etc.
The present worth rework cost at $t = T_1$ can be expressed approximately as

$$RC_p = d_i E(N)e^{-rT_1}$$

$$RC_p = d_i\Big\{\int_0^{T_1} \lambda P(T_1-X)\mu e^{-\mu X}e^{rT_1}dX\Big\}$$

$$RC_p = d_i\lambda P\mu e^{-rT_1}\Big[T_1+\Big(\frac{e^{-\mu T_1}-1}{\mu^2}\Big)\Big] \tag{4.11}$$

Therefore average total cost of the producer is

$$\Pi_p = \chi_1 \tag{4.12}$$

Where, $\chi_1 = \frac{1}{T}[SC_p + DC_p + HC_p + RC_p]$ The technology investment for the reduction of carbon emissions is L. Because there is a partnership between the two members of the supply chain for this technology investment,
The ratio of producer investment to a total of $0 \leq \gamma \leq 1$ reduces carbon emissions per unit for the producer by indicating the investment by γL. So the producer's total cost is

$$\Pi_{CEP} = \sigma_1 \tag{4.13}$$

$$\sigma_1 = \frac{1}{T}[SC_p + DC_p + HC_p + RC_p] + \gamma L$$

At the reduction rate of carbon emissions $m(L)$, the total amount for carbon emissions produced by the manufacturer per unit time.

$$\overline{\overline{\Pi_p(T,L)}} = \frac{1 - m(L)}{T}[\overline{\overline{SC_p}}] + \overline{\overline{DC_p}} + \overline{\overline{HC_p}} + \overline{\overline{RC_p}}] \tag{4.14}$$

For each unit of carbon emitted due to carbon tax regulation α amount is paid for the total. Therefore, the total cost per unit producer with carbon tax.

$$\Pi_{CP}(T,L) = \Pi_{CEP}(T,L) + \alpha\overline{\overline{\Pi_p(T,L)}} \tag{4.15}$$

4.2. Model Development for Supplier: During the interval $[0, T_2]$, the inventory level decreases due to deterioration as well as demand. The supplier's inventory system can be represented by the linear differential equation,

$$\frac{dI_s(t)}{dt} = -D - \theta_2 I_s(t), \qquad 0 \leq t \leq T_2 \tag{4.16}$$

Using the boundary condition $I_s(T_2) = 0$, the solution of (4.16) is

$$I_s(t) = \frac{D}{\theta_2}[e^{\theta_2(T_2-t)} - 1] \tag{4.17}$$

Supplier total cost depends on following factors:
(a) Holding Cost: It is the cost associated with holding one unit of inventory for one unit of time. Holding cost varies with the level of inventory and occasionally with the length of time an item is held.

$$HC_s = \sum_{j=1}^{n} h_s \Big[\int_{(j-1)T_2}^{jT_2} I_s(t)e^{-rt}dt\Big]$$

$$HC_s = \sum_{j=1}^{n} \frac{D}{\theta_2} h_s \Big[e^{\theta_2 T_2}\Big(\frac{e^{-(\theta_2+r)(j-1)T_2} - e^{-(\theta_2+r)jT_2}}{\theta_2 + r}\Big) + \Big(\frac{e^{-rjT_2} - e^{-r(j-1)T_2}}{r}\Big)\Big] \tag{4.18}$$

(b) Deterioration Cost: Deterioration is defined as damage, change, decay and loss of goods or loss of original value and price in a commodity that results in the decreasing usefulness from the original one.

$$DC_s = \sum_{j=1}^{n} \theta_2 d_s \Big[\int_{(j-1)T_2}^{jT_2} I_s(t)e^{-rt}dt\Big]$$

$$DC_s = \sum_{j=1}^{n} Dd_s\Big[e^{\theta_2 T_2}\Big(\frac{e^{-(\theta_2+r)(j-1)T_2} - e^{-(\theta_2+r)jT_2}}{\theta_2 + r}\Big) + \Big(\frac{e^{-rjT_2} - e^{-r(j-1)T_2}}{r}\Big)\Big] \tag{4.19}$$

(c) Ordering Cost: It is the cost of placing an order to an outside supplier. Ordering costs include details like counting items and calculating order quantities. It may or may not depend on the amount ordered. Due to inflation in economy, ordering cost may change from time.

$$OS_c = A_s \tag{4.20}$$

Therefor average total cost of the supplier is

$$\Pi_s = \chi_2 \tag{4.21}$$

$$\chi_2 = \frac{1}{T}[OC_s + DC_s + HC_s]$$

The supplier's total technical investment in carbon emission reduction is L. This is because of the partnership between the manufacturer and the supplier in this investment, where the supplier ratio is $1-\gamma$, in this investment the total carbon emissions per unit time by the supplier are reduced by $(1-\gamma)L$.
Therefore, the producer's total cost is

$$\Pi_{CES} = \sigma_2 \tag{4.22}$$

$$\sigma_2 = \frac{1}{T}[OC_s + DC_s + HC_s] + (1-\gamma)L$$

The total amount of carbon emissions produced per unit time

$$\overline{\overline{\overline{\Pi_p(T,L)}}} = \frac{1-m(L)}{T}[\overline{\overline{\overline{OC_s}}}] + \overline{\overline{\overline{DC_s}}} + \overline{\overline{\overline{HC_s}}}] \tag{4.23}$$

Total α amount is paid for each unit of carbon emissions that accompany the carbon tax regulation. Therefore, the supplier's total cost along with carbon tax per unit is

$$\Pi_{CS}(T,L) = \Pi_{CES}(T,L) + \alpha\overline{\overline{\overline{\Pi_s(T,L)}}} \tag{4.24}$$

Total cost for integrated model

$$\Pi_C = \Pi_{cp}(T,L) + \Pi_{CS}(T,L) \tag{4.25}$$

5. Numerical Example

Based on the previous studies, the following values of various parameters are considered in appropriate units for the model:
$d_i = 0.6$(\$/unit), $n = 2$,$h_s = 0.2$(\$/unit), $d_s = 0.1$(\$/unit), $A_s = 18$(\$/order), $r = 0.2$, $h_p = 0.2$(\$/unit), $d_p = 0.1$(\$/unit), $c_p = 2$(\$/unit), $A_p = 25$(\$/setup), $\theta_1 = 0.1$, $\theta_2 = 0.1$, $\lambda = 0.1$, $\alpha = 0.2$, $\mu = 0.3$, $\gamma = 0.3$, $A = 12$, $m(L) = a + bL$, $a = 0.1$, $b = 0.002$, $\bar{\bar{A}}_p = 25$, $\bar{\bar{h}}_p = 0.2$, $\bar{\bar{d}}_p = 0.1$, $\bar{\bar{d}}_l = 0.6$, $\bar{\bar{h}}_s = 0.2$, $\bar{\bar{d}}_s = 0.1$, $\bar{\bar{A}}_s = 18$, $D = 10$ units, $P = 15$ units,
And the solution of the above problem is minimum $\Pi = \$10.2479$, and optimum values $L = \$1.2752, T = 22.2986$ days

FIGURE 1. Convexity of total cost function w.r.t. T and L

5.1. Sensitivity Analysis with Key Parameters.

Parameter	Change(%)	L	T	Total Cost
A_p	-50	0.12678	16.1822	6.33601
	-25	0.23445	19.14	8.29298
	+25	0.32445	26.927	13.2889
	+50	1.27522	30.3751	15.9028

Parameter	Change(%)	L	T	Total Cost
r	-50	2.8753	27.4238	18.1087
	-25	2.6149	23.9463	12.858
	+25	1.1855	21.6384	8.6914
	+50	1.03294	21.5377	7.6512

Parameter	Change(%)	L	T	Total Cost
A_s	-50	1.08521	21.6357	9.8381
	-25	1.16279	21.9711	10.044
	+25	1.29336	22.6201	10.4482
	+50	1.38731	23.5482	11.033

5.2. Observations: Sensitivity analysis for the proposed model is performed by increasing or decreasing the value of the parameter by ±25% and ±50% from the original values.

(1) If setup cost for producer (A_P) is increases from -50% to + 50%, then technology investment for carbon emission reduction (L) increases from 0.12678 to 1.27522, cycle length (T) increases from 16.1822 to 30.3751 and total cost increases from 6.33601 to 15.9028.
(2) If constant rate of inflation (r) is increases from -50% to + 50%, then technology investment for carbon emission reduction (L) decreases from 2.8753 to 1.03294, cycle length (T) decreases from 27.4328 to 21.5377 and total cost decreases from 18.1087 to 7.6512.
(3) If ordering cost for supplier (A_s) increases from -50% to + 50%, then technology investment for carbon emission reduction (L) increases from 1.08521 to 1.38731, cycle length (T) increases from 21.6357 to 23.5482 and total cost increase from 9.8381 to 11.033.
(4) If reliability rate to good items (A) increases from -50% to + 50%, then technology investment for carbon emission reduction (L) constant having value 1.27522, cycle length (T) constant, having value 22.29 and total cost remains constant, having constant value 10.2479.
(5) If deterioration rate for producer (θ_1) increases from -50% to + 50%,then technology investment for carbon emission reduction (L) decreases from

Parameter	Change(%)	L	T	Total Cost
A	-50	1.27522	22.29	10.2479
	-25	1.27522	22.29	10.2479
	+25	1.27522	22.29	10.2479
	+50	1.27522	22.29	10.2479

Parameter	Change(%)	L	T	Total Cost
θ_1	-50	1.2877	23.3823	10.9091
	-25	1.2465	22.8018	10.5524
	+25	1.2198	21.8385	10.9844
	+50	1.1962	21.4698	10.7542

Parameter	Change(%)	L	T	Total Cost
θ_2	-50	1.19822	30.748	8.69806
	-25	1.20346	25.4808	9.52234
	+25	1.27833	20.0861	10.9158
	+50	1.3298	18.4214	11.5458

1.2877 to 1.1962, cycle length (T) decreases from 23.3823 to 21.4698 and total cost decreases from 10.9091 to 10.7452.

(6) If deterioration rate for producer θ_2 increases from -50% to + 50%,then technology investment for carbon emission (L) increases from 1.19882 to 1.3298, cycle length (T) decreases from 30.748 to 18.4212 and total cost increases from 8.69806 to 11.5458.

6. Conclusion

In this article, we have worked on a green inventory model under inflation. We have also discussed about the imperfect production process but the damaged goods during the imperfect production process are subjected to re-manufacture through the rework process by sending those items back to the production plant. we have explained the numerical example, and the variation of the possible parameters that affect this model. It has been found that, if constant rate of inflation increases then technology investment for carbon emission reduction decreases, and total cost decreases.When deterioration rate for producer increases then technology investment for carbon emission and total cost also increases. This model is not valid when demand rate is not constant and if shortage occur in the market. The model can be extended with different kinds of reduction technology for carbon emissions.

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