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DISCRETE ESSCHER TRANSFORMED LAPLACE DISTRIBUTION AND ITS APPLICATION

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Abstract. Discrete distributions play a very important role in modeling real life situations. Though a lot of discrete distributions are introduced so far, the existing distributions are not able to model many real life scenarios. Recently, various discretization methods are proposed in the literature and discrete analogues of some continuous distributions were also derived especially for modeling lifetime data. Here, we study the discrete version of the one parameter Esscher transformed Laplace distribution namely, discrete Esscher transformed Laplace Distribution. Esscher transformed Laplace distribution is introduced by Sebastian and Dais (2012). It is a heavy tailed and asymmetric distribution and is a member of one parameter exponential family. Most of the properties of the newly constructed distribution are studied. The parameter estimation is also done. Application of the distribution using a real data set is also considered.

1. Introduction

Recently discrete distributions play a crucial role in modeling lifetime data instead of continuous measurement and based on which many research papers were published. Usually life time data are of continuous nature and hence a number of continuous lifetime distributions are proposed by many researchers. But in practice, it is not always possible to measure the length of lifetime of an equipment on a continuous scale. When a discrete distribution is used to model life time data, it results in a multinomial distribution. But there exists many real situations which demand more lifetime distributions to model and hence many continuous distributions have to be discretized. There are a lot of research works available in the literature related to this. Many researchers such as Lisman and van Zuylen (1972), Kemp (1997), Das gupta (1993) and Szablowski (2001)) studied discrete version of Normal distribution. Discrete normal distribution and discrete Rayleigh distribution was studied by Dilip Roy (2003 & 2004). Inusah and Kozubowski (2006) introduced Discrete laplace distribution for integer values. A discrete version of skew Laplace distribution was studied by Kozubowski and Inusah (2006). Hare Krishna and Pramendra Singh Pundir, (2009) studied discrete Burr and discrete Pareto distributions using the general approach of discretization. A few methods for developing discrete analogues of continuous distributions are discussed by Ayman Alzaatrech et al. (2012). Also discrete analogue of the Generalized Exponential

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distribution and Exponentiated Weibull distribution were derived by Nekoukhou et al. (2012) and Nekoukhou Hamed Bidram (2015) respectively. Lekshmi and Sebastian (2014) proposed a generalized form of discrete Laplace distribution by taking the difference of two independent negative binomial variables with common dispersion parameter, called generalised discrete Laplace distribution. A detailed description for the study of discrete version of continuous probability distributions was seen in Chakraborty (2015). Chakraborty and Chakravarthy (2016) introduced discrete Logistic distribution. Berhane Abebe and Rama Shanker (2018) proposed and studied discrete version of continuous Lindley distribution which finds applications in biological sciences.
In this work, we proposed and studied a new lifetime distribution called discrete Esscher transformed Laplace distribution which is the discrete analogue of one parameter Esscher transformed Laplace distribution . The remaining part of the work is organized as follows. Section 2 is devoted to Esscher transformed Laplace distribution. Discrete analogue of esscher transformed distribution was proposed and studied in Section 3. Also in this section expressions for distribution function, moment generating function, moments etc are obtained. Estimation of the parameter is also carried out. A real data analysis is done in Section 4.

2. Esscher Transformed Laplace Distribution

Esscher (1932) introduced a concept known as Esscher transformation. Using this concept Sebastian and Dais (2012) introduced and studied a new heavy tailed and asymmetric distribution namely Esscher transformed Laplace (ETL) distribution. This is a tilted version of the classical Laplace distribution and it belongs to one parameter exponential family. The probability density function , distribution function and characteristic function of the Esscher transformed Laplace (ETL) distribution are respectively

$$g(x,\theta) = \begin{cases} \frac{1-\theta^2}{2} e^{x(1+\theta)}; & x < 0 \\ \frac{1-\theta^2}{2} e^{-x(1-\theta)}; & x \geq 0 \end{cases} \quad \text{and} \tag{2.1}$$

$$G(x) = \begin{cases} \frac{1-\theta}{2} e^{x(1+\theta)]}; & x < 0 \\ \frac{1-\theta}{2} + \frac{1+\theta}{2}[1 - e^{-x(1-\theta)}]; & x \geq 0 \quad . \end{cases} \quad . \tag{2.2}$$

$$\phi_X(t) = \left(1 + \frac{t^2}{1-\theta^2} - \frac{2it\theta}{1-\theta^2}\right)^{-1}; \ t \in R \tag{2.3}$$

Plots of ETL distribution for different values of θ are given in Figure 1.

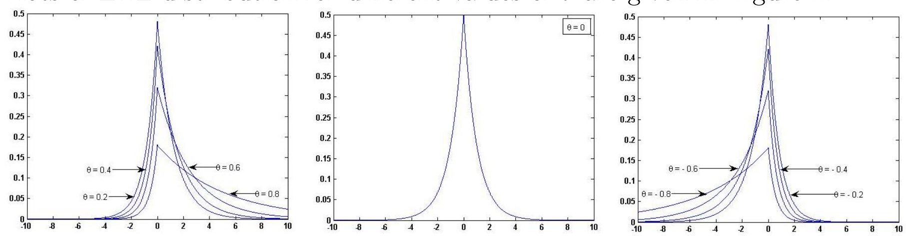

Figure 1. Plots of ETL distribution for (i) $\theta \in (-1, 0)$, (ii) $\theta = 0$ (classical Laplace) and (iii) $\theta \in (0, 1)$

The mgf, mean, variance, measure of dispersion, skewness and kurtosis, and quantile function of (2.1) are

$$\text{MGF} = \left(1 - \frac{t^2}{1-\theta^2} - \frac{2t\theta}{1-\theta^2}\right)^{-1}; \ |\theta| < 1$$

$$\text{Mean} = \frac{2\theta}{1-\theta^2}$$

$$\text{Variance} = \frac{2(1+\theta^2)}{1-\theta^2}$$

$$\text{Coefficient of variation} = \frac{\sqrt{1+\theta^2}}{\sqrt{2}\theta}$$

$$\text{Coefficient of skewness} = \frac{2\theta(3-\theta^2)}{(1+\theta^2)3/2}$$

$$\text{Coefficient of kurtosis} = 3 + \frac{12\theta^2}{(1+\theta^2)}$$

and α quantile

$$q_\alpha = \begin{cases} \frac{1}{1+\theta} log \frac{2\alpha}{1-\theta} \ for \ \alpha \ \in \ (0, \frac{1-\theta}{2}) \\ \frac{1}{1+\theta} log \frac{2(1-\alpha)}{1+\theta} \ for \ \alpha \ \in \ (\frac{1-\theta}{2}, 1) \end{cases}.$$

The ETL distribution is unimodel and possess the properties like infinite divisibility, geometric infinite divisibility, stability and maximum entropy property. This distribution finds application in modeling asymmetric data. For details (see Sebastian and Dais (2012) Dais and Sebastian (2013), Rimsha and Dais (2019) and Dais et al. (2016)).

3. Discrete Esscher transformed Laplace distribution

Recently discrete distributions are getting special attention in survival analysis. In this section discrete analogue of (2.1) was proposed and studied by the method mentioned below. If X is a continuous random variable on R, having pdf f(x) its discrete probability mass function is

$$P(X = x) = \frac{f(x)}{\sum_{u=-\infty}^{+\infty} f(u)};\ x = ..., -2, -1, 0, 1, 2, ... \tag{3.1}$$

By insertting (2.1) in (3.1), the probability mass function of Discrete Esscher transformed Laplace distribution(DETL) is obtained as

$$P(x = x) = \begin{cases} \frac{(1-u)(1-v)u^{|x|}}{1-uv};\ x = 0, -1, -2, ... \\ \\ \frac{(1-u)(1-v)v^{x}}{1-uv};\ x = 0, 1, 2, ... \end{cases}. \tag{3.2}$$

where $u = e^{-(1+\theta)}$ and $v = e^{-(1-\theta)}$, u *and* $v \in (0, 1)$.
Plots of DETL distribution for different values of θ are given in Figure 2.

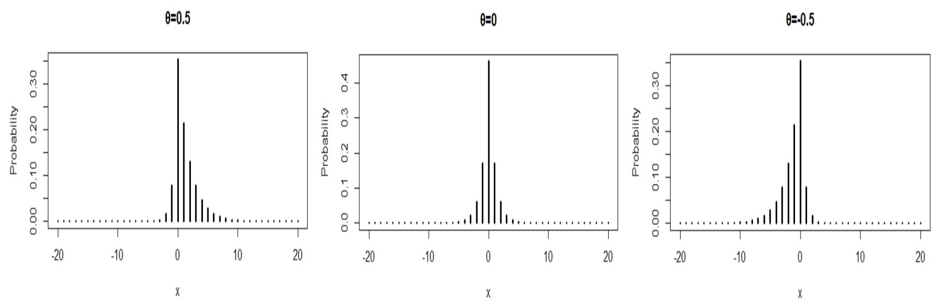

Figure 2. Plots of DETL distribution for (i) $\theta = 0.5$ (ii) $\theta = 0$ (iii) $\theta = -0.5$

The distribution function and characteristic function of this density can be easily derived.

3.1. Distribution function: Let X ∼ DETL(u, v), then the distribution function is

$$F(x) = \begin{cases} \frac{(1-v)u^{-[x]}}{1-uv};\ x = 0, -1, -2, ... \\ \\ 1 - \frac{(1-u)v^{[x]+1}}{1-uv};\ x = 0, 1, 2, ... \end{cases}. \tag{3.3}$$

where [.] is the greatest integer function.

3.2. Characteristic function: The characteristic function of X is given by

$$\phi_X(t) = \frac{(1-u)(1-v)}{(1-e^{-it}u)(1-e^{it}v)},\ t \in R \tag{3.4}$$

3.3. Moments: If X∼ DETL(u, v) then for $n \geq 1$,

$$E|X|^n = \frac{1}{1-uv} \sum_{x=-\infty}^{+\infty} S(n, x)x! \left((1-v) \left(\frac{u}{1-u} \right)^x + (1-u) \left(\frac{v}{1-v} \right)^x \right)$$

where S(n, x)is the Stirling number of the second type and n is any integer. Also

$$E(X^n) = \frac{1}{1-uv}\sum_{x=-\infty}^{+\infty} S(n,x)x!\left((-1)^n(1-v)\left(\frac{u}{1-u}\right)^x + (1-u)\left(\frac{v}{1-v}\right)^x\right)$$

In particular

$$E(X) = \frac{v}{1-v} - \frac{u}{1-u},$$

$$E(X^2) = \frac{(1-u)\left[\left(\frac{v}{1-v}\right) + 2\left(\frac{v}{1-v}\right)^2\right] + (1-v)\left[\left(\frac{u}{1-u}\right) + 2\left(\frac{u}{1-u}\right)^2\right]}{1-uv},$$

$$V(X) = \frac{1}{(1-u)^2(1-v)^2}\left[\frac{(1-v)^3u(1+u) + (1-u)^3v(1+v)}{1-uv} - (v-u)^2\right].$$

$$E(X^3) = \frac{(1-u)\left[\left(\frac{v}{1-v}\right) + 6\left(\frac{v}{1-v}\right)^2 + 6\left(\frac{v}{1-v}\right)^3\right] - (1-v)\left[\left(\frac{u}{1-u}\right) + 6\left(\frac{u}{1-u}\right)^2 + 6\left(\frac{u}{1-u}\right)^3\right]}{1-uv}$$

$$\text{and } E(X^4) = \frac{(1-u)\left[\left(\frac{v}{1-v}\right) + 14\left(\frac{v}{1-v}\right)^2 + 36\left(\frac{v}{1-v}\right)^3 + 24\left(\frac{v}{1-v}\right)^4\right]}{1-uv}$$
$$+\frac{(1-v)\left[\left(\frac{u}{1-u}\right) + 14\left(\frac{u}{1-u}\right)^2 + 36\left(\frac{u}{1-u}\right)^3 + 24\left(\frac{u}{1-u}\right)^4\right]}{1-uv}.$$

Also

$$E|X| = \frac{1}{(1-uv)(1-u)(1-v)}\left[u(1-v)^2 + v(1-u)^2\right].$$

3.4. Quantile function: The α th quantile $q(\alpha)$ is

$$q(\alpha) = \begin{cases} \frac{log(1-uv)-log(1-v)+log\alpha}{logu}; & \alpha\epsilon(0, \frac{1-v}{1-uv}) \\ \frac{log(1-uv)+log(1-\alpha)-log(1-u)-logv}{logv}; & \alpha\epsilon(\frac{1-v}{1-uv}, 1). \end{cases}.$$

3.5. Order statistics: Let $Z_{(1:n)} \le Z_{(2:n)} \le Z_{(3:n)} \le ... \le Z_{(n:n)}$ represents the order statistics obtained from a from the i.i.d. DETL(u, v) distribution of size n. Then probability mass function of first order statistics is given by

$$f_{Z_{(1:n)}}(z) = \begin{cases} \left(1-\frac{(1-v)u^{-[z-1]}}{(1-uv)}\right)^n - \left(1-\frac{(1-v)u^{-[z]}}{(1-uv)}\right)^n & ;\ z = 0,-1,-2,... \\ \left(\frac{(1-u)(v^{[z-1]+1})}{(1-uv)}\right)^n - \left(\frac{(1-u)(v^{[z]+1})}{(1-uv)}\right)^n & ;\ z = 0,1,2,... \end{cases}$$

and the probability mass function of n th order statistics is given by

$$f_{Z_{(n:n)}}(z) = \begin{cases} \left(\frac{(1-v)(u^{-[z]})}{1-uv}\right)^n - \left(\frac{(1-v)(u^{-[z-1]})}{1-uv}\right)^n & ;\ z = 0,-1,-2,... \\ \left(1-\frac{(1-u)(v^{[z]+1})}{(1-uv)}\right)^n - \left(1-\frac{(1-u)(v^{[z-1]+1})}{(1-uv)}\right)^n & ;\ z = 0,1,2,... \end{cases}.$$

3.6. Reliability characteristics:

$$\text{Reliability function}, R(x) = 1 - F(x) = \frac{(1-u)v^{[x]+1}}{1-uv}\ ; x = 0, 1, 2...,$$

$$\text{Failure rate}, r(x) = \frac{f(x)}{R(x)} = \frac{1-v}{v} \text{ and}$$

$$\text{Second rate of failure } is\ r^{*}(x) = log\frac{R(x)}{R(x+1)} = log\frac{1}{v}.$$

3.7. Infinite divisibility: Definition: A probability distribution with characteristic function ϕ is infinitely divisible if for any integer $n \geq 1$,

$$\phi(t) = (\phi_n(t))^n.$$

Theorem 3.1. *An DETL (u, v) with characteristic function $\phi(t)$ is infinitely divisible.*

Proof.

$$\phi(t) = \begin{cases} ([(\frac{1-u}{1-ue^{-it}})^{\frac{1}{n}}][(\frac{1-v}{1-ve^{it}})^{\frac{1}{n}}])^n \\ \\ (\phi_n(t))^n \end{cases}.$$

The characteristic function $\phi_n(t)$ corresponds to

$$Z = X - Y$$

where X and Y are NB $(\frac{1}{n}, 1-v)$ and NB $(\frac{1}{n}, 1-u)$ random variables respectively and are independent. Hence the result. □

3.8. Convolution property: Let $Z_1, Z_2, ...Z_n$ be independently and identically distributed DETL (u, v) variables. The characteristic function of

$$Z = Z_1 + Z_2 + ... + Z_n \text{ is}$$

$$\phi_Z(t) = \left(\frac{(1-u)(1-v)}{(1-e^{-it}u)(1-e^{it}v)}\right)^n$$

which is the characteristic function of skewed generalized discrete Laplace distribution having parameters (n, u, v). Thus DETL (u, v) is closed under convolution.

3.9. Entropy: The Shannon's entropy of random variable X having probability density function f(x) is given by

$$H(X) = E(-logf(x)).$$

For DETL distribution, H(X) is obtained as

$$H(X) = -log\frac{(1-u)(1-v)}{1-uv} - \frac{(1-u)(1-v)}{1-uv}\left(\frac{ulogu}{(1-u)^2} + \frac{vlogv}{(1-v)^2}\right).$$

3.10. Estimation: In this section, we estimate the parameters u and v of DETL distribution using the method of moments and method of maximum likelihood. Let us take a random sample$X_1, X_2 ... X_n$ of size n from an DETL(u, v) distribution. Then the logarithm of likelihood function is

$$logL = n(log(1-u) + log(1-v) - log(1-uv) + \bar{X_n}^- logu + \bar{X_n}^+ logv)$$

where

$$\bar{X_n}^+ = \frac{\sum X_i^+}{n},$$
$$\bar{X_n}^- = \frac{\sum X_i^-}{n}$$

and x^+ and x^- are the positive and negative parts of x, respectively.

$$x^+ = x$$

if $x \geq 0$ and zero elsewhere.

$$x^- = (-x)^+.$$

3.11. Case(i). Consider the case as all sample observations are zero. Then logL is maximised by $\hat{u} = \hat{v} = 0$.

3.12. Case(ii). Suppose the sample has only positive values, then

$$\hat{v} = \frac{\bar{X_n}^+}{1 + \bar{X_n}^+} \text{ and } \hat{u} = 0.$$

3.13. Case(iii). If the sample has only negative values, then

$$\hat{u} = \frac{\bar{X_n}^-}{1 + \bar{X_n}^-} \text{ and } \hat{v} = 0.$$

3.14. Case(iv). Finally consider the case that the sample must contain atleast one positive and negative observation. Then logL takes the maximum value at the point (u,v) in the interior of an open unit square where the partial derivatives are zero. This will give the following set of equations.

$$\frac{v}{1-uv} + \frac{\bar{X_n}^-}{u} = \frac{1}{1-u} \; and$$
$$\frac{u}{1-uv} + \frac{\bar{X_n}^+}{v} = \frac{1}{1-v}.$$

The solution to these set of equations will give the maximum likelihood estimators of u and v.

For that we take
$C_1 = \bar{X_n}^+ - \bar{X_n}^-$ and $C_2 = \bar{X_n}^+ + \bar{X_n}^-$ where C_1 and C_2 are the sample mean and the sample first absolute moment respectively.
When $C_1 \geq 0$,

$$\hat{u} = \frac{C_1^2 - C_1C_2 - 1 \pm \sqrt{1 + (C_2 + C_1)(C_2 - C_1)}}{(C_1 - 1)(C_1 - C_2)}$$

and

$$\hat{v} = \frac{C_1(1-\hat{u})+\hat{u}}{1+C_1(1-\hat{u})}.$$

When $C_1 \leq 0$,

$$\hat{v} = \frac{C_1^2 + C_1C_2 - 1 \pm \sqrt{1-(C_2+C_1)(C_2-C_1)}}{(C_1+1)(C_1+C_2)}$$

and

$$\hat{u} = \frac{\hat{v} - C_1(1-\hat{v})}{1-C_1(1-\hat{v})}.$$

3.15. Moment Method: The estimators obtained by both the moment method and maximum likelihood method are the same.

4. Real data Analysis

An application of DETL distribution is done by considering the following data set used by Suttida Sangpoom and Winai Bodhisuwan (2016). The data was taken from the stock price of petroleum published in the official website of the Stock Exchange of Thailand PTT Public Limited Company in 2014. The proposed data was obtained by noting the difference in stock price by comparing it with the previous day's closing price for a period from April 1, 2014 to October 20, 2014 .

The data set is

-3, 9, -6, -5, 4, -12, -11, 7, 1, -3, 6, -5, 3, 6, 7, -4, 5, -1, -6, 1, 4, 4, 0, -2,

0, -2, -2, 6, 2, 14, 3, 0, 4, 7, 1, -2, 0, -9, -3, -1, -2, 0, 0, 2, 0, -1, 3, 1, 9

-4, 1, -3, 5, 9, -5, -7, -5, 2, -5, 0, 1, 2, -4, 4, 1, 6, 0, -2, 0, 4, 1, -5, 7, 0

0, 1, 6, 4, 8, 3, -4, 3, 1, 2, -4, 1, 6, -2, -5, 1, 3, -3, 1, -4, -1, 4, 4, 1, -2,

1, -2, -8, -4, -2, 5, -3, 3, 2, -1, 3, -3, -1, -5, 1, 0, -4, 10, -3, -1, -2, 1, 1,

-3, -1, -1, -3, 8, 0, 3, 2, -4, -2, 1, 5, 3.

Figure 3 shows the observed data.

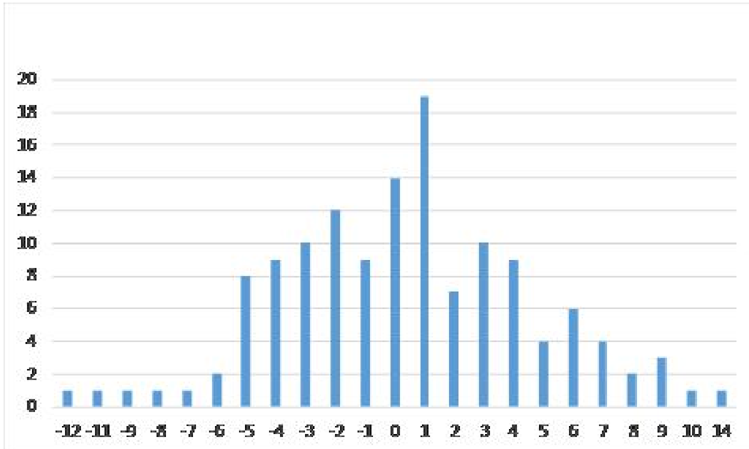

Figure 3. Observed data.

Since it shows asymmetric nature as that of the DETL distribution, we fit the empirical data using DETL distribution. For that we estimate the parameter of DETL distribution from the empirical data using MLE method and it is given in Table 1. Then we embed the theoretical pmf over the empirical pmf and the graph is given in Figure 4.

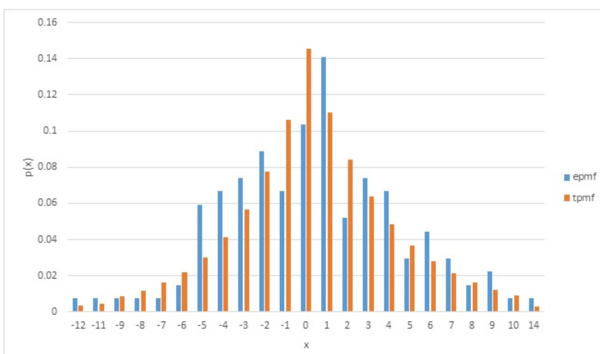

Figure 4. Theoretical and Empirical pmf of DETL

Also we check the goodness of fit using the Chi-square test of goodness of fit. It is found that the DETL distribution is adequate for studying differences of stock price data. More over, we compare DETL, DAL and DN distribution. For that the values of Log-likelihood, AIC , BIC are computed and is shown in Table 1.

Table 1: MLE values,Chi-square value, Log-likelihood value, AIC value and BIC value for the difference of stock price data.

Distribution fitted	estimators	Chi-square	LL value	AIC value	BIC value
DAL distribution	$\hat{\mu} = 1.000$ $\hat{\beta} = .732$ $\hat{\lambda} = 0.724$	11.467	-384.534	775.069	783.785
DN distribution	$\hat{\mu} = 0.884$ $\hat{\sigma} = 4.261$	12.511	-387.539	779.078	784.888
DETL distribution	$\hat{\theta} = -0.685$	14.213	-117.247	236.495	236.625

Table shows that the proposed DETL distribution provides a better fit than Discrete Asymmetric Laplace distribution and Discrete Normal distribution.

5. Summary

We usually come across situations where lifetimes are suitable to measure as discrete random variable rather than continuous scale. Rather than Negative binomial and Geometric distributions which are often used to model discrete lifetime data, discrete analogues of continuous distributions are recently used. In this paper, we introduced and studied discrete Esscher transformed Laplace distribution (DETL). A real data analysis is also carried out using stock price data and finds that DETL distribution provides a better model than DAL and DL distribution..

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