Stochastic Modeling and Applications
Vol. 25 No. 1 (January-June, 2021)

Received: 21st January 2021 *Revised: 10th February 2021* *Selected: 23rd February 2021*

RANGE-BASED POSTERIOR AVERAGE CONTROL CHART

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Abstract. Control charts are instrumental in process monitoring while deciding about whether the process is in control or not. In the presence of prior information, Bayesian control charts have been more cogent. In this paper, we propose posterior average control chart when process standard deviation (SD) is estimated in terms of range. The performance of the proposed control chart is analysed in terms of its power, average run length (ARL), standard deviation of run length (SDRL), coefficient of variation of run length (CVRL) and process capability ratio (C). It is compared with Shewhart's $\bar{X}$ control chart and $\bar{X}$ posterior control chart due to Bhat and Gokhale (2014). The control chart is illustrated.

Keywords: control limits, process average, posterior distribution, range, ARL, CVRL

AMS Classification: 62P30, 62C10.

1 Introduction

Statistical process control is one of the emphatic techniques in monitoring the quality of a product which uses sampling and graphical stratagem known as control charts. Hence it forms the basis for taking decision about, whether the process out of control is due to chance cause or assignable cause. Shewhart (1931) initiated the theory of control charts based on the assumption of normal distribution.

The adequacy of a control chart is gauged on how early it gives alarms whenever process goes out of control. The performance is examined through various measures like power, ARL, SDRL, CVRL, C, etc. The past information available in the manufacturing industry, due to considerable growth in production process is worth utilizing. Hence considering control charts using Bayesian approach are highly sensible.

A detailed study on control charts including Shewhart's $\underline{X}$ control chart (S_A) is given in Montgomery (1996). Bayesian methods and Bayesian process monitoring are exhaustively studied in Berger (1986) and Colosimo and Del Castillo (2006). The cost modeling of quality control systems is devised in Girshick and Rubin (1952) and a predictive control chart to detect the shift in process mean is developed in Saghir (2007).

Bhat and Gokhale (2014) and Gokhale (2017) developed posterior control chart (P_B) for process average using conjugate prior distribution. They established that P_B control chart performs better than Saghir's predictive control chart and S_A control chart which does not use prior information. Also, in case of unknown variance, they show that maximum likelihood estimator (MLE) is better estimator than unbiased estimator for SD.

The range is a lucid measure of variability and can also be used as a substitute for measure of variation. It is desirable to construct posterior control chart for process average based on range as an estimate of SD. The traces of estimation of variation in terms of range is found in Tippett (1925). He observed that the estimator of SD (σ) is

$$\hat{\sigma} = \underline{R}/d_2 \tag{1}$$

where $\underline{R}$ is the average of ranges of sample subgroups,
$d_2 = \int_{-\infty}^{+\infty} [1-(1-\Phi(x))^n - (\Phi(x))^n]dx$ and $\Phi(\cdot)$ is the cumulative distribution function of standard normal variate. Nelson (1975) used this relation and discussed about the use of $d_2^* = \sqrt{\frac{d_3^2}{k} + d_2^2}$ where $d_3^2 = V(\sigma)$ and when k the number of subgroups are less. Luko (1996) compared the use of d_2^* and d_2 on the basis of mean squared error.

In Section 2, we propose range based posterior average control chart (P_R). In section 3, we evaluate its performance and carry out a comparative study. In section 4, we illustrate the proposed control chart and record our concluding remarks in section 5. The tables supporting our study are given in appendix.

2 Proposed control chart

In this section, we propose P_R control chart, discuss about the underlying basis and obtain its control limits.

Suppose $X_1, X_2, \ldots, X_n$ is a random sample of size n from $N(\mu, \sigma^2)$, then the density function of X_i is given by

$$f_{X_i}(\mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} x, \mu \epsilon R, \sigma > 0. \tag{2}$$

The joint distribution of $\underline{X} = (X_1, X_2, \ldots, X_n)$ is given by

$$f_{\underline{X}}(\mu, \sigma^2) = \prod_{i=1}^{n} f_{X_i}(\mu, \sigma^2). \tag{3}$$

We assume that, σ^2 is unknown and the prior distribution of μ is $N(\theta, \lambda^2)$ where θ and λ^2 are hyper-parameters. The prior density function of μ is given by

$$\pi(\mu) = \frac{1}{\lambda\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\mu-\theta}{\lambda}\right)^2} \mu, \theta \epsilon R, \lambda > 0. \tag{4}$$

The posterior distribution of $(\mu|\underline{x})$ is given by

$$\pi(\mu|\underline{x}) = \sqrt{\frac{\rho}{2\pi}} e^{-\frac{\rho}{2}\left(\mu - \frac{\rho}{2}\left(\frac{n\underline{x}}{\sigma^2} + \frac{\theta}{\lambda^2}\right)\right)^2} \tag{5}$$

where $\rho = \frac{\sigma^2 + n\lambda^2}{\sigma^2\lambda^2}$.

That is,

$$(\mu|\underline{x}) \sim N\left(\frac{n\underline{x}\lambda^2 + \theta\sigma^2}{n\lambda^2 + \sigma^2}, \frac{\lambda^2\sigma^2}{n\lambda^2 + \sigma^2}\right)$$
$$\sim N\left(\underline{x}\zeta + \theta(1-\zeta), \frac{\zeta}{n}\sigma^2\right) \tag{6}$$

where $\zeta = \frac{n\lambda^2}{n\lambda^2 + \sigma^2}$.

Substituting (1) in (6), we observe that, $(\mu|\underline{x}) \sim N(\eta, \upsilon)$ where

$$\eta = \frac{n\underline{x}\lambda^2 d_2^2 + \theta \underline{R}^2}{n\lambda^2 d_2^2 + \underline{R}^2} \quad (7)$$

$$\text{and } v = \frac{\lambda^2 \underline{R}^2}{n\lambda^2 d_2^2 + \underline{R}^2} \ . \quad (8)$$

The posterior mean and variance are respectively given by

$$\eta = \underline{x}\xi + \theta(1-\xi) \quad (9)$$

$$\text{and } v = \frac{\xi \underline{R}^2}{n d_2^2} \quad (10)$$

where $= \frac{n\lambda^2 d_2^2}{n\lambda^2 d_2^2 + \underline{R}^2}$.

Under squared error loss function, the limits for proposed P_R control chart are given by

$$Upper\ limit\ (UL) = \eta + \gamma\sqrt{v}, \quad (11)$$

$$Central\ limit\ (CL) = \eta \quad (12)$$

$$\text{and } Lower\ limit(LL) = \eta - \gamma\sqrt{v} \quad (13)$$

where γ is multiplier to decide upon the nature of limits of the proposed control chart. Suppose in (11) and (13), we take $\gamma = 3$, we get upper control limit (UCL) and lower control limit (LCL) respectively. Taking $\gamma = 2$, we get upper warning limit (UWL) and lower warning limit (LWL) respectively and taking γ as specified by the consumer, we get upper specification limit (USL) and lower specification limit (LSL) respectively. Also, the control limits of P_B control chart due to Bhat and Gokhale (2014) were obtained by them using (6).

3 Evaluation of P_R control chart

In this section, we evaluate the performance of the proposed control chart in terms of its power, ARL, SDRL, CVRL and C. We carry out a comparative study of P_R, S_A and P_B control charts. Let the 'in control mean' be μ. Whenever there is a shift $(b \neq 0)$ in the process, let the target mean be $\mu^{'} = \mu + b$, process variable $X \sim N(\mu^{'}, \sigma^2)$ and the density function of $(\underline{x}|\mu^{'}, \sigma^2)$ is given by

$$f_{(\underline{x}|\mu^{'},\sigma^2)}(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu^{'}}{\sigma}\right)^2} x, \mu^{'} \epsilon R\ , \sigma > 0.$$

The power of any chart A, to detect the shift in process mean is given by

$$P(A) = 1 - \beta_A \quad (14)$$

where β_A is the probability of not detecting the shift of size α and is given by

$$\beta_A = \Phi\left(\frac{UCL_A - \mu^{'}}{\sqrt{V_A}}\right) - \Phi\left(\frac{UCL_A - \mu^{'}}{\sqrt{V_A}}\right) \quad (15)$$

where V_A is variance of A.

The ARL gives average number of samples required to indicate 'out of control' signal whereas SDRL measures dispersion of the run length distribution, CVRL shows the extent of variability in relation to mean of the run length and C is the measure of goodness of process to the given specifications. For any chart A, it is desirable to have smaller values of ARL, SDRL, CVRL which are respectively given by

$$ARL_A = \frac{1}{1-\beta_A} = \frac{1}{P(A)}, \quad (16)$$

$$SDRL_A = \frac{\sqrt{\beta_A}}{1-\beta_A} \quad (17)$$

$$\text{and } CVRL_A = \frac{SDRL_A}{ARL_A} \ . \quad (18)$$

And the process capability ratio of any chart is given by

$$C_A = \frac{USL_A - LSL_A}{6\sigma_A}. \quad (19)$$

To evaluate and compare the performance of P_R control chart with S_A and P_B control charts, we consider the following example.

Suppose a company is manufacturing packets of half kilogram capacity and packs groceries weighing 500 grams, we take $X \sim N(500,5)$. Assuming $\mu \sim N(500,20)$ and
$\underline{x} = 499.5$, a random sample of size n is generated from normal distribution with mean 500 and variance 5 using R programming. The power, ARL, SDRL of S_A, P_B and P_R control charts are respectively computed using (15), (16) and (17) for n=4, 6, and 9 and are given in table 1 in the appendix. The CVRL of S_A control chart for various values of n and σ^2 are given in table 2. The computation of CVRL for P_B and P_R control charts for various values of n, λ^2 and σ^2 are given in table 3. The process capability ratio for P_B and P_R control chart are given in table 4 for various values of specification limits, n, $\sigma^2 = 5$ and $\lambda^2 = 20$. The power and ARL of S_A, P_B and P_R control charts using table 1 are presented in figure 1.

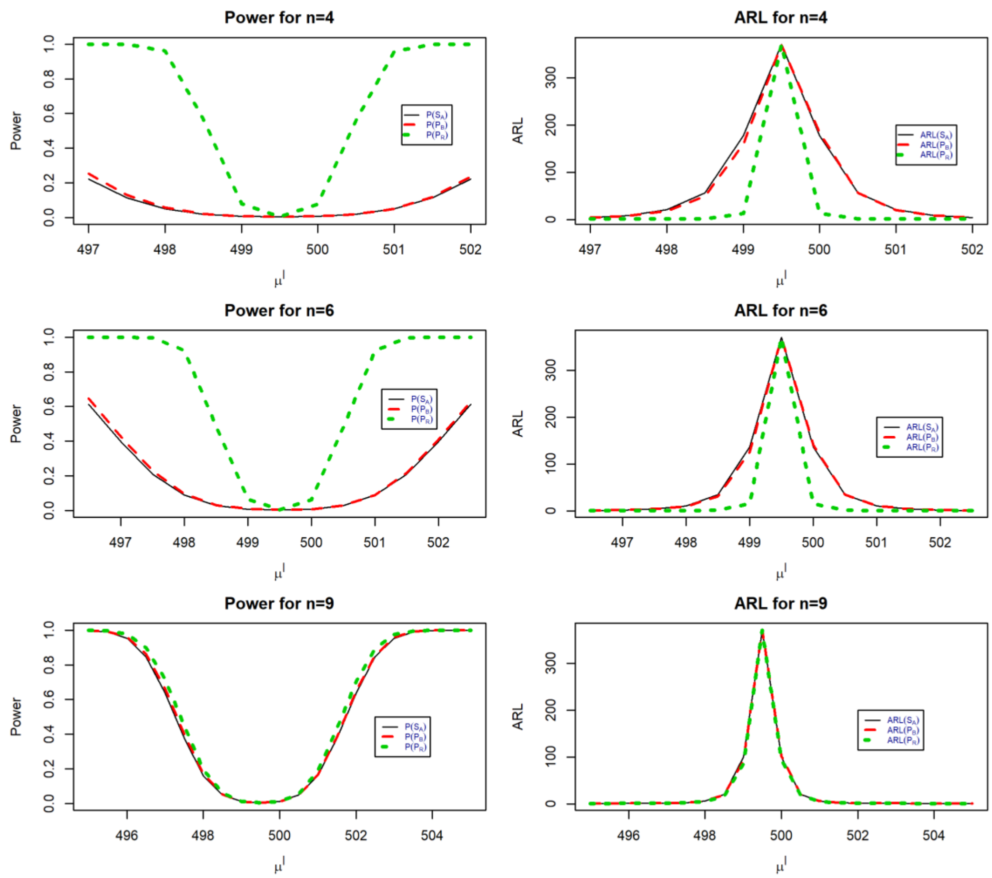


Figure 1: Power and ARL of S_A, P_B and P_R control charts

From figure 1, we observe that, P_R control chart performs better than S_A and P_B control charts for smaller values of n. Table 1 reflects that, the power of P_R control chart considerably increases, ARL and SDRL values decrease for smaller shifts when n is small. For moderate values of n, the power increases, ARL and SDRL decrease as shift increases. For a given shift, the power of P_R control chart is higher, ARL and SDRL values are lesser than S_A and P_B control charts. Table 2 shows that CVRL of S_A control chart increases as σ^2 increases and decreases as n increases. From table 3, it is seen that, as σ^2 increases CVRL for both P_B and P_R control charts increase. Also, as λ^2 increases CVRL of both the control charts increase. However, CVRL of P_R control chart is lesser than CVRL of P_B control chart for all given shifts and values of n, λ^2 and σ^2. Table 4 reveals that as n increases, the values of CP_B and CP_R increase. Also, for all given values of specification limits and n, CP_R is higher than CP_B. To interpret, suppose n=4, USL=503, LSL=497, we have CP_B=0.9218544 and CP_R=1.344609 which means that P_R control chart uses 74.37% of the specification band whereas, P_B control chart uses 108.47% of specification band. The width of the control limits of P_B and P_R control charts are respectively given by

$$WP_B = \frac{6\lambda\sigma}{\sqrt{n\lambda^2 + \sigma^2}} \tag{20}$$

and $$WP_R = \frac{6\lambda \underline{R}}{\sqrt{n\lambda^2 d_2^2 + \underline{R}^2}}. \tag{21}$$

From table 4, we also notice that, width of the control limits, that is, UCL-LCL becomes narrower as n increases and WP_R is always smaller than WP_B.

4 Illustration of P_R chart

In this section, we illustrate P_R control chart using flow width measurement data on hard-bake process and piston ring data of automobile engine given in Montgomery (1996) and are respectively presented in figure 2 and figure 3.

Example

Flow width measurement data (in microns) of sample size 225 has been taken from a hard-bake process which is used in conjunction with photolithography in semiconductor manufacturing and the data is given as follows.

1.3235	1.4128	1.6744	1.4573	1.6914	1.4314	1.3592	1.6075	1.4666	1.6109
1.4284	1.4871	1.4932	1.4324	1.5674	1.5028	1.6352	1.3841	1.2831	1.5507
1.5604	1.2735	1.5265	1.4363	1.6441	1.5955	1.5451	1.3574	1.3281	1.4198
1.6274	1.5064	1.8366	1.4177	1.5144	1.4190	1.4303	1.6637	1.6067	1.5519
1.3884	1.7277	1.5355	1.5176	1.3688	1.4039	1.6697	1.5089	1.4627	1.5220
1.4158	1.7667	1.4278	1.5928	1.4181	1.5821	1.3355	1.5777	1.3908	1.7559
1.2856	1.4106	1.4447	1.6398	1.1928	1.4951	1.4036	1.5893	1.6458	1.4969
1.3589	1.2863	1.5996	1.2497	1.5471	1.5747	1.5301	1.5171	1.1839	1.8662
1.3680	1.7269	1.3957	1.5014	1.4449	1.4163	1.3864	1.3057	1.6210	1.5573
1.5796	1.4185	1.6541	1.5116	1.7247	1.7106	1.4412	1.2361	1.3820	1.7601
1.4371	1.5051	1.3485	1.5670	1.4880	1.4738	1.5936	1.6583	1.4973	1.4720
1.5917	1.4333	1.5551	1.5295	1.6866	1.6399	1.5243	1.5705	1.5563	1.5530
1.5797	1.3663	1.6240	1.3732	1.6887	1.4483	1.5458	1.4538	1.4303	1.6206
1.5435	1.6899	1.5830	1.3358	1.4187	1.5175	1.3446	1.4723	1.6657	1.6661
1.5454	1.0931	1.4072	1.5039	1.5264	1.4418	1.5059	1.5124	1.4620	1.6263
1.4301	1.2725	1.5945	1.5397	1.5252	1.4981	1.4506	1.6174	1.5837	1.4962
1.3009	1.5060	1.6231	1.5831	1.6454	1.4132	1.4603	1.5808	1.7111	1.7313
1.3817	1.3135	1.4953	1.4894	1.4596	1.5765	1.7014	1.4026	1.2773	1.4541
1.4936	1.4373	1.5139	1.4808	1.5293	1.5729	1.6738	1.5048	1.5651	1.7473
1.8089	1.5513	1.8250	1.4389	1.6558	1.6236	1.5393	1.6738	1.8698	1.5036
1.4120	1.7931	1.7345	1.6391	1.7791	1.7372	1.5663	1.4910	1.7809	1.5504
1.5971	1.7394	1.6832	1.6677	1.7974	1.4295	1.6536	1.9134	1.7272	1.4370
1.6217	1.8220	1.7915	1.6744	1.9404					

We treat the first 215 sample measurements as prior information and last 10 sample measurements as the present data. Hence n=10, $\theta = 1.5240$, $\lambda^2 = 0.01274$, $\underline{x} = 1.77$,
$\underline{R} = 0.3187$, $d_2 = 3.078$, UCL=1.7964, LCL= 1.5640, CL=1.6802, LWL=1.6027 and
UWL= 1.7576. The P_R control chart with these values are given in exhibit-1 in figure 2.

Also, taking first 220 sample measurements as prior information and last 5 sample measurements as current data, we get n=5, $\theta = 1.5264$, $\lambda^2 = 0.0131$, $\underline{x} = 1.7011$,
$\underline{R} = 0.4013$, $d_2 = 2.326$, UCL=1.8777, LCL= 1.5535, CL=1.7156, LWL=1.6076 and UWL= 1.8236. We give P_R control chart for these values in exhibit-2 of figure 2.

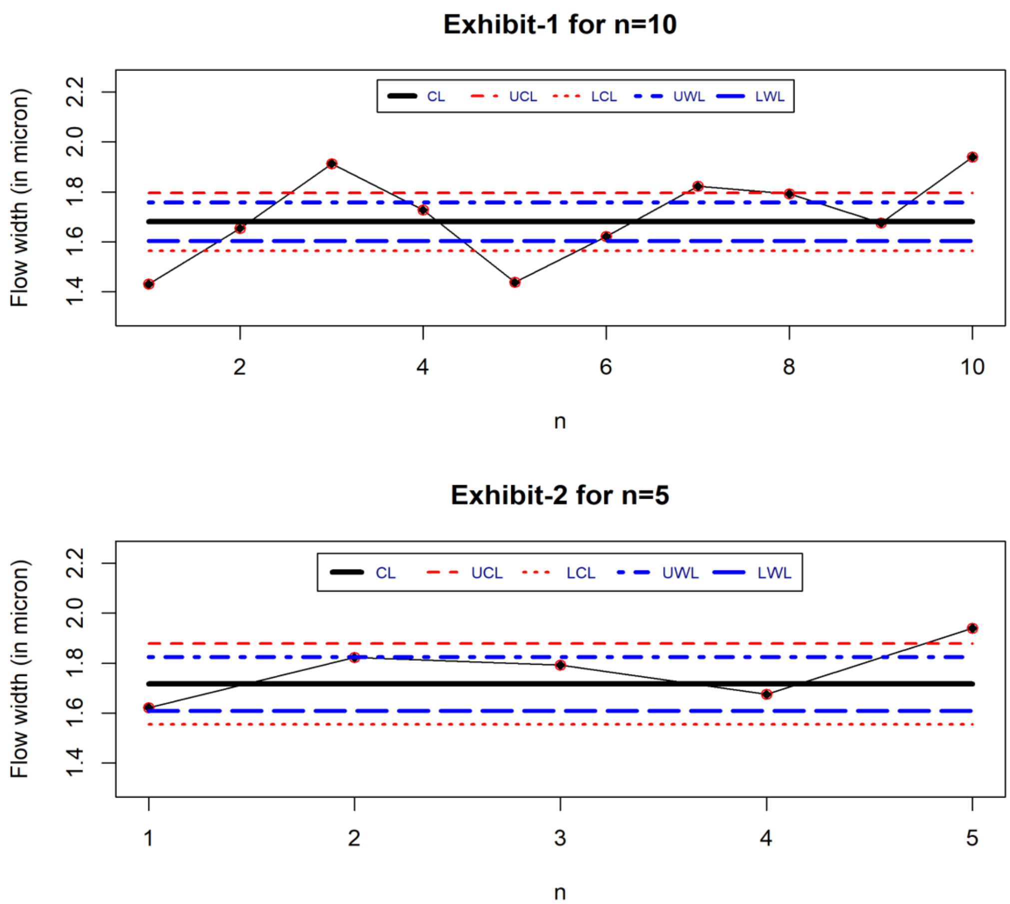


Figure 2: P_R control chart for hard-bake process

From figure 2, it is noted that in exhibit-1, the samples 1, 3, 5, 7 and 10 fall outside the control limits. In exhibit-2, the sample 5 falls outside the control limits.

Example

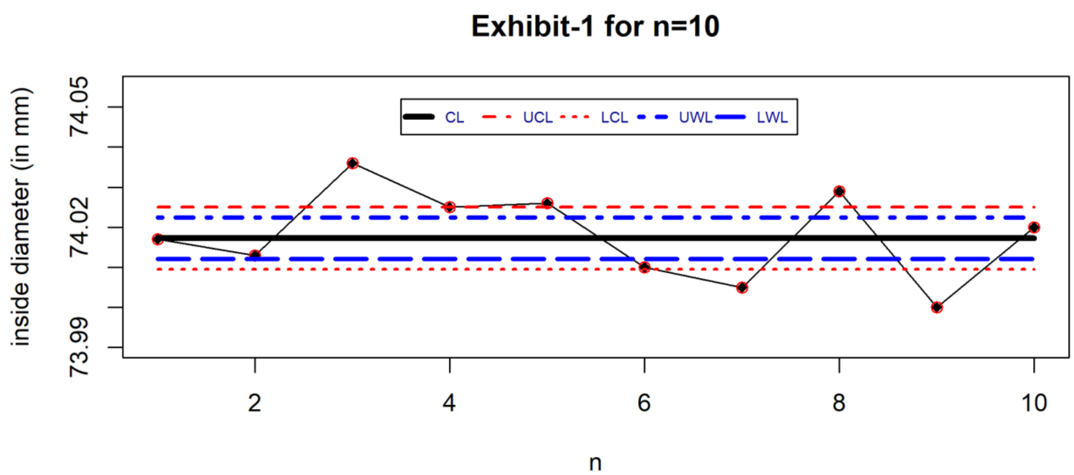
The data of sample size 200 on 'inside diameter measurements (in mm)' for automobile engine piston ring produced by a forging process is given as below.

74.030	74.002	74.019	73.992	74.008	73.995	73.992	74.001	74.011	74.004
73.988	74.024	74.021	74.005	74.002	74.002	73.996	73.993	74.015	74.009
73.992	74.007	74.015	73.989	74.014	74.009	73.994	73.997	73.985	73.993
73.995	74.006	73.994	74.000	74.005	73.985	74.003	73.993	74.015	73.988
74.008	73.995	74.009	74.005	74.004	73.998	74.000	73.990	74.007	73.995
73.994	73.998	73.994	73.995	73.990	74.004	74.000	74.007	74.000	73.996
73.983	74.002	73.998	73.997	74.012	74.006	73.967	73.994	74.000	73.984
74.012	74.014	73.998	73.999	74.007	74.000	73.984	74.005	73.998	73.996
73.994	74.012	73.986	74.005	74.007	74.006	74.010	74.018	74.003	74.000

73.984	74.002	74.003	74.005	73.997	74.000	74.010	74.013	74.020	74.003
73.988	74.001	74.009	74.005	73.996	74.004	73.999	73.990	74.006	74.009
74.010	73.989	73.990	74.009	74.014	74.015	74.008	73.993	74.000	74.010
73.982	73.984	73.995	74.017	74.013	74.012	74.015	74.030	73.986	74.000
73.995	74.010	73.990	74.015	74.001	73.987	73.999	73.985	74.000	73.990
74.008	74.010	74.003	73.991	74.006	74.003	74.000	74.001	73.986	73.997
73.994	74.003	74.015	74.020	74.004	74.008	74.002	74.018	73.995	74.005
74.001	74.004	73.990	73.996	73.998	74.015	74.000	74.016	74.025	74.000
74.030	74.005	74.000	74.016	74.012	74.001	73.990	73.995	74.010	74.024
74.015	74.020	74.024	74.005	74.019	74.035	74.010	74.012	74.015	74.026
74.017	74.013	74.036	74.025	74.026	74.010	74.005	74.029	74.000	74.020

First 190 sample measurements are treated as prior information and last 10 sample measurements are treated as present data. Hence n=10, $\theta = 74.002842$, $\lambda^2 = 0.000119$, $\underline{x} = 74.0181$, $\underline{R} = 0.026$, $d_2 = 3.078$, UCL=74.025025, LCL= 74.009456, CL=74.01724, LWL=74.012051 and UWL= 74.02243.The P_R control chart is given in exhibit-1 of figure 3.

Again, first 195 sample measurements are treated as prior information, last 5 sample measurements are treated as current data, n=5, $\theta = 74.0033$, $\lambda^2 = 0.000129$, $\underline{x} = 74.0128$, $\underline{R} = 0.029$, $d_2 = 2.326$,UCL=74.0259, LCL= 73.9959, CL=74.01096, LWL=74.000957 and UWL= 74.020972 and P_R control chart is given in exhibit-2 of figure 3.



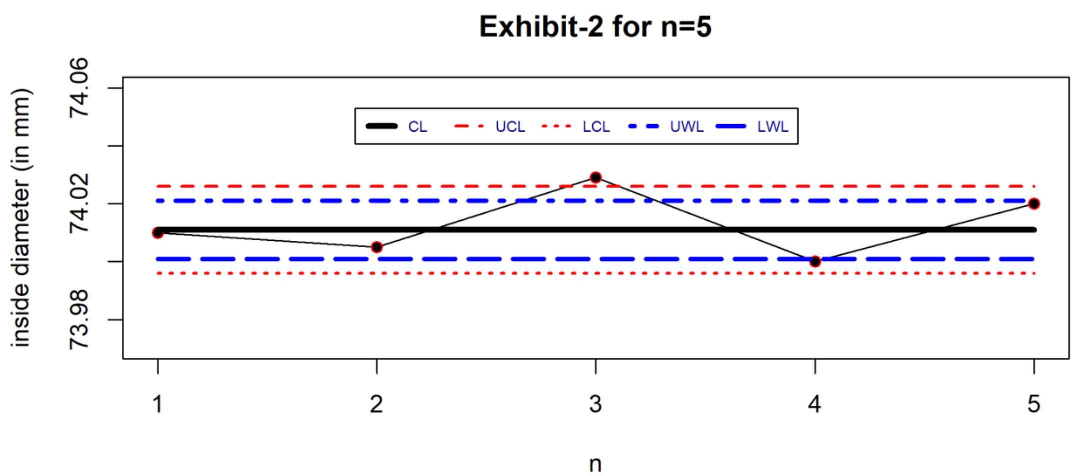


Figure 3: P_R control chart for inside diameter of piston ring

Figure 3 shows that, in exhibit-1, the sample 5 falls almost on UCL, the samples 3, 7, 8 and 9 fall outside the control limits. In exhibit-2, the sample 3 falls outside the control limits.

5 Conclusions

- We propose P_R, a range-based posterior control chart for process average under the assumption of normality and unknown variance.
- The control limits of P_R control chart are obtained under squared error loss function and conjugate prior distribution.
- The proposed control chart performs better than S_A, Shewhart's $\underline{X}$ control chart for fewer samples.
- The P_R control chart performs better than P_B, $\underline{X}$ posterior control chart due to Bhat and Gokhale (2014) for smaller shifts when sample size is small.
- The process capability of P_R control chart is always higher than that of P_B control chart.
- The width of both P_R and P_B control charts get narrower as sample size increases and width of P_R control chart is always smaller than P_B control chart.
- The P_R control chart is useful whenever the process variation is observed in terms of range and the process demands few samples.

Appendix

Table 1: Power, ARL and SDRL of various control charts for different values of n, $\sigma^2 = 5$ and $\lambda^2 = 20$.

n	μ'	Power			ARL			SDRL		
		P(S_A)	P(P_B)	P(P_R)	ARLS$_A$	ARLP$_B$	ARLP$_R$	SDRLS$_A$	SDRLP$_B$	SDRLP$_R$
	497.0	0.222450	0.252070	1.000000	4.495392	3.967152	1.000000	3.963983	3.430910	0.000000
	497.5	0.112920	0.129460	0.999580	8.855827	7.724394	1.000420	8.340854	7.207070	0.020503
	498.0	0.048630	0.055930	0.960340	20.563438	17.879492	1.041298	20.057207	17.372298	0.207373
	498.5	0.017670	0.020180	0.568430	56.593096	49.554014	1.759232	56.090867	49.051466	1.155709
	499.0	0.005630	0.006250	0.079290	177.619893	160.000000	12.611931	177.119188	159.499216	12.101606
4	499.5	0.002700	0.002710	0.002700	370.370370	369.003690	370.370370	369.870032	368.503351	369.870032
	500.0	0.005630	0.005440	0.076990	177.619893	183.823529	12.988700	177.119188	183.322848	12.478687
	500.5	0.017670	0.017690	0.562220	56.593096	56.529112	1.778663	56.090867	56.026881	1.176852
	501.0	0.048630	0.050080	0.958970	20.563438	19.968051	1.042785	20.057207	19.461629	0.211225
	501.5	0.112920	0.118360	0.999550	8.855827	8.448800	1.000450	8.340854	7.933059	0.021223
	502.0	0.222450	0.235080	1.000000	4.495392	4.253871	1.000000	3.963983	3.720423	0.000000
	496.5	0.612690	0.646710	1.000000	1.632147	1.546288	1.000000	1.015754	0.919086	0.000000
	497.0	0.396900	0.427570	0.999990	2.519526	2.338798	1.000010	1.956652	1.769514	0.003162
	497.5	0.209230	0.229170	0.998240	4.779429	4.363573	1.001763	4.250119	3.831083	0.042026
6	498.0	0.087420	0.096700	0.925150	11.439030	10.341262	1.080906	10.927597	9.828552	0.295722
	498.5	0.028440	0.031490	0.485320	35.161744	31.756113	2.060496	34.658138	31.252114	1.478225
	499.0	0.007290	0.007960	0.065000	137.174211	125.628141	15.384615	136.673297	125.127142	14.876215
	499.5	0.002700	0.002710	0.002700	370.370370	369.003690	370.370370	369.870032	368.503351	369.870032
	500.0	0.00729	0.00709	0.06288	137.1742	141.0437	15.90330	136.67329	140.5428	15.39519

		0	0	0	11	24	8	7	34	1
	500.5	0.028440	0.028460	0.478580	35.161744	35.137034	2.089515	34.658138	34.633425	1.508826
	501.0	0.087420	0.089260	0.922730	11.439030	11.203227	1.083741	10.927597	10.691541	0.301253
	501.5	0.209230	0.215850	0.998140	4.779429	4.632847	1.001863	4.250119	4.102490	0.043208
	502.0	0.396900	0.410110	0.999990	2.519526	2.438370	1.000010	1.956652	1.872773	0.003162
	502.5	0.612690	0.629960	1.000000	1.632147	1.587402	1.000000	1.015754	0.965631	0.000000
9	495.0	0.998810	0.999150	0.999690	1.001191	1.000851	1.000310	0.034537	0.029180	0.017612
	495.5	0.991020	0.993030	0.996640	1.009061	1.007019	1.003371	0.095622	0.084073	0.058161
	496.0	0.955030	0.962370	0.977170	1.047088	1.039101	1.023363	0.222047	0.201570	0.154626
	496.5	0.847300	0.864080	0.900940	1.180220	1.157300	1.109952	0.461193	0.426666	0.349344
	497.0	0.638370	0.662300	0.717470	1.566490	1.509890	1.393786	0.942019	0.877427	0.740847
	497.5	0.375730	0.396920	0.445830	2.661486	2.519399	2.243007	2.102860	1.956521	1.669753
	498.0	0.161690	0.173250	0.198280	6.184674	5.772006	5.043373	5.662643	5.248242	4.515777
	498.5	0.048630	0.052460	0.059470	20.563438	19.062143	16.815201	20.057207	18.555407	16.307538
	499.0	0.010050	0.010790	0.011670	99.502488	92.678406	85.689803	99.001225	92.177050	85.188336
	499.5	0.002700	0.002700	0.002700	370.370370	370.370370	370.370370	369.870032	369.870032	369.870032
	500.0	0.010050	0.009810	0.010670	99.502488	101.936799	93.720712	99.001225	101.435567	93.219371
	500.5	0.048630	0.048640	0.055420	20.563438	20.559211	18.044027	20.057207	20.052978	17.536901
	501.0	0.161690	0.164000	0.188650	6.184674	6.097561	5.300822	5.662643	5.575185	4.774713
	501.5	0.375730	0.382820	0.431980	2.661486	2.612194	2.314922	2.102860	2.052160	1.744690
	502.0	0.638370	0.648770	0.705470	1.566490	1.541378	1.417495	0.942019	0.913493	0.769283
	502.5	0.847300	0.855900	0.894670	1.180220	1.168361	1.117731	0.461193	0.443516	0.362755

	503.0	0.955030	0.959260	0.975200	1.047088	1.042470	1.025431	0.222047	0.210414	0.161485
	503.5	0.991020	0.992290	0.996260	1.009061	1.007770	1.003754	0.095622	0.088489	0.061385
	504.0	0.998810	0.999040	0.999650	1.001191	1.000961	1.000350	0.034537	0.031014	0.018715
	504.5	0.999900	0.999920	0.999980	1.000100	1.000080	1.000020	0.010001	0.008945	0.004472
	505.0	0.999990	1.000000	1.000000	1.000010	1.000000	1.000000	0.003162	0.000000	0.000000

Table 2: CVRL of S_A control chart for various values of n and σ^2.

n	σ^2 / μ'	**1**	**2**	**3**	**4**	**5**
4	497.0	0.150831	0.544188	0.738298	0.831542	0.881786
	497.5	0.398316	0.753733	0.868960	0.917248	0.941849
	498.0	0.707107	0.900118	0.947413	0.966017	0.975382
	498.5	0.917248	0.971391	0.983609	0.988543	0.991126
	499.0	0.988543	0.994469	0.996053	0.996774	0.997183
	499.5	0.998649	0.998649	0.998649	0.998649	0.998649
	500.0	0.988543	0.994469	0.996053	0.996774	0.997183
	500.5	0.917248	0.971391	0.983609	0.988543	0.991126
	501.0	0.707107	0.900118	0.947413	0.966017	0.975382
	501.5	0.398316	0.753733	0.868960	0.917248	0.941849
	502.0	0.150831	0.544188	0.738298	0.831542	0.881786
	497.0	0.029881	0.302883	0.544188	0.689446	0.776597
	497.5	0.169657	0.566822	0.753733	0.842030	0.889255
	498.0	0.500081	0.810018	0.900118	0.936782	0.955291
	498.5	0.842030	0.947413	0.971391	0.980847	0.985677

6	499.0	0.980847	0.991727	0.994469	0.995675	0.996347
	499.5	0.998649	0.998649	0.998649	0.998649	0.998649
	500.0	0.980847	0.991727	0.994469	0.995675	0.996347
	500.5	0.842030	0.947413	0.971391	0.980847	0.985677
	501.0	0.500081	0.810018	0.900118	0.936782	0.955291
	501.5	0.169657	0.566822	0.753733	0.842030	0.889255
	502.0	0.029881	0.302883	0.544188	0.689446	0.776597
9	497.0	0.001843	0.103107	0.302883	0.476054	0.601358
	497.5	0.036741	0.327109	0.566822	0.707107	0.790108
	498.0	0.258471	0.654063	0.810018	0.879416	0.915593
	498.5	0.707107	0.900118	0.947413	0.966017	0.975382
	499.0	0.966017	0.986785	0.991727	0.993824	0.994964
	499.5	0.998649	0.998649	0.998649	0.998649	0.998649
	500.0	0.966017	0.986785	0.991727	0.993824	0.994964
	500.5	0.707107	0.900118	0.947413	0.966017	0.975382
	501.0	0.258471	0.654063	0.810018	0.879416	0.915593
	501.5	0.036741	0.327109	0.566822	0.707107	0.790108
	502.0	0.001843	0.103107	0.302883	0.476054	0.601358

Table 3: CVRL of P_B and P_R control charts for various n, $\lambda^2$and σ^2.

λ^2	**n**	μ'	**Values of σ^2**					
			1		**2**		**5**	
			CVRLP$_B$	**CVRLP$_R$**	**CVRLP$_B$**	**CVRLP$_R$**	**CVRLP$_B$**	**CVRLP$_R$**
1	4	497.0	0.049479	0.000561	0.229663	0.035433	0.483405	0.270427
		497.5	0.212055	0.017774	0.475176	0.177188	0.691149	0.516532
		498.0	0.530783	0.178697	0.738298	0.492115	0.854796	0.763104
		498.5	0.839949	0.621622	0.914482	0.823390	0.949064	0.922179
		499.0	0.975382	0.948145	0.983609	0.973520	0.987305	0.984449

		499.5	0.998308	0.998477	0.998071	0.998334	0.997660	0.998018
		500.0	0.991126	0.971946	0.996053	0.989934	0.998031	0.996519
		500.5	0.915593	0.715776	0.969647	0.902195	0.989363	0.974437
		501.0	0.669393	0.250149	0.868960	0.627733	0.955572	0.889399
		501.5	0.325451	0.031214	0.656028	0.276296	0.868742	0.701342
		502.0	0.094747	0.001250	0.385789	0.069124	0.712327	0.444370
	9	497.5	0.015755	0.000000	0.168760	0.000989	0.509233	0.108607
		498.0	0.169163	0.000477	0.482064	0.046660	0.758828	0.399185
		498.5	0.611830	0.091302	0.818906	0.415260	0.920866	0.778593
		499.0	0.946731	0.775232	0.973012	0.912149	0.984306	0.968365
		499.5	0.998481	0.998590	0.998340	0.998534	0.998028	0.998385
		500.0	0.970873	0.819569	0.989604	0.943184	0.996440	0.986520
		500.5	0.705931	0.117139	0.898499	0.497508	0.973635	0.864131
		501.0	0.237590	0.000776	0.616710	0.069368	0.885942	0.522936
		501.5	0.027777	0.000000	0.264214	0.001862	0.693487	0.175539
5	4	497.0	0.122138	0.001721	0.467992	0.089850	0.809581	0.000000
		497.5	0.354361	0.034970	0.697480	0.301882	0.903294	0.000744
		498.0	0.671641	0.250813	0.872032	0.629804	0.958771	0.038637
		498.5	0.903561	0.698119	0.962484	0.889361	0.985527	0.378107
		499.0	0.986386	0.963782	0.992826	0.984903	0.995813	0.893391
		499.5	0.998633	0.998642	0.998619	0.998635	0.998582	0.998173
		500.0	0.989184	0.968249	0.994920	0.987841	0.997498	0.955616
		500.5	0.917170	0.716921	0.971297	0.903745	0.991003	0.542336
		501.0	0.699937	0.267547	0.894574	0.658034	0.972212	0.085283
		501.5	0.383226	0.039012	0.734984	0.327710	0.929695	0.002635
		502.0	0.137870	0.002013	0.510750	0.101885	0.851908	0.000000
	9	497.5	0.031145	0.000000	0.288933	0.002115	0.734963	0.001163
		498.0	0.238213	0.000777	0.618735	0.069517	0.889070	0.047766
		498.5	0.688216	0.111626	0.885463	0.481102	0.967016	0.404184
		499.0	0.962565	0.802483	0.984496	0.931951	0.993371	0.899550
		499.5	0.998642	0.998647	0.998635	0.998644	0.998616	0.998148
		500.0	0.967102	0.811261	0.987472	0.937918	0.995396	0.959838
		500.5	0.707056	0.117261	0.900042	0.498218	0.975286	0.573854
		501.0	0.254291	0.000856	0.646897	0.075129	0.910378	0.103905
		501.5	0.034776	0.000000	0.313951	0.002396	0.771829	0.004050
10	4	497.0	0.135841	0.001974	0.505449	0.100327	0.847085	0.573051
		497.5	0.375932	0.037967	0.725808	0.321161	0.923647	0.770543
		498.0	0.689418	0.261197	0.886430	0.647507	0.967613	0.906022
		498.5	0.910528	0.707570	0.967100	0.896706	0.988506	0.972225
		499.0	0.987493	0.965507	0.993680	0.986062	0.996539	0.994308
		499.5	0.998645	0.998647	0.998641	0.998645	0.998631	0.998640

		500.0	0.988874	0.967723	0.994707	0.987512	0.997359	0.995293
		500.5	0.917228	0.716960	0.971367	0.903802	0.991092	0.976160
		501.0	0.703551	0.269722	0.897399	0.661648	0.973863	0.916197
		501.5	0.390745	0.040093	0.744432	0.334462	0.935961	0.788620
		502.0	0.144236	0.002135	0.527401	0.106776	0.867216	0.596084
	9	497.5	0.033836	0.000000	0.307603	0.002323	0.762865	0.206915
		498.0	0.248187	0.000825	0.636388	0.072983	0.902721	0.541334
		498.5	0.697681	0.114417	0.892902	0.489645	0.971364	0.857681
		499.0	0.964316	0.805798	0.985669	0.934232	0.994201	0.982036
		499.5	0.998647	0.998649	0.998646	0.998648	0.998640	0.998646
		500.0	0.966568	0.810181	0.987138	0.937201	0.995193	0.983667
		500.5	0.707094	0.117265	0.900099	0.498242	0.975357	0.865654
		501.0	0.256382	0.000866	0.650504	0.075864	0.913041	0.554846
		501.5	0.035747	0.000000	0.320498	0.002472	0.781061	0.216413

Table 4: CP_B and CP_R for various values of n, $\sigma^2 = 5$ and $\lambda^2 = 20$.

n		4		6		9	
UCL		502.7834	501.745	502.2033	500.998	501.7192	500.7876
USL	**LCL**	496.2755	497.282	496.8387	498.0144	497.3079	498.2215
	LSL	**CP_B**	**CP_R**	**CP_B**	**CP_R**	**CP_B**	**CP_R**
506	494	1.843909	2.689219	2.236068	4.021922	2.720294	4.676305
505	495	1.536591	2.241015	1.863390	3.351602	2.266912	3.896921
504	496	1.229272	1.792812	1.490712	2.681281	2.328566	4.692911
503	497	0.921854	1.344609	1.118034	2.010961	1.746425	3.519683
502	498	0.614636	0.896406	0.745356	1.340640	0.906764	1.558768
501	499	0.307318	0.448203	0.372678	0.670320	0.453384	0.779384

References

Berger, J.O. (1986). Statistical Decision Theory and Bayesian Analysis. *Springer-Verlag.*

Bhat, S. V. and Gokhale, K. D. (2014). Posterior control chart for process average under conjugate prior distribution. Economic Quality Control, 29(1):19–27.

Colosimo, B. M. and Del Castillo, E. (2006). Bayesian Process Monitoring Control and Optimization. *Chapman and Hall /CRC New York.*

Girshick, M. A. and Rubin, H. (1952). A Bayesian approach to a quality control model. Annals of Mathematical Statistics, 23(1):114–125.

Gokhale, K. D. (2017). Studies in Statistical Quality Control Using Prior Information. *An Unpublished thesis submitted to the Karnatak, University Dharwad.*

Luko, S. N. (1996). Concerning the estimators r/d2 and r/d2* in estimating variability in a normal universe. Quality Engineering, 8(3):481–487.

Montgomery, D. C. (1996). Introduction to Statistical Quality Control (6th Edition ed.). John Wiley and Sons, New York.

Nelson, L. S. (1975). Use of the range to estimate variability. Journal of Quality Technology, 7(1):46–48.

Saghir, A. (2007). Evaluation of and s chart when standards vary randomly. Published Online:interstat.statjournals.net/YEAR/, 2007.

Shewhart, W. A. (1931). Economic Control of Quality of Manufactured Product. D Van Nostrand Company Inc, New York.

Tippett, L. H. C. (1925). On the extreme individuals and the range of samples taken from a normal population. Biometrika, 17(1):364–387.

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