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NEOTERIC RELATIONSHIP BETWEEN VARIOUS PRIMES AND THEIR ANALYSIS

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ABSTRACT. After all numbers have been represented as the smallest possible arrangement of integers, primes are the set of numbers that remain. When no more factoring is possible, prime numbers are the only ones left. Number theorists love primes because they are the fundamental units of whole numbers, while the rest of the world values primes because of their unique mathematical properties, which make them suitable for today's demands. When it comes to factorization, primes have special characteristics. One of these characteristics is that, while it is relatively easy to find greater prime numbers, factoring gigantic numbers back into primes is invariably difficult. In this paper, novel primes such as the digital super prime, super twin primes, and digital multi-reverse prime have been defined, as well as the correlation between them has been found.

1. Introduction

Eratosthenes, a Greek mathematician, devised the Sieve's method for determining primes up to a given number in the year 200 B.C. Furthermore, Gauss, Fermat, and Mersenne made significant contributions to the study of prime numbers. The most recent prime number researcher was G.H. Hardy, an English mathematician. As a result, there exist many different types of prime numbers, such as mersenne primes, fermat primes, twin primes, and so on. Two prime numbers $P_{\alpha,\beta,\gamma}$ = pqr and $P_{\gamma,\beta,\alpha}$ = rqp such that $P_{\alpha,\beta,\gamma} \neq P_{\gamma,\beta,\alpha}$ are called Multi-reverse primes (Sharma, D.K., Agarwal, S. and Uniyal A.S., 2021), where p, q, and r be three different primes having α, β, and γ digits respectively. According to (Agarwal, S. and Uniyal A.S., 2016), digital primes are the primes having sum of its digits is a prime number. (Agarwal, S., Sharma D.K. and Uniyal A.S., 2021) defined perfect super primes and non-super primes and discovered their distribution inside a given interval, as well as developed a Java programme to locate super primes, perfect super primes, and non-super primes within the given interval.

We know there are an unlimited number of primes, but can we come up with a formula to determine the n^{th} prime? Nonetheless, it is a problem that must be resolved. Composites do not have a perfect pattern of primes; hence they are random. Gauss, on the other hand, approximated a pattern that is the way to the prime number theorem in 1792. The Prime Number Theorem (PNT) addresses the asymptotic distribution of prime numbers in number theory. The PNT describes how primes are distributed among the positive integers in general. It formalizes the intuitive assumption that as primes grow larger, they become less common.

Agarwal, S., Sharma D.K. and Uniyal A.S. (2021) defined super primes, perfect super primes, non-super primes and found their distribution in a particular interval. Patel, M., Patel, A.M. and Gandhi, R.B. (2020) defined twenty new sorts of

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prime numbers and generated them using Python programmes. Soltan, S.S. (2022) discovered a novel approach to visualize the distribution of prime numbers in the number system and use a simpler algorithm to find prime numbers in a subset of numbers. Gunasekara, A.R.C., Jayathilake A.A. and Perera A.A.I. (2015) outlined distinct forms of primes and developed MATLAB scripts to detect whether a given positive integer is a prime or not.

Because the fundamental theorem states that any number may be factorized as the product of primes, Agarwal, S. and Uniyal, A.S. (2015) created the role of primes in a unique way. The research looked at pairs of composite numbers that can be factorized using non-repeating primes. In the realm of cryptography key systems, certain distributions have been produced and their uses outlined. Agarwal, A., Agarwal, S. and Singh, B.K. (2021) created a Java programme to detect Fibonacci primes inside a given interval and discovered the Fibonacci prime distribution. Fibonacci primes were also used to suggest a data encryption and decryption technique.

In comparison to existing encryption approaches, the suggested method provides a high level of protection from unauthorized access. This research aims to discover the esoteric relationship between numerous primes and their analysis in various intervals, as well as the correlation between them.

2. New Definitions

2.1 Digital Super Prime

A number which is super prime as well as a digital prime is called a digital super prime.

Example: 2, 11, 101, 139, 151 are super primes as well as digital primes, therefore these numbers are digital super primes.

2.2 Super Twin Prime

The twin primes which are also super prime numbers are known as super twin primes.

Example: (3, 5), (11, 13), (101, 103) are pairs of twin primes which are also super prime, therefore are called super twin primes.

2.3 Digital Multi-Reverse Prime

A multi-reverse prime number which is also a digital prime is called digital multi-reverse prime.

Example: $P_{1,1,2} = 3217$ and $P_{2,1,1} = 1723$ are multi-reverse prime number and the sum of their digits = 3+2+1+7 = 13 (prime), therefore the numbers 3217 and 1723 are digital multi-reverse primes.

3. Analysis and Representation of Various Primes

3.1 Analysis of Super primes and Digital Super primes in different intervals

Table 1 represents the analysis of super primes and digital super primes in different intervals.

Table 1

	1 to 10000	10000 to 20000	20000 to 30000	30000 to 40000	40000 to 50000	50000 to 60000	60000 to 70000	70000 to 80000	80000 to 90000	90000 to 100000
No. of Super primes	551	368	449	334	331	376	344	252	360	288
No. of Digital Super primes	243	161	196	142	130	151	98	92	146	103

3.2 Graphical Representation of Super Primes and Digital Super Primes in different Intervals

Fig. 1 shows the graphical representation of super primes and digital super primes in different intervals. In Fig. 1 blue colour represents the no. of super primes and red colour represents the no. of digital super primes.

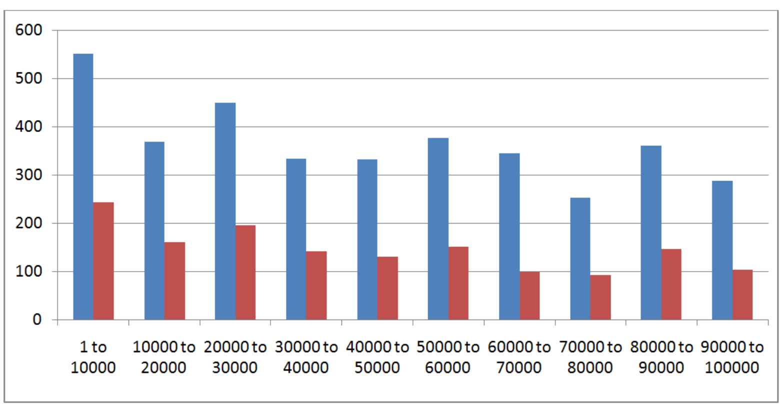

Fig. 1

3.3 Analysis of Twin Primes and Super Twin Primes in different Intervals

Table 2 represents the analysis of twin primes and super twin primes in different intervals.

Table 2

	1 to 100000	100000 to 200000	200000 to 300000	300000 to 400000	400000 to 500000	500000 to 600000	600000 to 700000	700000 to 800000	800000 to 900000	900000 to 1000000
No. of Twin primes	1224	936	834	810	761	766	730	705	706	697
No. of Super Twin primes	194	118	140	71	88	104	67	52	91	78

3.4 Graphical Representation of Twin Primes and Super Twin Primes in different Intervals

Fig. 2 shows the graphical representation of twin primes and super twin primes in different intervals. In Fig. 2 blue colour represents the no. of twin primes and red colour represents the no. of super twin primes.

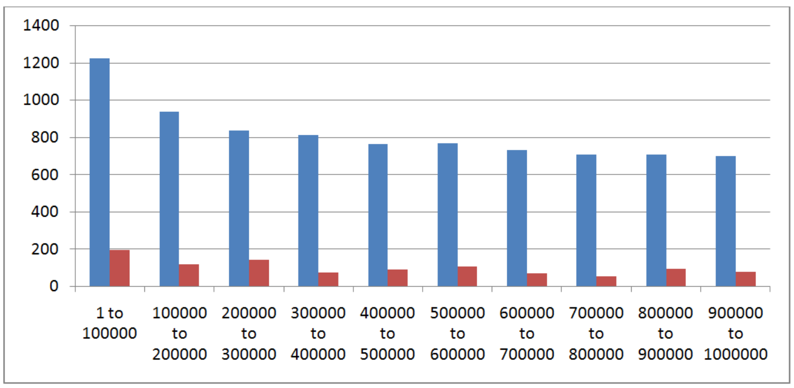

Fig. 2

3.5 Analysis of Multi-reverse Primes and Digital Multi-reverse Primes

Table 3 represents the analysis of multi-reverse primes and digital multi-reverse primes in the interval [1-10000].

Table 3

Interval	1 to 10000
No. of Multi-reverse primes	24
No. of Digital Multi-reverse primes	16

3.6 Graphical Representation of Multi-reverse Primes and Digital Multi-reverse Primes

Fig. 3 shows the graphical representation of multi-reverse primes and digital multi-reverse primes in the interval [1-10000]. In Fig. 3 blue colour represents the no. of multi-reverse primes and red colour represents the no. of digital multi-reverse primes.

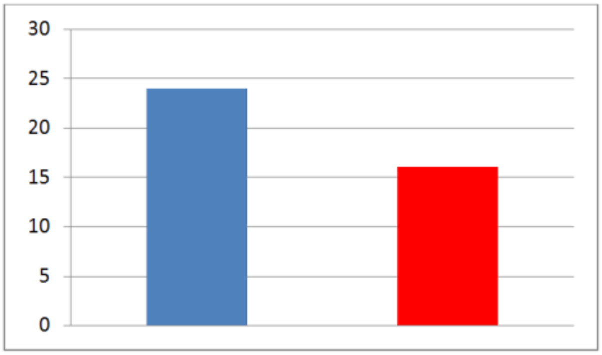

Fig. 3

3.7 Analysis of Primes and Multi-reverse Primes in the interval [1, 10000]

Table 4 represents the analysis of multi-reverse primes and digital multi-reverse primes in the interval [1-10000].

Table 4

Interval	1 to 10000
No. of primes	1229
No. of Multi-reverse primes	24

3.8 Graphical Representation of Primes and Multi-reverse Primes in the interval [1, 10000]

Fig. 4 shows the graphical representation of primes and multi-reverse primes in the interval [1, 10000]. In Fig. 4 blue colour represents the no. of primes and red colour represents the no. of multi-reverse primes.

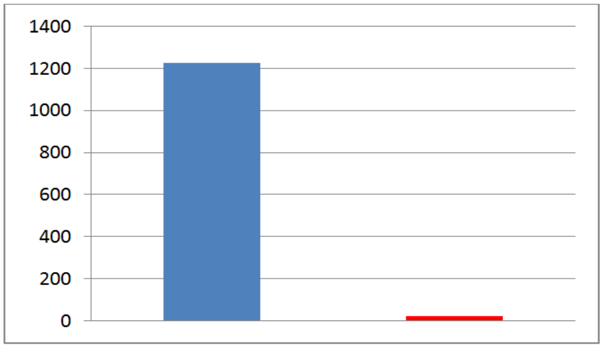

Fig. 4

4. Correlation between different Primes

4.1 Correlation between Super Primes and Digital Super Primes

Number of Super primes (x)	Number of Digital Super prime (y)	$d_x = x - A$ where A = 331	$d_y = y - B$ where B = 130	d_x^2	d_y^2	d_xd_y
551	243	220	113	48400	12769	24860
368	161	37	31	1369	961	1147
449	196	118	66	13924	4356	7788
334	142	3	12	9	144	36
331	130	0	0	0	0	0
376	151	45	21	2025	441	945
344	98	13	-32	169	1024	416
252	92	-79	-38	6241	1444	3002
360	146	29	16	841	256	464
288	103	-43	-27	1849	729	1161
		$\sum d_x = 343$	$\sum d_y = 162$	$\sum d_x^2 = 74827$	$\sum d_y^2 = 22124$	$\sum d_xd_y = 38987$

Here n = 10

The correlation coefficient is given by,

$$r = \frac{n\sum dx\,dy - \sum dx \sum dy}{\sqrt{[n\sum \mathbf{dx}^2 - (\sum \mathbf{dx})^2]} \times \sqrt{[n\sum \mathbf{dy}^2 - (\sum \mathbf{dy})^2]}}$$

$$r = \frac{10 \times 38987 - 343 \times 162}{\sqrt{748270 - 117649} \times \sqrt{221240 - 26244}}$$

$$r = \frac{389870 - 55566}{\sqrt{630621} \times \sqrt{194996}}$$

$$r = \frac{334304}{794.116 \times 441.583}$$

$$r = \frac{334304}{350668.125}$$

r = 0.95334538

There is a strong positive correlation between super primes and digital super primes.

4.2 Correlation between Twin Primes and Super Twin Primes

Number of Twin primes (x)	Number of Super twin prime (y)	$d_x = x - A$ where A = 761	$d_y = y - B$ where B = 88	d_x^2	d_y^2	$d_x d_y$
1224	194	463	106	214369	11236	49078
936	118	175	30	30625	900	5250
834	140	73	52	5329	2704	3796
810	71	49	-17	2401	289	-833
761	88	0	0	0	0	0
766	104	5	16	25	256	80
730	67	-31	-21	961	441	651
705	52	-56	-36	3136	1296	2016
706	91	-55	3	3025	9	-165
697	78	-64	-10	4096	100	640
		$\sum d_x = 559$	$\sum d_y = 123$	$\sum d_x^2 = 263967$	$\sum d_y^2 = 17231$	$\sum d_x d_y = 60513$

Here n = 10

The correlation coefficient is given by,

$$r = \frac{n \sum dx\, dy - \sum dx \sum dy}{\sqrt{[n\sum \mathbf{dx^2} - (\sum \mathbf{dx})^2]} \times \sqrt{[n\sum \mathbf{dy^2} - (\sum \mathbf{dy})^2]}}$$

$$r = \frac{10 \times 60513 - 559 \times 123}{\sqrt{2639670 - 312481} \times \sqrt{172310 - 15129}}$$

$$r = \frac{536373}{\sqrt{2327189} \times \sqrt{157181}}$$

$$r = \frac{536373}{1525.512 \times 396.460}$$

$$r = \frac{536373}{604804.4875}$$

r = 0.88685

There is a strong positive correlation between twin primes and super twin primes.

5. Conclusion

This research demonstrates the relationship between various primes and their analysis in various intervals, which is beneficial for furthering prime research and determining their distribution. The correlation between different primes has been observed and found that there is a strong positive correlation between them, which will also aid in the investigation of the complex relationship between various primes.

References

1. Soltan, S.S. (2022). *Step Pyramid Distribution for Prime Numbers*. Journal of Mathematics Research; Vol.14, No.1, pp.55-61.
2. Sharma, D.K., Agarwal, S. and Uniyal A.S. (2021). *Distribution of Multi-Reverse Primes within the Given Interval & Their Application in Asymmetric Cryptographic Algorithm*. International Journal of Applied Engineering & Technology 3(1), pp.29-33.
3. Agarwal, S., Sharma, D.K. and Uniyal, A.S. (2021). *Formulation & Distribution of Super Primes*. Global and Stochastic Analysis Vol.8 No.2, pp.155-166.
4. Agarwal, A., Agarwal, S. and Singh, B.K. (2021). *Analysis of Fibonacci Primes & Their Application in Cryptography*. Stochastic Modeling and Applications, 25(2), pp.73-82.
5. Patel, M., Patel, A.M. and Gandhi, R.B. (2020). *Prime Numbers and Their Analysis*. Journal of Emerging Technologies and Innovative Research, Vol.7, Issue 2, pp.1-5.
6. Agarwal, S. and Uniyal, A.S. (2016). *Computational Algebraic Number Theory & its Relevance in Modern Cryptography*. Thesis, Kumaun University, Nainital.
7. Gunasekara, A.R.C., Jayathilake A.A. and Perera A.A.I. (2015). *Survey on Prime Numbers*. Elixir Appl. Math. 88, pp.36296-36301.
8. Agarwal, S. and Uniyal, A.S. (2015). *Multiprimes Distribution within a Given Norms*. International Journal of Applied Mathematical Sciences, Vol.8, No.2, pp.126-132.

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