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MEDICAL CONSULTATION AND PANDEMIC INVESTIGATION: PERTINENCE TO GAME THEORY

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ABSTRACT. Game theory is a theory centered on assumptions of rational choice and aiming on interactive decision making has the prospective to offer a new conceptual and theoretical basis for future empirical work in medical consultation. The medical consultation can be demarcated as a two-way social interaction encompassing interactive decision making. In a typical consultancy the doctor elicits information from the patient, then offers a diagnosis or opinion and may also discuss and offer treatment. The outcome of the consultancy is affected by the actions and choices of both participants. Vaccination prevents the spread of infectious diseases but is driven by individual decision making. Further, if herd immunity is built than a free−rider can get away from infection without paying any cost for vaccination. Clearly, there is a conflict between individual and social paybacks i.e., a conflict between individual rational choices: trying to escape vaccination, or everyone getting vaccinated. This study aims to describe a new perspective for medical consultation and Pandemic investigation in context with game theory.

1. Introduction

The game theory was structured by Morgenstern and Neumann in 1947 to analyze the strategic interaction between players (organizations, entities, etc). Anthony Kelly (2003) stated that it is the theory of independent and interdependent decision making. It is concerned with decision making in organizations where the outcome depends on the decisions of two or more autonomous players and where no single decision maker has full control over the outcomes. Classical models fail to deal with interdependent decision making. A Game Theory model, on the other hand, is constructed around the strategic choices available to players, where the preferred outcomes are clearly defined and known. Game theory aims to find optimal solutions to situations of conflicts and cooperation under the assumption that players are instrumentally rational and act in their own best interests.

The consultation can be seen as an exercise between the patient and the doctor in gaining information from each other. Decisions are based on which information is disclosed and how it is presented. In game theory both patient and doctor can be conceptualized as players in a game who develop strategies in order to get what they feel are the best outcome for themselves. Repeated interactions between the same doctor and patient provide knowledge and experience about the other. If used, this in turn helps predict future behaviour, trust and better outcomes.

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In the last twenty–five years, game theory has been applied to a growing number of practical problems. The purpose of game theory and medical consultation is to expand the applications of game theory into a broad and meaningful view of the way consultation decisions can be modelled and analyzed. In the language of game theory, players take actions consistent with the given rules of thegame, and these joint actions determine final outcomes and payoffs. This demonstrates that the game theory provides a persuasive guide for medical consultation. It has the potential to provide models of the consultancy that can be used to generate empirically testable predictions about the factors that promote quality of care. Different game structures provide insights into the possible underlying dynamics of the doctor and patient interaction.

2. Strategic Notations of Game Theory

Game theory is a formal theoretic framework for evaluating strategic interactions among two or more persons who, in the jargon of game theory, are called players, and where each player has two or more methods of acting, called strategies. Each player is presumed to have clear predilections amongst the likely outcomes. The players might mark either immediate or consecutive moves, and each player at the end of the game obtains a definite payoff, which can be in the form of money, influence or other items of value, depending on the game. This theoretical tactic epitomizes are evolutionary viewpoint to describe the physiognomies of trust and cooperation.

John Von Neumann and Oscar Morgenstern's 1944 work Theory of Games and Economic Behavior initially deliberated zero-–sum games, which necessitate one player to win and the other to lose for a net payoff of zero, and provided the framework for future embellishments on the notion. They revealed with their 'Minimax theorem' that a finite, two–person, zero–sum game would end in an equilibrium where both players, in seeing each likely move the other player could make, have pursued a strategy to minimize their own losses.

John Nash consequently presented the notion of cooperative and non–cooperative (or non–zero–sum) games. Additionally, he revealed that similar predicted equilibrium outcomes, which are now referred to as Nash equilibria, exist in non–cooperative games, and proposed the Nash bargaining solution for cooperative games. The non–zero–sum games end up with a less–than–maximal result, as the two players do not cooperate, even though cooperation would produce the best possible outcome for both.

3. Critical Elements of Game Theory

- Players are the decision makers in the game who can be an individual, group or organization.
- Strategies are the courses of action out to the players.
- Payoffs are the concluding earnings to players.

3.1 Two–Player & Two–Strategy (2x2) Games

The two–by–two game is a branch of applied mathematics that models human decision making.

Table I
Payoff Matrix of 2x2 Games

Player 2

Player 1

	COOPERATION	DEFECTION
COOPERATION	R,R	S,T
DEFECTION	T,S	P,P

The game uses two discrete strategies (as shown in Table I); cooperation (C) and defection (D). The pair of players receives payoffs in each of the four combinations of C and D. A symmetrical structure between the two players is assumed. In Table II, the payoff of player 1 is represented by the entries preceding the commas; the payoff of player 2 by the entries after the commas. The payoff matrix is denoted by,

$$\begin{pmatrix} R & S \\ T & P \end{pmatrix}$$

Depending on the relative magnitudes of the matrix elements P, R, S, and T, the game can be divided into four classes:

- The Trivial game with no dilemma
- The Prisoner's Dilemma (PD)
- Chicken (also known as Snow Drift Game or Hawk– Dove Game) and
- Shag Hunt (SH)

In PD, Nash equilibrium indicates the grouping of rational strategies adopted by a player, where both agents exhibit D behaviour, and defect each another to accept low profit P. In Trivial game, Nash equilibrium is the R outcome (C, C). The Nash equilibria in Chicken are the S and T outcomes (C, D) and (D, C), while in SH, they are the R and P outcomes (C, C) and (D, D). A dilemma is introduced whenever the Pareto optimum does not match the Nash equilibria. In PD, Chicken, and SH, the fair Pareto optimums differ from the Nash equilibria. SH yields only partial match ((C, C) is one of Nash equilibria), but causes dilemma because other outcomes are also possible. Table II shows the games with their respective magnitude of outcome & equilibrium states.

Table II

Games with their Magnitude of Outcome & Equilibrium States

GAME	MAGNITUDE OF OUTCOME	EQUILIBRIUM STATE
Prisoner's Dilemma	T > R > P > S	D-dominate
Trivial game	R > T > S > P	C-dominate
Chicken game	T > R > S > P	Coexistence or Polymorphic
Shag Hunt	R > T > P > S	Bi-stable

4. Prisoner's Dilemma

Prisoner's Dilemma is one of the utmost prevalent and elementary game theory strategies encompassing cooperation and competition, or conviction and betrayal. This concept reconnoiters the decision–making strategy taken by two entities that, by acting in their personal distinct finest interest, end up with worse consequences than if they had cooperated with each other in the first place.

In the prisoner's dilemma, two accused seized for a crime are detained in isolated cells for interrogation and cannot communicate with each other. Both care more about their personal independence than about the well- being of their co-conspirator. The police have insufficient evidence for a conviction unless at least one of the prisoners discloses incriminating information. Each of them may pick to confess or stay quiet. The prosecutor apprises both accused individually that if he admits and affirms against the other, he can go free, but if he does not cooperate and the other suspect does, he will be penalized to three years in prison. If both confess, they will get a two-year sentence, and if neither confesses, they will be sentenced to one year in prison.

The “dilemma” encountered by the prisoners is that, whatever the other does, and each is affluent admitting rather than remaining quiet. While cooperation is the preeminent strategy for the two accused, when confronted with such a dilemma, research shows most rational people prefer to confess and testify against the other person than stay silent and take the chance the other party confesses. It is assumed players within the game are rational and will strive to maximize their payoffs in the game.

5. Connexion Bridging Prisoner’s Dilemma and Doctor–Patient Relationship–Medical Consultation

Medical consultations may have an underlying structure that corresponds to the Prisoner’s Dilemma game. In any consultation it is likely for the doctor either to act in the patient’s preeminent interests (Cooperate) or to take a course of action that is not in the best interests of the patient (Defect), leading to poor quality care. The patient, in any given consultation, has to decide whether to follow the doctor’s advice of prescribed course of treatment (Cooperate), or not (Defect).

The ensuing proposed scenario will provide an instinctive elucidation of a prisoner’s dilemma in a medical consultation. A patient with a severe cough in chest visits doctor's office and asks for antibiotics. The doctor examines and explicates that the patient has dire flu which is nearly undoubtedly affected by a virus. Doctor said that cough won’t respond to antibiotics but the patient perseveres. Finally, the doctor jot downs the prescription as possibly since doctor knows the patient will just keep looking for second opinions until he gets the treatment hedesires. This could be perhaps because of new strategies in action that means physicians are being appraised relatively centered on patient gratification. Perchance the doctor instructs needless tests to diagnose infection. The patient is contented and the doctor can switch examining next patient. But both choices are far from perfect for the patient, the doctor and for society.

In nutshell, the patient can select either to follow the course of treatment/advice or not and to take other action which might comprise consulting another doctor for a second opinion. There are four possible outcomes:

- **(Cooperate, Cooperate):** doctor gives advice and the patient chooses to follow it.
- **(Cooperate, Defect):** doctor gives advice but the patient chooses not to follow it.
- **(Defect, Cooperate):** doctor gives prescription and the patient follows the course of treatment.

- **(Defect, Defect):** doctor gives prescription but the patient does not follow the course of treatment.

Here (Cooperate, Cooperate) is best—the doctor gives best advice to the patient and the patient follows the doctor's advice without taking up valuable time of other doctors. But this outcome is not Nash equilibrium. The unique Nash equilibrium is joint defection (Defect, Defect). By choosing Defect, both doctor and patient avoid the risk of the worst possible outcome for themselves i.e., cooperating when the other player chooses to defect. If the doctor selects to deal with the patient rapidly by issuing a prescription rather than spending time to find a more appropriate management option, then the best outcome for the patient is achieved by choosing not to follow through with the treatment and getting a second opinion; and if the patient chooses this way, then the best option for the doctor is to deal with the patient quickly. However, (Defect, Defect) would clearly not make for good quality care and is obviously not the most desirable outcome. That is why the Prisoner's Dilemma game, and actual interactions which have its strategic structure, are considered somewhat contradictory and challenging.

6. Aspects Encouraging Cooperation

In a single–play Prisoner's Dilemma game as defined above, game theoretic principles illustrated that cooperation is not a rational strategy. However, the condition is changed when we consider interactions that are estimated to continue indefinitely in the future. Evolutionary game theory, enthused mainly by the efforts of Maynard Smith and Axelrod, emphases specifically on such indefinitely repeated games. In this perspective, it is likely to find cooperative strategies that are Nash equilibrium by analyzing indefinitely repeated Prisoner's Dilemma game. This recommends that cooperation is a rational strategy only when interactions are entrenched in a series of repeated contacts that are expected to continue indefinitely in the future. Significant aspects that encourage cooperation comprise players' anticipation of future interactions and their skill to recognize each other and recall past interactions. If players anticipate interacting again in the future, then they can foresee future payoffs from mutual cooperation.

In the perspective of the consultation, mutual cooperation becomes a more attractive prospect if future interactions are anticipated. There are incentives for the doctor to spend time finding an appropriate management approach: consultations with the same patient in the future are likely to take up less time and the doctor will have the satisfaction of carrying a management plan through to completion. The patient is likely to follow through with the treatment if there is an expectation that the doctor will monitor his progress in the future. Both the doctor and the patient can anticipate future payoffs from this mutual cooperation, and this model implies that higher quality of care can be achieved when the patient sees the same doctor repeatedly.

7. Pandemic Investigation

The world is fronting the vilest pandemic in a century, triggered by a new form of corona virus. Both cooperation and coordination are needed to address this pandemic. Idyllically, one would look at earlier practices to acquire lessons on how deal such a pandemic. However, serious pandemics are rare events and there is simply not enough evidence to draw statistically significant conclusions. Furthermore, COVID–19 may very well be the first serious pandemic in a truly interconnected world. Hence, theory and experimental results provide the best guidance available.

8. How Pandemic is allied to Prisoner's Dilemma?

The methodology of study of Pandemic is rather unusual as it applied game theory for determining how people make strategic decisions within a group. Each individual has choices, but the payoff for each choice depends on choices made by others. This is what's called a "prisoner's dilemma game"– players weigh cooperation against betrayal, often producing a less than optimal outcome for the common good. The pandemic presents an everyday complexity of such choices. Imagine, if everyone followed public health recommendations. They wore masks, socially distanced, washed their hands, and followed stay–at–home orders. In that case there is a significantly reduced risk of infection. But there are always trade–offs and temptations to defect from the regimen. Masks are annoying. Hand–washing is tedious.

Now the vaccine adds one more protective layer. The perceived benefits and costs of vaccination are often expressed as concerns about safety and side effects. If you are on the fence about vaccination, you might decide–noticing lower infection rates as vaccination campaigns gain speed – that it no longer seems so critical to get the job. Some people might play a "wait–and–see" game. People who choose not to be vaccinated effectively get a free ride, reaping the benefits of reduced virus transmission generated by the people who do opt for vaccination. But the free rides generate a collective threat. That is what the prisoner's dilemma is. When infection levels are low, people feel less at risk, let down their guard, and then infection levels again rise; the ebb and flow between our behavior and the virus causes the pandemic waves.

Vaccination decisions based purely on self- interest can lead to vaccination coverage that is lower than what is optimal for society overall. The self–interest strategy maximizing individual payoff is called the Nash equilibrium. Vaccination decisions can be influenced by altruism, thereby boosting uptake beyond the Nash equilibrium and serving the common good. Game theory assumes people are rational in their decision-making. Fear can suppress vaccination to precarious levels insufficient to prevent the spread of an outbreak.

Vaccine hesitancy could be explained by a mechanism called "hysteresis". In general terms, hysteresis occurs when the effects of a force persist even after the force is removed – the response lags. For example, paper clips exposed to a magnetic field still cling together after the field is turned off. Similarly, even after a vaccine is deemed safe and efficacious, uptake rates often remain low. The hysteresis effect makes the population hysterical, or sensitive, to the perceived risks of the vaccine. To overcome the hysteresis effect, vaccination should be promoted as an act of altruism – one's personal contribution to defeating the pandemic.

9. Predicting People's Behaviour in Pandemic Using Game Theory

Epidemiologists consider many factors to assess the impact and severity of any disease, along with its transmission rate and infection length, contact and human mobility patterns. This may be understood as they classify human beings in two main categories:

- **Responsive:** who will take measures to limit their risk of infection and
- **Non–responsive:** who will continue as normal without changing their behaviour.

Individuals react to epidemics in a way that's not coordinated by public policymakers. The way people perceive risk – how likely they think they are to be infected– depends on the severity of the disease and how much it spreads through the population. Risk perception is not static, and it affects the way people behave; individuals will behave according to how at risk they think they are.

Now let us see how to apply game theory in studying strategic decision–making – to model the behaviour. Individuals change their behaviour based on that of other people around them, and on their perceived risk of getting infected. People tend to change their behaviour if they talk about the risk with people who think and behave differently. Because of this, the biggest changes in behaviour happen when you have an equal division of reactive and un–reactive people.

When there's no epidemic, everyone behaves normally. In the event of an outbreak, changing behavior will be costly in some way– perhaps people stay home from work or avoid crowded environments, change their commute or buy preventative medication. But there's also a payoff: people who change behavior are less likely to be infected. At a certain point in the epidemic, those people have an advantage in the population, and others begin to copy their behavior.

Game theory can be applied to these dynamics to produce a mathematical model that shows how people's responses can affect the spread of a disease during an epidemic. It was found that reducing the number of people an individual is in contact with, even by a small amount, can make a difference to the spread of disease. It was also established that people are more responsive when the symptoms of the disease are more evident, like coughing and sneezing.

One of the other things model revealed is that when accurate information about the infection is disseminated quickly, people's responses to the information are effective and can limit the spread of a disease. The effectiveness of a response depends on the disease and the best thing to do is not always clear. When information is available and communicated quickly, people are more likely to act in the right way.

10. Demonstrating the Spread of Virus and Vaccination Behaviour

In the Pandemic vaccine rollout,

- the "players" would be individuals looking for care
- the "actions" would be the individuals' selection of a facility and
- the "payoffs" would be measured in terms of how individuals perceive the risk of vaccination, distance travelled, and level of service available at a chosen facility.

Pre–emptive vaccination is one of the best public health measures for preventing epidemics of infectious diseases as well as reducing morbidity and mortality (Anderson and May 1991). Regardless of the fact that few country wide or local governments providing subsidies for it, it still remains the choice of individuals. Decision making at the individual level may be the result of a trade–off between protection and perceived risks and costs of vaccination and infection. Furthermore, it might be that an individual's decision is influenced by the vaccination behaviors of others (Chapman and Coups 1999, 2006; Basu et al. 2008). Thus Vaccination can be viewed as a game.

One key cause for the difficulty in eliminating vaccine–preventable diseases is herd immunity. As vaccination increases, we achieve herd immunity; which occurs when the share of immunized people in the end exceeds an important level above which the disease can no longer persist. Once the herd immunity is attained, the remaining unvaccinated people have less chance to be infected as they're indirectly protected through those who have been vaccinated. Thus; unvaccinated people achieve profits from the herd immunity without thinking about the perceived dangers related to vaccination, consisting of complications, facet outcomes, and monetary prices. As a consequence, there may be much less incentive for them to get vaccinated and then, the so–called first–order free–rider problem arises. Some reports suggest that the welfare of a society can be threatened if too many individuals perceive the herd immunity as a public good (Asch et al. 1994). Excess of self-interest subverts the herd immunity state, and the disease resurges. This paradox results in spread of disease despite enough voluntary vaccination coverage, and it results in a conflict between the optimal vaccination behaviour for every individual and the adequate level of vaccination desired to shield the whole society via the herd immunity (e.g., Cullen and West 1979; Fine and Clarkson 1986; Geoffard and Philipson 1997; Bauch et al. 2003; Bauch and Earn 2004).

Linkage between vaccination coverage, disease occurrence, and the vaccination behaviours of individuals are complex, so if we plan to develop effective public health measures for avoiding waves of infectious diseases then we should dynamically as well as quantitatively project the consequences of these interrelations. Many studies of the vaccination dilemma have applied a game theoretic framework to a population in this regard, wherein every individual attempts to maximize their payoff. These studies have provided highly fruitful results (e.g., Bauch 2005). Earlier analyses of vaccination behaviour by game theory have presumed a static game wherein individuals always act with perfect information on their probability of becoming infected. In reality, individuals cannot precisely know this probability. Moreover, the game should allow individuals to update their strategies through learning by imitating others who appear to have adopted more successful strategies. In this context, imitating others means adapting one's strategy based on his or her own personal experience and based on information from media (the former and latter can be called active and passive information, respectively). To describe this process explicitly, we should construct a model that combines mathematical epidemiologic dynamics with game–theoretic dynamics. For example, Bauch constructed and analyzed a model that combines epidemiological dynamics with replicator dynamics of evolutionary game theory to capture the imitative behavior of individuals during outbreaks of diseases; he found that imitative behavior provokes periodic outbreaks of diseases (e.g., Bauch 2005). Vardavas et al. (2007) proposed an individual–level adaptive decision–making model that was inspired by Minority Game methodology. By solving the model numerically and analytically, they showed that incentive-based vaccination programs are indispensable to control epidemics of infectious disease but that misuse of these programs may lead to a severe epidemic.

An individual has a strong incentive to exploit the public good by free–riding on the herd immunity, wherein the individual pays nothing but still obtains benefit. But this works only as long as the majority in the community spontaneously receives the vaccination. On the contrary, if the majority neglects vaccination, then doing nothing is no longer a better choice as infection is likely. Spontaneous vaccination becomes the rational choice in this case. This difference infers that the best option for an individual is to always adopt the strategy of the social minority i.e., either free–ride when the herd immunity is well established or take the vaccination when most people neglect to do so. This situation obviously contains the structure of a Minority Game, as Vardavas pointed out (Vardavas et al. 2007).

A Minority Game (e.g., Challet et al. 2005), is a social dilemma where any individual has the incentive to adopt the strategy of the minority under any circumstance. This duality might be interpreted as a Chicken–type dilemma wherein the fair Pareto optimum is realized when two strategies coexist. From the perspective of evolutionary game theory, a simple, honest question might be raised: does a Chicken–type dilemma really appear in a particular expected situation wherein individuals live in a complex social network on which an epidemic is spreading? If so, what impact does the dilemma strength have? Furthermore, what social provisions can be taken to prevent a pandemic? For example, can the government provide subsidies to encourage people to take the vaccine? And do such actions really contribute to preventing outbreaks of a disease?

11. Formulating Dynamics

Let us suppose that a population is infinite and well mixed so dynamics can be formulated by a set of ordinary differential equations (ODEs). To model disease transmission, we apply the susceptible-infectious-recovered (SIR) model, wherein the population is divided into three groups:

- Susceptible individuals (S), who are currently healthy but may or may not be infected with the disease

- Infectious individuals (I), who are currently infected and will recover
- Recovered individuals (R), who are never infected again
- S': Rate of change of S
- I': Rate of change of I
- R': Rate of change of R

Here the following assumptions are made:

- The units of S, I, and R are persons.
- The units of time are days.
- The units of S', I', and R' are persons per day, written person/day.
- The system is closed; this simply means that the total size of the population, which equals the sum $S + I + R$, does not change.

The basic purpose of SIR model is to help us understand the way a contagious disease spreads through a population. Immunity is acquired by either recovering from the disease or by pre–emptive vaccination. The immunity is presumed to be effective over an individual's life span. The SIR model is expressed by rate equations. In this model, the rate of change of the recovered population is proportional to the size of the infected population.

Let's begin by addressing S'. Suppose that each susceptible individual comes into contact with a proportion, call it p, of the infected population each day. This implies that each susceptible person has contact with pI infected persons per day. This in turn implies that there are $pI \times S = pSI$ total contacts between susceptible and infected individuals each day. Further suppose that a certain proportion, call it q, of the above contacts cause infection. The above tells us that there are $q \times pSI$ new infections occurring each day, which in turn implies that the size of the susceptible population decreases by qpSI persons each day. In other words, in persons/day, we have

$$\mathbf{S' = -\beta .\ S.\ I}$$

This is Rate equation for the susceptible population. Where $\boldsymbol{\beta = q.p}$, which indicate the disease transmission rate per capita. The minus sign denotes a negative rate of change, meaning a decrease in the quantity in question.

Next consider R'. Suppose infection lasts for k days. On any given day, one k^{th} of the infected population will recover. In other words, the rate of recovery, in individuals per day, is equal to 1/k times **I.**

$R' = \gamma.I$, where **$\gamma = 1/k$**, which indicate the rate of recovery. This is Rate equation for the recovered population.

Finally consider **I'**. Since S + I + R is assumed to be constant, the sum $S' + I' + R'$ of the rates of change of the three subpopulations must be zero– any change in one of these quantities is offset by changes in the others. That is, by the above formulas for S' and R',

$$0 = S' + I' + R' = -\beta SI + I' + \gamma I$$

or, solving for **I'**,

$$I' = \beta. S.I - \gamma. I$$

This is Rate equation for infected population.

Clearly,
$$S + I + R = 1 \quad ...(1)$$

Obviously, the SIR process always takes place in a unilateral direction, S → I →R, which is unlike the SIS model (Hethcote and vanden Driessche1995) wherein immunization efficacy is neglected. Therefore, we can deduce the final epidemic size at the equilibrium of the dynamics: R(∞) is the fraction of individuals who were once infected with the disease. According to Eq. (1) with initial conditions S(0) ≈ 1, R(0) = 0, I(∞) = 0, and S(∞) = 1–R (∞), we derive

$$R(\infty) = 1 - \exp[-R_0.R(\infty)] \quad ...(2)$$

Here, $R_0 \equiv \beta/\gamma$ is called the basic reproduction ratio, which is the number of secondary infections caused by a single infected individual. Let x be the fraction of the total population that is vaccinated, so the remaining fraction 1–x is not. Then, we can rewrite the final epidemic size at the equilibrium of the dynamics when the fraction of pre–emptive vaccination is x, R(x,∞), by solving the following equation:

$$R(x,\infty) = (1-x)(1- \exp[-R_0.R(\infty)]) \quad ...(3)$$

This equation is obviously nonlinear and transcendental. Technically, we cannot derive an exact analytic solution, but we can obtain a numerical solution by, for instance, using the Newton–Raphson method. The solution is in Figure 1, which shows the relation between R(x,∞) and x. One important factor wherein we are interested is the infection point at which R (x,∞) rises from zero with a decreasing vaccination rate. Let us call this value the critical vaccination rate, x_{cr}, because the epidemic cannot be suppressed if the vaccination rate decreases below this critical value. The value of x_{cr} can be obtained by solving the nonlinear Equation (3) to obtain,

$$x_{cr} = 1 - 1/R_0 \quad ...(4)$$

The critical value x_{cr} is the so–called herd immunity threshold; if x exceeds x_{cr}, further propagation of the disease cannot occur in the population (**$R(x_{cr},\infty) = 0$**).

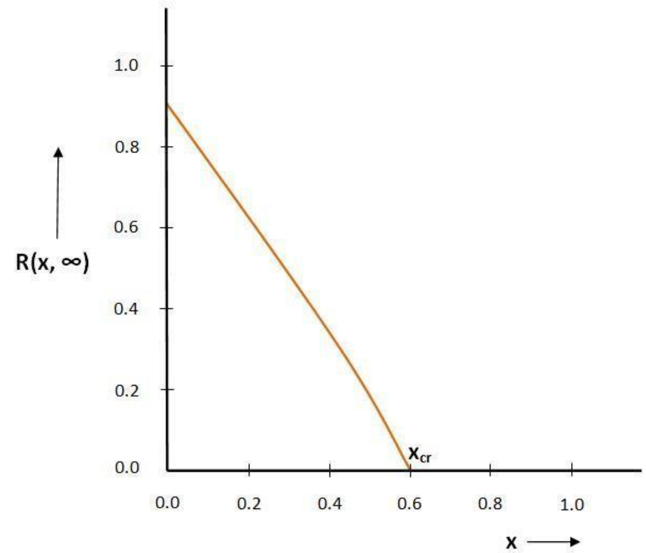

Figure 1
Relation between R(x, ∞) and x when R_0=2.5

A fair Pareto optimum means a state wherein the accumulated social payoff is maximal but fairness is maintained among all individuals. The fair Pareto optimum can be regarded as a state at which the maximum social flux appeared. By determining whether or not this maximum social payoff is consistent with Nash equilibrium, we can identify the dilemma class; whether a PD (defection–dominate), Chicken (polymorphic), or SH (bi–stable) dilemma exists in the model.

We can derive a vaccination rate at which a social payoff is maximized, $x_{social-max}$, which identifies a state at which the epidemic is successfully suppressed ($R(x_{social-max},\infty) = 0$) by a minimum vaccination rate. We want the minimum vaccination rate because both vaccination and infection are costly. The maximum social payoff is attained when the vaccination rate is

$$x_{social-max} = \begin{cases} x_{cr} = 1-(1/R_0) & \text{if } R_0 > 1, \\ 0 & \text{if } R_0 \leq 1 \end{cases} \quad ...(5)$$

The next step is to find the Nash equilibrium. The vaccination rate, which is the strategy in this game, is defined as continuous. In a game defined with a continuous strategy, the Nash equilibrium was provided by Doebeli et al. (2004). We follow their development. Let us presume a resident strategy x. Suppose, here, a small proportion of the resident, defined as ε, which is called mutant, converts from x to y (x ≠ y). The new average social strategy, p, for example, the new vaccination rate, is obtained from p = x.(1−ε) + y.ε. The expected payoffs (average social payoffs) of mutants y are as follows:

$$E(y, p) = -y.C_r + (1-y).R(p,\infty) \quad ...(6)$$

where, again for confirmation, C_r is the vaccination cost normalized by the infection cost of 1. The necessary and sufficient conditions that X_{NE} is as table equilibrium are the following:

At y = X_{NE},

$$[\partial E(y,p)/\partial y] = 0 \quad \text{and} \quad [\partial^2 E(y,p)/\partial y^2] \leq 0 \quad ...(7)$$

These two equations lead us to the explicit form for X_{NE},

$$X_{NE} = \begin{cases} 1 & \text{if } C_r = 0, \\ 1 + \ln(1-C_r)/(C_r.R_0) & \text{if } 0 < C_r \leq R_0(0,\infty), \\ 0 & \text{if } R_0(0,\infty) < C_r \end{cases} \quad ...(8)$$

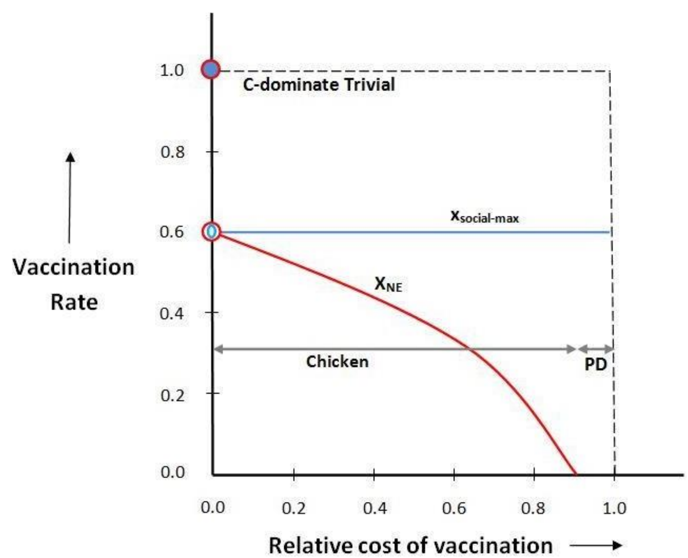

Figure 2
Vaccination Rate at the Social Maximum Payoff and Nash Equilibrium when R_0= 2.5

Figure 2 shows how $x_{social-max}$ and X_{NE} are affected by the vaccination cost (relative cost of vaccination to infection) when we assume R_0= 2.5. At the point C_r= 0, the social maximum is consistent with Nash equilibrium; thus, we would say it is a Trivial game, or, more precisely, a C–dominate Trivial game. At the point C_r=0, Nash equilibrium becomes discontinuous, as described in Equation (8). If the vaccination cost increases, the Chicken game class appears because its dynamics are absorbed by an internal equilibrium. With additional increases in cost, the Prisoner's Dilemma (PD) class is finally observed, and the absorbed state is an all–defectors–state.

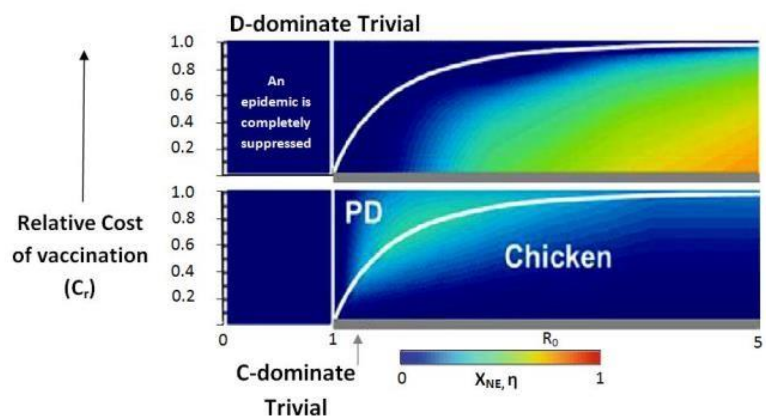

Figure 3
Phase Diagram on the Cost–R_0 Plane with Colored Contours indicating Nash Equilibrium X_{NE} (Upper Panel) and Dilemma Strength η (Bottom Panel)

Figure 3 shows the phase diagram on the cost–R_0 plane. In the figure, the upper panel shows Nash equilibrium with X_{NE} indicating the vaccination rate as the final game result. The lower panel shows the normalized dilemma strength, which varies from 0 to1 according to the definition by Nakata et al. (2010), and is formulated by

$$\eta = E(x_{social\text{-}max}) - E(X_{NE}) \quad \text{...(9)}$$

In Figure 3, the region $R_0 \leq 1$ has no dilemma because an epidemic is completely suppressed even though $x_{social-max}$= 0. Thus, this game belongs to the defectors–dominate (D–dominate) but still Trivial game class (hereafter, D–dominate Trivial or D–Trivial). The region C_r = 0 and R_0 >1, highlighted by the bold gray line, is a cooperators–dominate (C–dominate) and Trivial game, as mentioned in Figure 2 (hereafter, C–dominate Trivial or C–Trivial), where in all individuals take a vaccine, so no infection occurs.

A social dilemma only appears in the region $C_r > 0$ and $R_0 > 1$. As the relative cost of vaccination increases, the PD replaces the Chicken–type game. In the PD region at extremely high cost, the dilemma strength is rather relaxed because too much increased vaccination cost does not differ from the cost of infection, which, for the individual, is not only socially unacceptable but also rather hopeless. The highest social dilemma occurs on the boundary between PD and Chicken in the middle range of the basic reproduction ratio, $1:5 < R_0 < 2:5$. This range of R_0 corresponds to typical epidemic.

Large values of R_0 make the entire region (except for $C_r = 0$) belong to a Chicken–type game, and the game becomes less sensitive to further increases in R_0. Sensitivity to cost also becomes less than that for small R_0. Moreover, the dilemma strength with large R_0 gradually becomes small. This decrease occurs because large R_0, implying higher likelihood to be infected, instinctively motivates more people to take the vaccine (letting X_{NE} increase with the increase in R_0, which is manifestly understood by Equation (9) as well as the upper panel of Figure 3). Thus, the discrepancy between X_{NE} and $x_{social-max}$ becomes relatively small. At the extreme limit of $R_0 \rightarrow \infty$, there is only a C–dominate trivial game for any C_r, although Figure 3 does not extend that far. In this sense, a middle level of epidemic infection, rather than a high level, evokes a social dilemma (this claim can be confirmed by the fact that the light yellow region spreads around $1:5 < R_0 < 2:5$). This interesting result crucially affects people's decisions regarding whether or not to be vaccinated.

12. Conclusion

In this paper, we have seen how game theory continues to be an effective tool to model decision–making by individuals with respect to medical consultation and vaccination. It can be valuable for predicting behavior and incentivizing decisions that improve a broader system. In game theory both patient and doctor can be conceptualized as players in a game who develop strategies in order to get what they feel are the best outcome for themselves. Repeated interactions between the same doctor and patient provide knowledge and experience about the other. If used, this in turn helps predict future behaviour, trust and better outcomes. Epidemics have ever been a great concern of human kind and game theory is an essential tool in studying a diverse range of such diseases to gain a better understanding of transmission mechanisms, and make predictions and determine and evaluate control strategies which includes a decision whether to be vaccinated or not.

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