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# SERVICE HALT IN M/M/1 QUEUE WITH FUNCTIONING VACATION AND CUSTOMER INTOLERANCE

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ABSTRACT. This study examines the operating characteristics pertaining to single-server Markovian queueing model possessing infinite capacity with functioning vacation. The server is subject to sudden halt while in normal busy period and immediately after repair process, its service resumes. The server failure and server repair follow the exponential distributions. Once the system becomes vacant, the server takes functioning vacation during which the customer waiting in the queue tends to renege and server serves the customer at a lower rate. The steady state probabilities are obtained in terms of transition rates of inflow and outflow processes by means of matrix geometric approach. Various measures of performance of the model are determined in terms of system state probabilities. Further impact of model parameters on these measures are shown graphically with the help of numerical illustration.

## 1. Introduction

In our daily life, we come across many situations where service delivery mechanism comes to a sudden halt and can resume working after repair process. This is very common in computers, manufacturing systems and communication networks. Many queue theorists and practitioners have examined various queueing systems with service halts from different point of views. Some notable works in this area have been published by Neuts and Lucantoni (1979), Wang (1990), Grey et a*l*. (2000), Hsieh and Hsieh (2003) and Ke (2006) and many more. The paper published by Haridass and Arumuganathan (2008) presents the analysis on an unreliable server and single vacation in a bulk queue. Further the work by Jain and Bhargava (2009) facilited the study of an unreliable server, continuous trials and customer intolerance in the $M/G/1$ queue by incorporating the theory of feedback based on Bernoulli's concept. Also Jain et a*l*. (2012b) presented the notion of optimal control where server is erratic with options of repair in multiple phases and startups by considering the (N, F) policy. Later on, the feature of unreliable server for the $M/G/1$ queue with multiple options of services and vacations were also studied by Jain et a*l*. (2013).

A vast literature based on queueing theory dedicated to the design and application of functioning vacation exists. The main feature of a functioning vacation in the queueing model is the slow pace of service at the time of vacation. The important contributions towards this direction are due to published works by Servi

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and Finn (2002), Baba (2005), etc. Jain and Jain (2010) studied a functioning vacation queueing model with various kinds of breakdowns of server. Jain and Chauhan (2012) analyzed operational vacation in the queueing system having the provision of second type of service facility and server failure. Jain et a*l*. (2012a) examined the approach of maximum entropy for the discrete time $Geo^x/Geo/1$ queueing system with functioning vacation. State reliant $M/M/1/WV$ queueing model by including the realistic concepts of inspection, restricted rates of arrival and prolonged repair was analyzed by Jain et a*l*. (2014). Murugan and Santhi (2015) studied an $M/G/1$ queue demonstrating the server failure and multiple functioning vacations. A survey on queueing models with working vacation was done by Chandrasekaran et a*l*. (2016).

There are many circumstances where the customer intolerance is due to the server vacation. Altman and Yechiali (2006) considered the customer's intolerance at the time of waiting in vacation models of various kinds of queues, wherein every arrival on finding server on vacation, triggers an impatient timer that is independent and random. Selvaraju and Goswami (2013) presented the analysis of single and multiple operational vacations for studying the customer's intolerance of waiting in an $M/M/1$ queue. Goswami (2015) studied customer's intolerance in $G1/M/1/N$ queueing system where the server can take functioning vacation.

This study deals with an infinite capacity single server queue subject to sudden halt and functioning vacation policy by including the realistic assumption of customer's intolerance. The customers join the system in Poisson fashion with rate of arrival that changes as per the state of the server. When the customer appears, he is attended on first-come-first-served basis. As soon as the system gets vacant, the server goes on functioning vacation at the time of which customer tends to renege and server provides service at a slower pace. After the server comes back from the vacation and spots at least one arriving customer, the service provider starts serving the customer; otherwise it immediately leaves for another vacation. The service is interrupted if sudden halt occurs and the server is fixed instantly to resume service. The steady state probabilities for the system states are solved in terms of rate matrix formulae. The queue size distribution is presented by implementing the matrix geometric method.

The remaining paper is arranged in various sections as follows. In the next section, we present the Mathematical Model. The model is described by stating the relevant assumptions and then after mathematical formulation of model is given. Generator matrix Q and steady state equations are given in the section on 'Matrix Geometric Solution'. The condition of stability to arrive at the steady state is determined in the section devoted to 'Stability Condition'. Matrix analytic approach to find the solution for queue size distribution state is presented in the section on 'Steady State Solution'. Several performance measures related to system's behaviour at steady state are featured in the section on 'Performance Measures'. The section on 'Numerical Experiment and Results'comprises the illustration and sensitivity analysis for the performance measures. Finally conclusion and some future aspects of research done are stated in section devoted to 'Conclusion'.

## 2. Mathematical Model

We study the operating characteristics of service halt in M/M/1 queue with functioning vacation, customer intolerance and state dependent arrivals. The assumption related to input process is that every customer appears in the system independently in the Poisson fashion with rate $\lambda_V$ in vacation period, with rate $\lambda_B$ in busy period, with rate $\lambda_D$ during service halt. When a customer appears, it is immediately taken to the point of service station wherein the service is offered on first-come-first-served basis. The service durations in the normal busy period are considered as exponentially distributed with rate $\mu_B$. If the system is vacant, the server starts taking working vacation with rate $\psi$, during the working vacation, the server will be functioning at slow rate $\mu_V$ and $\mu_V < \mu_B$. After returning back from the vacation, when the server spots at least one customer, he changes its service rate from $\mu_V$ to $\mu_B$ and a non-vacation period begins; on the contrarily, on finding no customers, he leaves for the next vacation. After joining the queue, the customer waits for a given time T for service to commence, and may quit without being offered service. This time T is a random variable that is taken to be distributed as per the exponential distribution with parameter $\xi$. The increase in the number of customers N in the system is limited and consequently the reneging rate $\xi_n$ is redefined as

$$\xi_n = \begin{cases} 0 & \text{n=0} \\ \min(n, N)\xi & n \geq 1 \end{cases} \tag{2.1}$$

The service rate during functioning vacation is given by

$$\mu_{V,n} = \begin{cases} \mu_V & \text{n=0} \\ \mu_V + \xi_n & n \geq 1 \end{cases} \tag{2.2}$$

The server may suffer a halt at any point during the busy period with rate $b$ and is instantly sent for repair to get a recovery with rate $r$. The service halt duration and recovery time distributions of server are taken as exponentially distributed. After the server comes back to the normal condition, it immediately begins to render the service to the customer with the normal rate.

The $M/M/1$ queue subject to service halt with functioning vacation and customer intolerance can be modeled by a two dimensional continuous-time Markov process $\{Y(t), N(t); t \geq 0\}$, where, $N(t)$ is the number of customers in the system at time t and $Y(t)$ is the server state at time t with

$$Y(t) = \begin{cases} V, & \text{if the server is in functioning vacation state.} \\ B, & \text{if the server is in normal busy state.} \\ D, & \text{if the service halt occurs during normal busy state.} \end{cases} \tag{2.3}$$

The state space of the Markov Process is

$$S = \{(V, 0) \cup (i, n) | i = V, B, D; 1 \leq n \leq N\}$$

For formulating the model mathematically, we define the following steady state probabilities:

$P_{V,n} \equiv$ Probability stating that number of customers is n in the system wherein

the server is functioning during vacation state, n = 0, 1, 2, ...
$P_{B,n} \equiv$ Probability stating that number of customers is n in the system wherein the server is in busy state, n = 1, 2, ...
$P_{D,n} \equiv$ Probability stating that number of customers is n in the system wherein the service halt occurs during busy state, n = 1, 2, ...

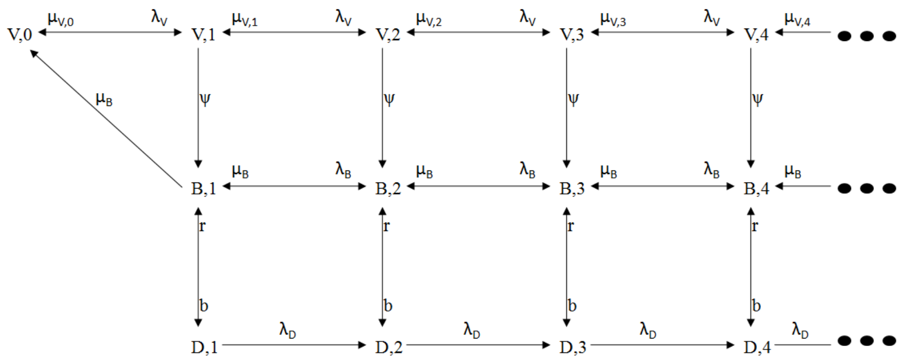


FIGURE 1. State Transition Rate Diagram

The transition flows for different system states can be balanced to build the steady state equations as shown below:

$$\lambda_V P_{V,0} = \mu_{V,0} P_{V,1} + \mu_B P_{B,1} \tag{2.4}$$

$$(\lambda_V + \mu_{V,n-1} + \psi) P_{V,n} = \lambda_V P_{V,n-1} + \mu_{V,n} P_{V,n+1} \quad n \geq 1 \tag{2.5}$$

$$(\lambda_B + \mu_B + b) P_{B,1} = \psi P_{V,1} + r P_{D,1} + \mu_B P_{B,2} \tag{2.6}$$

$$(\lambda_B + \mu_B + b) P_{B,n} = \lambda_B P_{V,n-1} + \psi P_{V,n} + r P_{D,n} + \mu_B P_{B,n+1} \quad n \geq 2 \tag{2.7}$$

$$(\lambda_D + r) P_{D,1} = b P_{B,1} \tag{2.8}$$

## 3. Matrix Geometric Solution

The matrix geometric approach has been developed to calculate the probabilities at stationary state for the Markov Process having structure that is repetitive in nature. By using matrix geometric method, we can easily get the closed form results for the probability vector at the steady state. The generator matrix $Q$ corresponding to probability vector $P$ is

$$Q=\begin{pmatrix} \Upsilon_0 & \Omega_0 & & & & & & & & \\ \Delta_1 & \Upsilon_1 & \Omega & & & & & & & \\ & \Delta_2 & \Upsilon_2 & \Omega & & & & & & \\ & & \ddots & \ddots & \ddots & & & & & \\ & & & & \Delta_N & \Upsilon_{N-1} & \Omega & & & \\ & & & & & \Delta_N & \Upsilon_N & \Omega & & \\ & & & & & & \Delta_N & \Upsilon_N & \Omega & \\ & & & & & & & \ddots & \ddots & \ddots \end{pmatrix}$$

The entries $\Upsilon_n$, $\Omega$ and $\Delta_n$ are square matrices having order 3 since the environmental states are 3 in this study. Also

$$\Upsilon_0 = \begin{pmatrix} -\lambda_V \end{pmatrix}, \Omega_0 = \begin{pmatrix} \lambda_V & 0 & 0 \end{pmatrix}, \Delta_1 = \begin{pmatrix} \mu_{V,0} \\ \mu_B \\ 0 \end{pmatrix},$$

$$\Upsilon_n = \begin{pmatrix} -(\lambda_V + \mu_{V,n-1} + \psi) & \psi & 0 \\ 0 & -(\lambda_B + \mu_B + b) & b \\ 0 & r & -(\lambda_D + r) \end{pmatrix}, n \geq 1,$$

$$\Omega = \begin{pmatrix} \lambda_V & 0 & 0 \\ 0 & \lambda_B & 0 \\ 0 & 0 & \lambda_D \end{pmatrix}, \Delta_n = \begin{pmatrix} \mu_{V,n-1} & 0 & 0 \\ 0 & \mu_B & 0 \\ 0 & 0 & 0 \end{pmatrix}, n > 1$$

The generator matrix Q is block-tridiagonal matrix representing the quasi birth and death process. The vector P is partitioned as $\mathbf{P} = \{\mathbf{P}_0, \mathbf{P}_1, \mathbf{P}_2, ...\}$ where $\mathbf{P}_n = \{\mathbf{P}_{V,n}, \mathbf{P}_{B,n}, \mathbf{P}_{D,n}\}$ where n = 0, 1, 2, ... The equations $\mathbf{P}$Q=$\mathbf{0}$ and $\mathbf{P}$e=1 is satisfied by $\mathbf{P}$ where e is a column vector that has all entries as '1'. After noticing that the stability condition is satisfied, the subvectors of P, corresponding to the different levels have the below mentioned properties:

$$P_{n+k} = P_n R^k, k \geq 1 \tag{3.1}$$

Here rate matrix R provides the minimal solution which is non-negative with the $Sp(R) < 1$ to the matrix quadratic equation

$$R^2 \Delta_N + R\Upsilon_N + \Omega = \mathbf{0} \tag{3.2}$$

Following the work of Neuts (1981) and Latouche and Ramaswami (1999), we know that R=$\lim_{n\to\infty} R_n$ , wherein sequence $\{R_n\}$ is given as

$$R_0 = \mathbf{0} \quad and \quad R_{n+1} = -\Omega\Upsilon_N^{-1} - R_n^2 \Delta_N \Upsilon_N^{-1}, n \geq 0 \tag{3.3}$$

R could be determined from (3.3) by successive substitutions since $\{R_n\}$ is monotonic sequence.

## 4. Stability Condition

According to the theorem 3.1.1 given by Neuts (1981), it is known that the necessary and sufficient condition for the existence of probability vector at steady state is

$$X\Omega e < X\Delta_N e \tag{4.1}$$

Here X represents the invariant probability of the matrix $E = \Delta_N + \Upsilon_N + \Omega$. The equations XE=$\mathbf{0}$ and Xe=1 are satisfied by X wherein e is a $1 \times 3$ order unit vector. It can be shown that the vector $X = [0, \frac{r}{r+b}, \frac{b}{r+b}]$ can be found easily. Some routine manipulation by substituting B and $C_N$ into (4.1) leads to

$$r\lambda_B + b\lambda_D < r\mu_B \tag{4.2}$$

The stability of the system is maintained if and only if the parameters r, b, $\lambda_B$, $\lambda_D$ and $\mu_B$ satisfies the above equation (4.2).

## 5. Steady State Queue Size Distribution

In the condition of stability, the stationary probability vector P of Q exists. Now we can convert the governing equations in matrix equations as follows the following recursive relations:

$$P_0\Upsilon_0 + P_1\Delta_1 = \mathbf{0} \quad 1 \le n \le N-1 \tag{5.1}$$

$$P_{n-1}\Omega + P_n\Upsilon_n + P_{n+1}\Delta_{n+1} = \mathbf{0} \tag{5.2}$$

$$P_{N-1}\Omega + P_N\Upsilon_N + P_{N+1}R\Delta_N = \mathbf{0} \tag{5.3}$$

$$P_N R^{n-N-1}\Omega + P_N R^{n-N}\Upsilon_N + P_N R^{n-N+1}\Delta_N = \mathbf{0} \qquad N+1 \le n \tag{5.4}$$

$$\sum_{n=0}^{\infty} P_n e = 1 \tag{5.5}$$

After manipulation, equations (5.1)-(5.3) yield

$$P_0 = P_1(-\Delta_1)\Upsilon_0^{-1} = P_1\theta_1 \tag{5.6}$$

$$P_{n-1} = P_n\Delta_n[-\phi_{n-1}\Omega + \Upsilon_{n-1}]^{-1} = P_n\theta_n \quad 2 \le n \le N \tag{5.7}$$

and

$$P_N\Theta_N\Omega + P_N\Upsilon_N + P_N R\Delta_N = \mathbf{0} \tag{5.8}$$

Consequently $P_n(0 \le n \le N-1)$ in equations (5.6) and (5.7) may be put in product form in terms of $P_N$ as $P_n = P_N\Pi_{i=N}^{n+1}\theta_i = P_N\Theta_{n+1}$, n=0,1,2,...,N-1 i.e.$\Theta_k = \Pi_{i=N}^{k}\theta_i$. The rest steady state vector $[\mathbf{P}_N, \mathbf{P}_{N+1}, \mathbf{P}_{N+2}, \ldots]$ can be determined recursively as $P_n = P_N R^{n-N}$, for $n \ge N$. After $\mathbf{P}_n$ is calculated, the solutions at steady state $\mathbf{P} = [\mathbf{P}_0, \mathbf{P}_1, \mathbf{P}_2, \ldots, \mathbf{P}_{N-1}, \mathbf{P}_N, \mathbf{P}_{N+1}, \ldots]$ are determined. The vector $P_N$ is obtained by equation (5.8) along with the normalizing equation mentioned below.

$$\begin{aligned}\sum_{n=0}^{\infty} P_n e &= P_N[\sum_{n=1}^{N}\prod_{i=N}^{n}\theta_i + (I-R)^{-1}]e \\ &= P_N[\sum_{n=1}^{N}\Theta_n + (I-R)^{-1}]e \end{aligned} \tag{5.9}$$

Solving (5.8) and (5.9) with the help of Cramer's rule, the vector $P_N$ can be calculated. The probabilities of the prior state $\mathbf{P} = [\mathbf{P}_0, \mathbf{P}_1, \ldots]$ may then be determined with the help of (3.1), (5.6) and (5.7).

## 6. Performance Measures

The validity of matrix geometric approach to the varying environment model can be checked by computing the system performance indices, in terms of the probabilities of system state. Some important performance indices of the system are mentioned below:

- Probability of the server is in functioning vacation mode is obtained as

$$P(V) = \sum_{n=0}^{\infty} P_{V,n} \tag{6.1}$$

- Probability of the server is in busy state is obtained as

$$P(B) = \sum_{n=1}^{\infty} P_{B,n} \tag{6.2}$$

- Probability of the halt in the service during busy state is obtained as

$$P(D) = \sum_{n=1}^{\infty} P_{D,n} \tag{6.3}$$

- Mean number of customers when the server is in functioning vacation mode is obtained as

$$E(V) = \sum_{n=0}^{\infty} nP_{V,n} \tag{6.4}$$

- Mean number of customers when the server is in normal busy mode is obtained as

$$E(B) = \sum_{n=1}^{\infty} nP_{B,n} \tag{6.5}$$

- Mean number of customers when the service halt occurs is obtained as

$$E(D) = \sum_{n=1}^{\infty} nP_{D,n} \tag{6.6}$$

- Mean count of customers in the system is obtained as

$$E(N) = E(V) + E(B) + E(D) \tag{6.7}$$

- Throughput is obtained as

$$TP = \mu_{V,n} \sum_{n=0}^{\infty} P_{V,n} + \mu_B \sum_{n=1}^{\infty} P_{B,n} \tag{6.8}$$

- Mean time of waiting is obtained as

$$E(W) = \frac{E(N)}{\lambda_{eff}} \tag{6.9}$$

where

$$\lambda_{eff} = \lambda P_{V,0} + \sum_{n=1}^{\infty} \lambda(P_{V,n} + P_{B,n} + P_{D,n}) \tag{6.10}$$

- Mean time of delay is calculated as

$$E(D) = \frac{E(N)}{TP} \tag{6.11}$$

## 7. Numerical Experiment and Results

A numerical experiment is executed by coding computer program in MATLAB software to test how sensitive the parameters are on different performance measures. For computational purpose, the system capacity is taken as N=20. Unless the system descriptions in the model are taken as variables or specific values are mentioned in the graphs drawn, the other model parameters are arbitrarily selected as $\lambda_B = 0.6$, $\lambda_V = 0.5$, $\lambda_D = 0.4$, $\xi = 0.3$, $\mu_B = 1$, $\mu_V = 0.9$, $b = 0.5$, $r = 2$, $\psi = 0.3$. The impact of parameters on the mean queue length, E(N) with the variation of $\lambda_B$, $\lambda_V$ and $\lambda_D$ are shown in Figs. 2(a)-2(h), Figs. 3(a)-3(h) and Figs. 4(a)-4(h). It is observed from the graphs that E(N) increases due to the increase in the arrival rate which is quite reasonable.

All the graphs for E(N) in Figs. 2(a)-2(h) reveal the increasing trend with respect to $\lambda_B$ and depict that the E(N) increases for lowering the values of $\xi$, $r$, $\mu_B$, $\mu_V$ and $\lambda_V$ and for the increasing values of $b$, $\psi$ and $\lambda_D$.

Figs. 3(a)-3(h) depict the linearly increasing trend of E(N) with respect to $\lambda_V$. From these graphs, one can easily see an increasing trend with the decrease in parameters $\xi$, $r$ and $\mu_B$ and an increasing trend with an increase in the parameter values of $b$, $\lambda_B$, $\psi$, $\mu_V$ and $\lambda_D$.

In Figs. 4(a)-4(h), with regard to $\lambda_D$, the expected number of customers increases linearly as $\xi$, $r$,$\mu_B$ and $\mu_V$ decreases. Also the expected number of customers decreases linearly as $b$, $\lambda_V$, $\psi$ and $\lambda_B$ decreases as we expect in real life situations.

## 8. Conclusion

In this paper, we have examined the adverse impact of service halt in single server queue with functioning vacation. In the model for dealing with randomly varying environment, we incorporated the noble and realistic assumption of the customer's tendency to renege during vacation period. With the help of matrix geometric method, queue size distribution at steady state are determined which are further employed to obtain various performance measures. The impact of various parameters on the average queue length has shown graphically.

For future work, one can develop a similar model wherein the service may be rendered in phases. The optimal service rate and repair rate can be evaluated to minimize the expected time of waiting for the customers at minimum cost.

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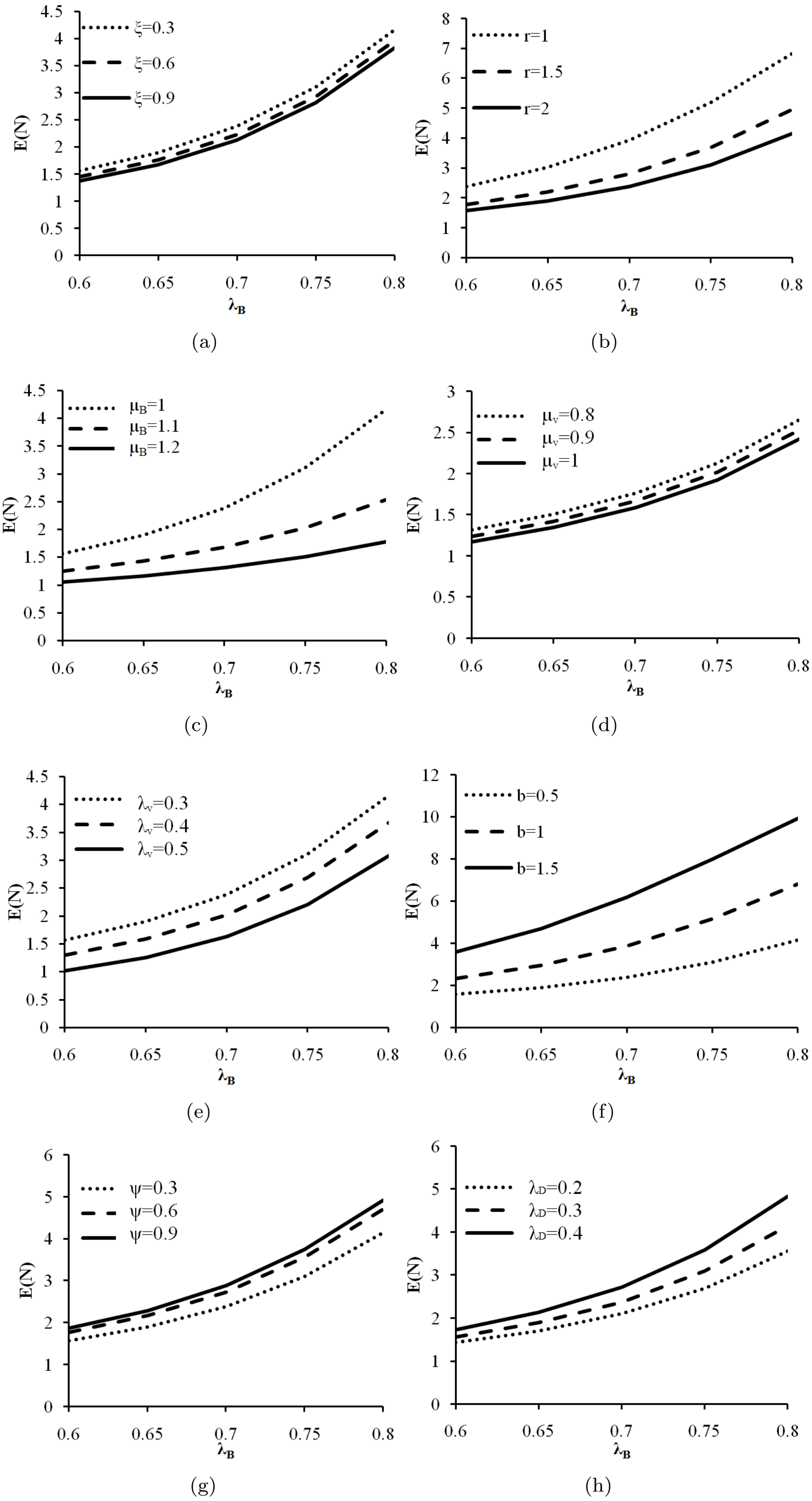


FIGURE 2. Impact of parameters on E(N) with the variation of $\lambda_B$

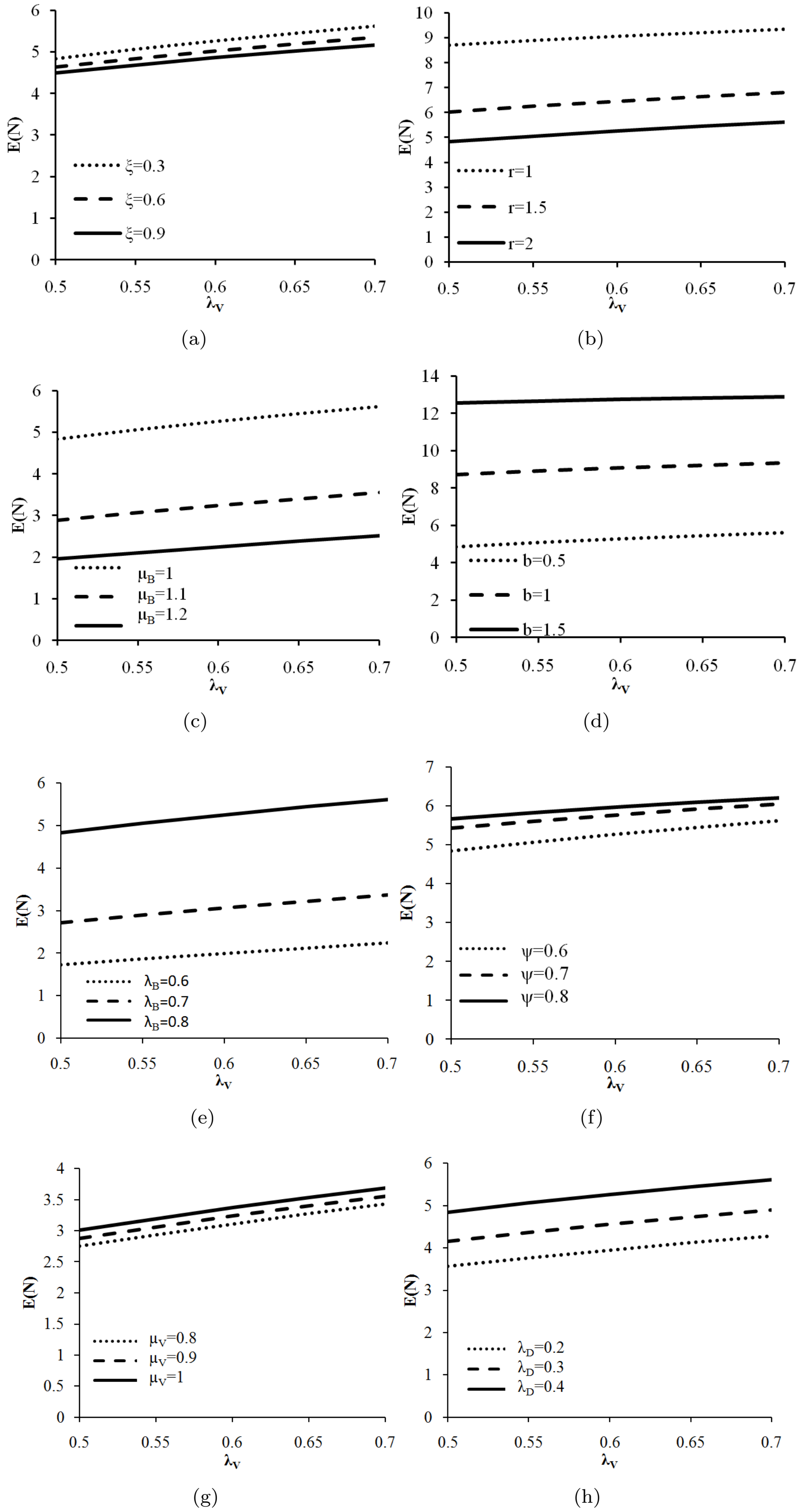


FIGURE 3. Impact of parameters on E(N) with the variation of $\lambda_V$

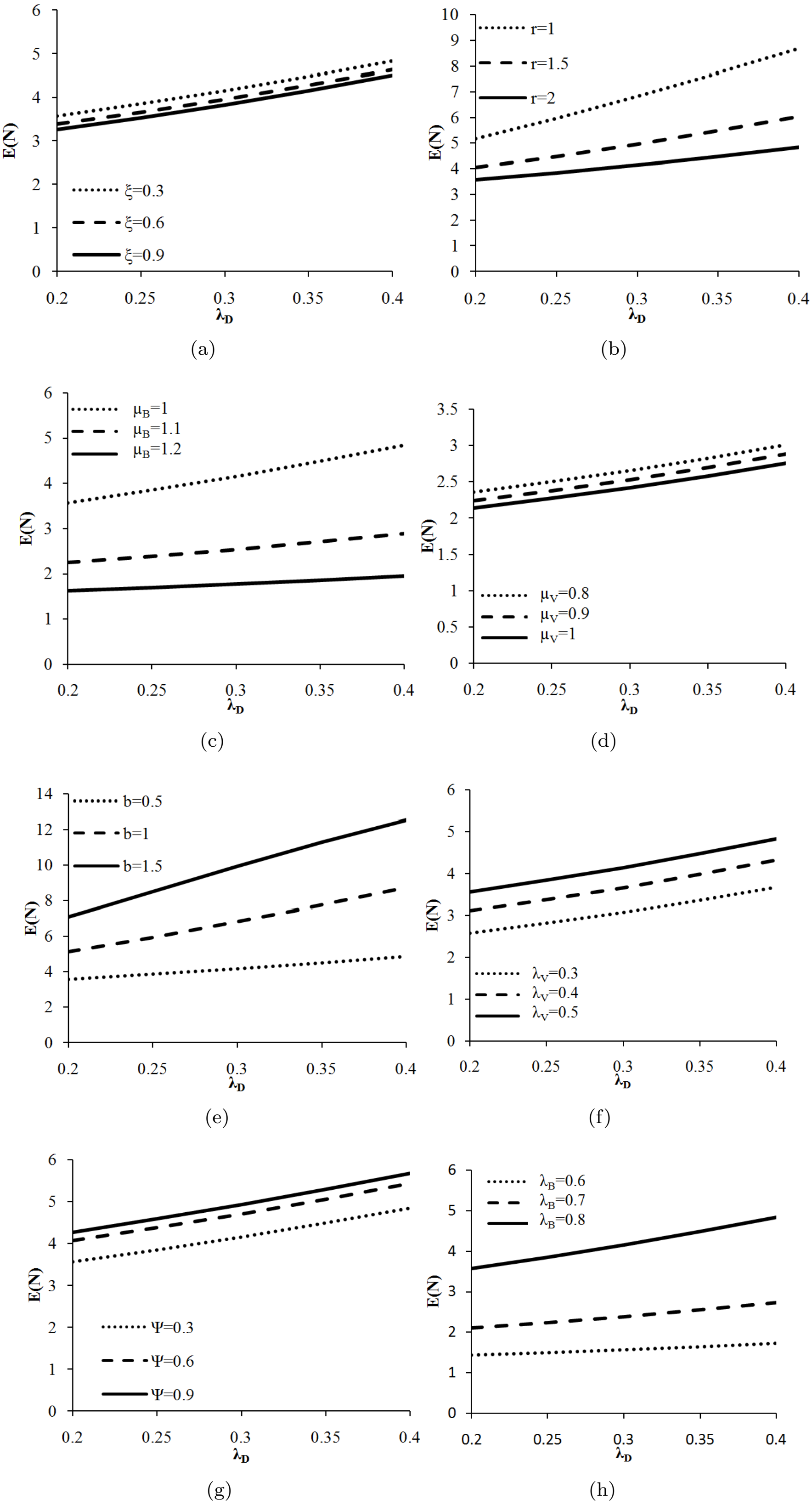


FIGURE 4. Impact of parameters on E(N) with the variation of $\lambda_D$

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