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FIXED POINT THEOREM IN FUZZY F-MENGER SPACES FOR OCCASIONALLY WEAKLY COMPATIBLE MAPPINGS

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Abstract. In this paper we prove fixed point theorems in fuzzy F-menger space with occasionally mappings and rational expression.

1. INTRODUCTION

The notion of probabilistic metric space is introduced by Menger in 1942 [9] and the first result about the existence of a fixed point of a mapping which is defined on a Menger space is obtained by Sehgel and Barucha-Reid[12].
A number of fixed point theorems for single valued and multivalued mappings in menger probabilistic metric space have been considered by many authors [1], [2], [3], [4], [5], [6]. In 1998, Jungck [7] introduced the concept weakly compatible maps and proved many theorems in metric space. Hybrid fixed point theory for nonlinear single valued and multivalued maps is a new development in the domain of contraction type multivalued theory ([3], [6], [8], [10], [11], [13], [15]). Jungck and Rhoades [7] introduced the weak compatibility to the setting of single valued and multivalued maps. Singh and Mishra[14] introduced (IT)-commutativity for hybrid pair of single valued and multivalued maps which need not be weakly compatible.

In this paper, we choose to utilize the notion of occasionally weakly compatibility to prove our results in fuzzy F-Menger space, which is a wider and suitable framework in many situations. Here one may observe that we need not impose the completeness requirement of the space or the containment of the ranges of the involved mappings.

2. PRELIMINARIES

Let us define and recall some definitions:

Definition 2.1. A fuzzy F probabilistic metric space (FPM space) is an ordered pair (X, $F^2{}_\alpha$) consisting of a nonempty set X and a mapping $F^2{}_\alpha$ from XxX into the collections of all distribution functions $F^2{}_\alpha \in$ RxR for all $\alpha \in [0, 1]$.For x, y $\in$ X we denote the distribution function $F^2{}_\alpha$(x,y) by $F^2{}_{\alpha(x,y)}$ and $F^2{}_{\alpha(x,y)}$(u)is the value of $F^2{}_{\alpha(x,y)}$ at u in R.The functions $F^2{}_{\alpha(x,y)}$ for all $\alpha \in [0, 1]$ assumed to satisfy the following conditions:

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(1) $F^2{}_{\alpha(x,y)}(u) = 1\ \forall\ u > 0$ iff x = y,
(2) $F^2{}_{\alpha(x,y)}(0) = 0\ \forall$ x , y in X,
(3) $F^2{}_{\alpha(x,y)} = F^2{}_{\alpha(y,x)} \forall$ x , y in X,
(4) $F^2{}_{\alpha(x,y)}(u) = 1$ and $F^2{}_{\alpha(y,z)}(v) = 1$ then $F^2{}_{\alpha(x,z)}(u+v) = 1\ \forall$ x, y, z in X and u, v > 0.

Definition 2.2. A commutative, associative and non-decreasing mapping t: [0,1] × [0,1]→ [0,1] is a t-norm if and only if t(a,1) = a for all a∈[0,1] , t(0,0)=0 and t(c,d) ≥ t(a,b) for c ≥ a, d ≥ b.

Definition 2.3. A Fuzzy F-Menger space is a triplet (X, $F^2{}_\alpha$, t), where (X, $F^2{}_\alpha$) is a F FPM-space, t is a t-norm and the generalized triangle inequality
$F^2{}_{\alpha(x,z)}(u+v) \geq t(F^2{}_{\alpha(x,y)}(u), F^2{}_{\alpha(y,z)}(v))$ holds for all x, y, z in X u, v > 0 and $\alpha \in [0,1]$
The concept of neighbourhoods in Fuzzy F-Menger space is introduced as

Definition 2.4. Let (X, $F^2{}_\alpha$, t)be a Fuzzy F-Menger space.If x∈X, $\epsilon > 0$ and $\lambda \in (0, 1)$, then (ϵ, λ) - neighbourhood of x, called $U_x(\epsilon,\lambda)$, is defined by
$U_x(\epsilon,\lambda) = \{y \in X: F^2{}_{\alpha(x,y)}(\epsilon) > (1-\lambda)\}$
An (ϵ,λ)-topology in X is the topology induced by the family $\{U_x\ (\epsilon,\lambda)$: x ∈ X, $\epsilon > 0$, $\alpha.\in [0,1]$ and $\lambda \in (0,1)\}$ of neighbourhood.

Remark: If t is continuous, then Fuzzy F-Menger space (X, $F^2{}_\alpha$, t)is a Housdorff space in (ϵ,λ)-topology.
Let (X, $F^2{}_\alpha$, t) be a complete Fuzzy F-Menger space and A ⊂ X. Then A is called a bounded set if

$$\lim_{u\to\infty} \inf_{x,y\in A} F^2_{\alpha(x,y)}(u) = 1$$

Definition 2.5. A sequence $\{x_n\}$ in (X, $F^2{}_\alpha$, t) is said to be convergent to a point x in X if for every $\epsilon>0$ and $\lambda>0$, there exists an integer N = N(ϵ,λ) such that $x_n \in U_x(\epsilon,\lambda)$ for all n ≥ N or equivalently $F^2{}_\alpha\ (x_n, x; \epsilon) > 1-\lambda$ for all n ≥ N and $\alpha \in [0,1]$.

Definition 2.6. A sequence $\{x_n\}$in (X, $F^2{}_\alpha$, t) is said to be Cauchy sequence if for every $\epsilon > 0$ and $\lambda > 0$, there exists an integer N = N(ϵ,λ) such that $F^2{}_\alpha(x_n, x_m; \epsilon) > 1-\lambda\ \forall$n, m ≥ N for all $\alpha \in [0,1]$.

Definition 2.7. A Fuzzy F-Menger space (X, $F^2{}_\alpha$, t) with the continuous t-norm is said to be complete if every Cauchy sequence in X converges to a point in X for all $\alpha \in [0,1]$.

Definition 2.8. Let (X, $F^2{}_\alpha$, t) be a Fuzzy F-Menger space. Two mappings f, g: X→X are said to be weakly compatible if they commute at coincidence point for all $\alpha \in [0,1]$.

Lemma 2.1. Let $\{x_n\}$ be a sequence in a Fuzzy F-Menger space (X, $F^2{}_\alpha$, t), where t is continuous and t(p,p) ≥ p for all p∈[0,1], if there exists a constant k(0,1) such that ∀p > 0 and n∈N
$t(F^2{}_\alpha\ (x_n,x_{n+1}; kp)) \geq t(F^2{}_\alpha(x_{n-1},x_n; p))$

for all α $\in$[0,1] then $\{x_n\}$ is cauchy sequence.

Lemma 2.2. If (X, d) is a metric space, then the metric d induces, a mapping $F^2{}_\alpha$: XxX→L defined by $F^2{}_\alpha$(p, q) = H_α(x- d(p, q)), p, q $\in$ R for all $\alpha\in$[0,1]. Further if t: [0,1] × [0,1]→ [0,1] is defined by t(a,b) = min{a,b}, then (X, $F^2{}_\alpha$, t) is a Fuzzy F-Menger space. It is complete if (X, d) is complete.

Definition 2.9. Let (X, $F^2{}_\alpha$, t) be a Fuzzy F-Menger space. Maps s: X → X and T: X→CB(X)

(1) s is said to be T weakly commuting at x∈X if ssx ∈ Tsx.
(2) are weakly compatible if the commute at their coincidence points, i.e. if sTx = Tsx whenever sx∈Tx.
(3) are (IT) commuting at x∈X if sTx ⊂ Tsx whenever sx∈Tx.

Definition 2.10. Two self maps f and g of a set X are occasionally weakly compatible (o.w.c.) iff there is a point x in X which is a coincidence point of f and g at which f and g commute.

Definition 2.11. A function ϕ: $[0,\infty)$→ [0,∞) is said to be a ϕ-function if it satisfies the following conditions:
(i) ϕ (t) = 0 if and only if t = 0,
(ii) ϕ (t) is strictly increasing and ϕ(t) → ∞ as t→ ∞
(iii) ϕ is left continuous in (0,∞) and
(iv) ϕ is continuous at 0.
An altering distance functions with the additional property that h(t)→∞ as t→∞ generates function ϕ in the following way.

$$\phi(t) = \begin{cases} \sup\{s : h(s) < t \ if \ t > 0 \\ 0 \quad \text{if } t = 0 \end{cases}$$

It can be easily seen that ϕ is a ϕ-function.

Lemma 2.3. Let (X, $F^2{}_\alpha$, t) be a fuzzy F-Menger space, A and B are occasion-ally weakly compatible self maps of X. If A and B have a unique point of coincidence, w = Ax = Bx, then w is the unique common fixed point of A and B.
Proof: Since A and B are occasionally weakly compatible, there exists a point x ∈ X such that Ax = Bx = w and ABx = BAx. Thus, AAx = ABx = BAx which says that Ax is also a point of coincidence of A and B. Since the point of coincidence w = Ax is unique by hypothesis, BAx = AAx = Ax and w = Ax is a common fixed point of A and B. Moreover, if z is any common fixed point of A and B, then z = Az = Bz = w by the uniqueness of the point of coincidence.

3. MAIN RESULTS

Theorem 3.1. *Let(X, $F^2{}_\alpha$, t) be a complete F-Menger space and let p, q, f and g be mappings of X on X.Let pairs* {p, f} *and* {q, g} *be occasionally weakly compatible (owc). If there exists* k∈ $(0,1)$ and $a\geq 1$ *such that*

$$F^2{}_{\alpha px,qy} \quad (kt)$$

$$\geq \min \left\{ \begin{array}{c} F^2{}_{\alpha fx,gy}(t), F^2{}_{\alpha px,fx}(t), F^2{}_{\alpha qy,gy}(t), F^2{}_{\alpha px,gy}(t), F^2{}_{\alpha fx,qy}(t), \\ \frac{F^2{}_{\alpha qy,gy}(t)+F^2{}_{\alpha fx,px}(t)}{2}, \frac{F^2{}_{\alpha fx,gy}(t)+F^2{}_{\alpha px,gy}(t)}{2} \end{array} \right\} \quad ..(1)$$

for all x, y∈ X *and for all* t > 0 *and* $\alpha \geq 1$, *then there exists a unique point* w ∈ X *such that* pw = fw = w.

Proof. Since the pairs {p, f} and {q, g} are occasionally weakly compatible (owc), there exists points x, y∈ X such that px = fx and qy = gy. We claim that px = qy If not, by inequality (1)

$$F^2{}_{\alpha px,qy}(kt)$$

$$\geq \min \left\{ \begin{array}{c} F^2{}_{\alpha fx,gy}(t), F^2{}_{\alpha px,fx}(t), F^2{}_{\alpha qy,gy}(t), F^2{}_{px,gy}(t), F^2{}_{\alpha fx,qy}(t), \\ \frac{F^2{}_{\alpha qy,gy}(t)+F^2{}_{\alpha fx,px}(t)}{2}, \frac{F^2{}_{\alpha fx,gy}(t)+F^2{}_{\alpha px,gy}(t)}{2} \end{array} \right\}$$

$$= \min \left\{ \begin{array}{c} F^2{}_{\alpha px,qy}(t), F^2{}_{\alpha px,px}(t), F^2{}_{\alpha qy,qy}(t), F^2{}_{\alpha px,qy}(t), F^2{}_{\alpha px,qy}(t), \\ \frac{F^2{}_{\alpha qy,qy}(t)+F^2{}_{\alpha px,px}(t)}{2}, \frac{F^2{}_{\alpha px,qy}(t)+F^2{}_{\alpha px,qy}(t)}{2} \end{array} \right\}$$

$$= \min \left\{ \begin{array}{c} F^2{}_{\alpha px,qy}(t), 1, 1, F^2{}_{\alpha px,qy}(t), F^2{}_{\alpha px,qy}(t), \\ \frac{1+1}{2}, \frac{2F^2{}_{\alpha px,qy}(t)}{2} \end{array} \right\}$$

$$= F^2{}_{\alpha px,qy}(t)$$

Thus, we have px = qy. Therefore, px = qy = fx = gy.

Suppose that there is another point z such that pz = fz then by (1) we have pz = fz = qy = gy, so px = pz and w = px = fx is the unique point of coincidence of p and f.

By Lemma **2.3** w is the only common fixed point of p and f. Similarly there is a unique point z ∈ X such that z = qz = gz

Uniqueness: Assume that w ≠ z and taking x = w, y = z in inequality(1), we get

$$F^2{}_{\alpha w,z}(kt) = F^2{}_{\alpha pw,qz}(kt)$$

$$\geq \min \left\{ \begin{array}{c} F^2{}_{\alpha fw,gz}(t), F^2{}_{\alpha pw,fw}(t), F^2{}_{\alpha qz,gz}(t), F^2{}_{\alpha pw,gz}(t), F^2{}_{\alpha fwqz}(t), \\ \frac{F^2{}_{\alpha qz,gz}(t)+F^2{}_{\alpha fw,pw}(t)}{2}, \frac{F^2{}_{\alpha fw,gz}(t)+F^2{}_{\alpha pw,gz}(t)}{2} \end{array} \right\}$$

$$= \min \left\{ \begin{array}{c} F^2{}_{\alpha w,z}(t), F^2{}_{\alpha w,w}(t), F^2{}_{\alpha z,z}(t), F^2{}_{\alpha w,z}(t), F^2{}_{\alpha w,z}(t), \\ \frac{F^2{}_{\alpha z,z}(t)+F^2{}_{\alpha w,w}(t)}{2}, \frac{F^2{}_{\alpha w,z}(t)+F^2{}_{\alpha w,z}(t)}{2} \end{array} \right\}$$

$$= \min \left\{ \begin{array}{c} F^2{}_{\alpha w,z}(t), 1, 1, F^2{}_{\alpha w,z}(t), F^2{}_{\alpha w,z}(t), \\ \frac{1+1}{2}, \frac{2F^2{}_{\alpha w,z}(t)}{2} \end{array} \right\}$$

$$= F^2{}_{\alpha w,z}(t)$$

Therefore we have z = w by lemma **2.3** and z is a common fixed point of p, f, q and g. The uniqueness of the fixed point holds from (1)

□

Theorem 3.2. *Let (X, $F^2{}_\alpha$, t) be a complete F-Menger space and let p,q,f and g be self mappings of X. Let pairs* {p, f} *and* {q, g} *be occasionally weakly compatible (owc).If there exists* k∈ (0, 1) *such that*

$$F^2{}_{\alpha \mathrm{px,qy}}(\mathrm{kt})$$
$$\geq \phi\left(\min\left\{\begin{array}{c} \boldsymbol{F}^2{}_{\alpha \mathrm{fx,gy}}(\mathrm{t}), \boldsymbol{F}^2{}_{\alpha_{\mathrm{px,fx}}}(\mathrm{t}), \boldsymbol{F}^2{}_{\alpha \mathrm{qy,gy}}(\mathrm{t}), \boldsymbol{F}^2{}_{\alpha \mathrm{px,gy}}(\mathrm{t}), \boldsymbol{F}^2{}_{\alpha \mathrm{fx,qy}}(\mathrm{t}), \\ \frac{\boldsymbol{F}^2{}_{\alpha \mathrm{qy,gy}}(\mathrm{t})+\boldsymbol{F}^2{}_{\alpha \mathrm{fx,px}}(\mathrm{t})}{2}, \frac{\boldsymbol{F}^2{}_{\alpha \mathrm{fx,gy}}(\mathrm{t})+\boldsymbol{F}^2{}_{\alpha \mathrm{px,gy}}(\mathrm{t})}{2} \end{array}\right\}\right) \quad ..(2)$$

for all x, y∈ X *and* $\phi \in \phi$ *for all* $0 < t < 1$ *and* α∈*[0,1]* , *then there exists a unique common fixed point of* p, f, q and g.

Proof. The proof follows from theorem 3.1. □

Theorem 3.3. *Let (X, $F^2{}_\alpha$, t) be a complete F-Menger space and let p, q, f and g be self mappings of X. Let pairs* {p, f} and {q, g} *be owc. If there exists* k∈ (0, 1) *such that*

$$F^2{}_{\alpha \mathrm{px,qy}}(\mathrm{kt})$$
$$\geq \phi\left\{\begin{array}{c} \boldsymbol{F}^2{}_{\alpha \mathrm{fx,gy}}(\mathrm{t}), \boldsymbol{F}^2{}_{\alpha \mathrm{px,fx}}(\mathrm{t}), \boldsymbol{F}^2{}_{\alpha \mathrm{qy,gy}}(\mathrm{t}), \boldsymbol{F}^2{}_{\alpha \mathrm{px,gy}}(\mathrm{t}), \boldsymbol{F}^2{}_{\alpha \mathrm{fx,qy}}(\mathrm{t}), \\ \frac{\boldsymbol{F}^2{}_{\alpha \mathrm{qy,gy}}(\mathrm{t})+\boldsymbol{F}^2{}_{\alpha \mathrm{fx,px}}(\mathrm{t})}{2}, \frac{\boldsymbol{F}^2{}_{\alpha \mathrm{fx,gy}}(\mathrm{t})+\boldsymbol{F}^2{}_{\alpha \mathrm{px,gy}}(\mathrm{t})}{2} \end{array}\right\} \ldots(3)$$

for all x, y∈ X *and* ϕ:$[0,1]^7 \to [0,1]$ *such that* ϕ (t, 1, 1, t, t, 1, t) > *t for all* $0 < t < 1$, *then there exists a unique common fixed point of* p, f, q and g.

Proof. Since the pairs { p, f } and {q, g } are occasionally weakly compatible (owc), there exists points x, y∈ X such that px = fx and qy = gy. We claim that px = qy. By inequality (3) we have

$$F^2{}_{\alpha \mathrm{px,qy}}(\mathrm{kt})$$
$$\geq \phi\left\{\begin{array}{c} \boldsymbol{F}^2{}_{\alpha \mathrm{fx,gy}}(\mathrm{t}), \boldsymbol{F}^2{}_{\alpha_{\mathrm{px,fx}}}(\mathrm{t}), \boldsymbol{F}^2{}_{\alpha \mathrm{qy,gy}}(\mathrm{t}), \boldsymbol{F}^2{}_{\alpha \mathrm{px,gy}}(\mathrm{t}), \boldsymbol{F}^2{}_{\alpha \mathrm{fx,qy}}(\mathrm{t}), \\ \frac{\boldsymbol{F}^2{}_{\alpha \mathrm{qy,gy}}(\mathrm{t})+\boldsymbol{F}^2{}_{\alpha \mathrm{fx,px}}(\mathrm{t})}{2}, \frac{\boldsymbol{F}^2{}_{\alpha \mathrm{fx,gy}}(\mathrm{t})+\boldsymbol{F}^2{}_{\alpha \mathrm{px,gy}}(\mathrm{t})}{2} \end{array}\right\}$$
$$= \phi\left\{\begin{array}{c} \boldsymbol{F}^2{}_{\alpha \mathrm{px,qy}}(\mathrm{t}), \boldsymbol{F}^2{}_{\alpha_{\mathrm{px,px}}}(\mathrm{t}), \boldsymbol{F}^2{}_{\alpha \mathrm{qy,qy}}(\mathrm{t}), \boldsymbol{F}^2{}_{\alpha \mathrm{px,qy}}(\mathrm{t}), \boldsymbol{F}^2{}_{\alpha \mathrm{px,qy}}(\mathrm{t}), \\ \frac{\boldsymbol{F}^2{}_{\alpha \mathrm{qy,qy}}(\mathrm{t})+\boldsymbol{F}^2{}_{\alpha \mathrm{px,px}}(\mathrm{t})}{2}, \frac{\boldsymbol{F}^2{}_{\alpha \mathrm{px,qy}}(\mathrm{t})+\boldsymbol{F}^2{}_{\alpha \mathrm{px,qy}}(\mathrm{t})}{2} \end{array}\right\}$$
$$= \phi\left\{\begin{array}{c} \boldsymbol{F}^2{}_{\alpha \mathrm{px,qy}}(\mathrm{t}), 1, 1, \boldsymbol{F}^2{}_{\alpha \mathrm{px,qy}}(\mathrm{t}), \\ \boldsymbol{F}^2{}_{\alpha \mathrm{px,qy}}(\mathrm{t}), 1, \boldsymbol{F}^2{}_{\alpha \mathrm{px,qy}}(\mathrm{t}) \end{array}\right\}$$
$$> \boldsymbol{F}^2{}_{\alpha \mathrm{px,qy}}(\mathrm{t})$$

This contradiction, thus, we have px = qy. Therefore, px = qy = fx = gy.

Suppose that there is another point z such that pz = fz , then by (3) we have pz = fz = qy = gy, so px = pz and w = px = fx is the unique point of coincidence of p and f.

By Lemma 2.3 w is the only common fixed point of p and f. Similarly there is a unique point z ∈ X such that z = qz = gz. Thus z is a common fixed point of p, f, q and g. The uniqueness of the fixed point holds from (3) .

Example: Let X = $[0,1]$ with metric d defined by $d(x,\ y) = |x-y|$ and for each $t\in[0,1]$ define

$$F^2{}_{\alpha x,\ y}\ (t) = \begin{cases} \alpha\ e^{\frac{-|x-y|^2}{t}}, & \text{if } t > 0; \\ 0, & \text{if } t = 0. \end{cases}$$

For all $x, y\in X$. Clearly $(X, F^2{}_\alpha, \triangle)$ is a Fuzzy F-Menger Space. Define p, q, f and g : $X \to X$ by

$$p(x) = \begin{cases} x^2, & \text{if } 0 \le x \le \frac{1}{2}; \\ 1, & \text{if } \frac{1}{2} < x \le 1. \end{cases} \qquad f(x) = \begin{cases} \frac{1}{2}, & \text{if } 0 \le x \le \frac{1}{2}; \\ 0, & \text{if } \frac{1}{2} < x \le 1. \end{cases}$$

$$q(X) = \begin{cases} \frac{1}{2}, & \text{if } 0 \le x \le \frac{1}{2}; \\ 1, & \text{if } \frac{1}{2} < x \le 1. \end{cases} \qquad g(X) = \begin{cases} \frac{1}{2}, & \text{if } 0 \le x \le \frac{1}{2}; \\ \frac{x^2}{4}, & \text{if } \frac{1}{2} < x \le 1. \end{cases}$$

Then p, q, f and g satisfy all the conditions of theorem 3.1 for $k \in (0,1)$ with respect to the distribution function $F^2{}_{\alpha(x,y)}$

Thus $\left(\frac{1}{2}\right)$ is the unique common fixed point of p, q, f and g and also we see that the mappings p, q, f and g are discontinuous at $\left(\frac{1}{2}\right)$. For all $\alpha\geq 1$.

Remark: In all the above theorems p, q, r, s all may be set valued functions then uniqueness cannot be obtained.

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