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CALCULATION OF OPTIMUM BACKORDER QUANTITY USING FUZZY INVENTORY MODEL

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Abstract

In this paper, a mathematical model is designed to calculate the economic backorder quantity. In this model, shortages are allowed and backlogged. To discuss the same case under fuzzy environment, pentagonal fuzzy numbers are applied. Graded Mean Integration Representation method is used for defuzzification. The arrived solutions are verified using numerical examples in both crisp and fuzzy senses.

Keywords

Backorder Quantity, Graded Mean Integration Representation Method, Pentagonal Fuzzy Numbers.

Introduction

There are plenty of mathematical models available for finding the economic order quantity for any business atmosphere. But, very few mathematical models are discussing the case of backorder quantity. When the nature of business allows shortages, it is mandatory to be prepared to face the issues regarding shortages. Any kind of shortage should be noticed quickly and necessary steps are to be taken as early as possible. Orders from the customers are to be backlogged whenever there is a shortage. At the same time, care must be taken while placing backorders. The backorder quantity should also be optimum. Sometimes, the excess of orders may lead to a heavy loss to the company.

The concept of fuzzy was initially developed by Zadeh. Later, solutions for inventory problems were derived using fuzzy concepts. Different solutions for different business problems are suggested with the help of fuzzy numbers. Chang et al. designed an EOQ model in 1998, in which backorder was a considerable factor in both crisp and fuzzy senses. In his model, the inventory quantities and back order quantities are subject to some kind of uncertainties. He used the extension principle to solve the model by fuzzifying backorder quantity by a fuzzy number.Chan and Wang used trapezoidal fuzzy numbers to fuzzify the order cost, inventory cost and backorder cost in the total cost of inventory model. Vujosevic et al. designed an inventory model in which shortages are allowed and backlogged. Further in Yao et al. framed mathematical models for cases with and without backorder.

Here, a mathematical model is formulated to calculate the economic backorder quantity. Shortage quantity, shortage cost and backorder cost are considered for this calculation. While calculating backorder quantity under fuzzy environment for the same business, shortage cost, backorder cost and setup cost are taken as fuzzy numbers. Pentagonal fuzzy numbers are used for the fuzzification of the problem. Graded mean integration representation method is applied to defuzzify the fuzzified equation. The solution is found by deriving the total cost equation with respect to Q and equating the same to zero in both the cases. The verification of reliability is done by checking the mathematical formulation with the help of numerical illustrations.

Definitions and Methodologies

i. Fuzzy Set

A fuzzy set $\tilde{A}$ is the following set of ordered pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$.The mapping $\mu_{\tilde{A}} : X \to [0,1]$ is called the membership function of $x \in X$ in $\tilde{A}$.

ii. Pentagonal Fuzzy Number

A Pentagonal Fuzzy Number $\tilde{A} = (a,b,c,d,e)$ is represented with membership function

$$\mu_{\tilde{A}}(x) = \begin{cases} L_1(x) = \dfrac{x-a}{b-a}, a \le x \le b \\ L_2(x) = \dfrac{x-b}{c-b}, \mathrm{b} \le x \le c \\ R_1(x) = \dfrac{d-x}{d-c}, \mathrm{c} \le x \le d \\ R_2(x) = \dfrac{e-x}{e-d}, \mathrm{d} \le x \le e \\ 0, Otherwise \end{cases}$$

iii. Graded Mean Integration Representation Method

The Graded mean integration representation method is a function that maps the set of all pentagonal numbers to the real line R. The real number that corresponds to the pentagonal fuzzy number using the graded mean representation method is given by

$$\mathrm{R}(\tilde{A}) = \frac{a_1 + 2a_2 + 2a_3 + 2a_4 + a_5}{8}$$

iv. Arithmetic Operations under Function Principle

Suppose $\tilde{A} = (a_1, a_2, a_3, a_4, a_5)$ and $\tilde{B} = (b_1, b_2, b_3, b_4, b_5)$ are two pentagonal fuzzy numbers, then the arithmetic operations are defined as follows:

- $\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5)$
- $\tilde{A} \otimes \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4, a_5 b_5)$
- $\tilde{A} \Theta \tilde{B} = (a_1 - b_5, a_2 - b_4, a_3 - b_3, a_4 - b_2, a_5 - b_1)$
- $\tilde{A} \oslash \tilde{B} = \left(\dfrac{a_1}{b_5}, \dfrac{a_2}{b_4}, \dfrac{a_3}{b_3}, \dfrac{a_4}{b_2}, \dfrac{a_5}{b_1} \right)$
- $\alpha \tilde{A} = \begin{cases} \alpha a_1, \alpha a_2, \alpha a_3, \alpha a_4, \alpha a_5, \alpha \ge 0 \\ \alpha a_5, \alpha a_4, \alpha a_3, \alpha a_2, \alpha a_1, \alpha < 0 \end{cases}$

Notations

D	- Total Demand
A	- Ordering Cost
S	- Setup Cost
B	- Backorder Cost
H	- Shortage Cost
a	- Shortage Quantity
T	- Length of the cycle
Q	- Economic Backorder Quantity

T_C - Total Cost
$\tilde{A}$ - Fuzzy Ordering Cost
$\tilde{S}$ - Fuzzy Setup Cost
$\tilde{B}$ - Fuzzy Backorder Cost
$\tilde{H}$ - Fuzzy Shortage Cost
$\tilde{Q}$ - Fuzzy Economic Backorder Quantity
$\tilde{T}_C$ - Fuzzy Total Cost

Assumptions

- Total demand is constant
- Length of time plan is constant
- Shortages are allowed and backlogged

Mathematical Model in Crisp Sense

The total cost of the given mathematical model is calculated as the sum of the ordering cost, setup cost, shortage cost and backorder cost. The total cost over the period [0, T] is given by

$$T_C = \frac{A}{Q} + \frac{SD}{Q} + \frac{HTQ}{2} + \frac{Ba^2T}{2Q} \quad \text{------------------------- (1)}$$

By differentiating equation (1) with respect to Q, we get

$$\frac{\partial T_C}{\partial Q} = \frac{-A}{Q^2} - \frac{SD}{Q^2} + \frac{HT}{2} - \frac{Ba^2T}{2Q^2} \quad \text{------------------------ (2)}$$

$$\frac{\partial T_C}{\partial Q} = 0 \Rightarrow \frac{-A}{Q^2} - \frac{SD}{Q^2} + \frac{HT}{2} - \frac{Ba^2T}{2Q^2} = 0$$

$$\Rightarrow Q = \sqrt{\frac{2(A+SD)+Ba^2T}{HT}}$$

Hence, the economic backorder quantity is given by

$$Q^* = \sqrt{\frac{2(A+SD)+Ba^2T}{HT}} \quad \text{------------------------ (3)}$$

And the total cost is given by the equation (1).

Mathematical Model in Fuzzy Sense

To consider the same mathematical model under fuzzy sense, the inventory costs like ordering cost, setup cost, shortage cost and backorder cost are taken as pentagonal fuzzy numbers. Let $\tilde{A} = (a_1, a_2, a_3, a_4, a_5)$, $\tilde{S} = (s_1, s_2, s_3, s_4, s_5)$, $\tilde{H} = (h_1, h_2, h_3, h_4, h_5)$ and $\tilde{B} = (b_1, b_2, b_3, b_4, b_5)$ be pentagonal fuzzy numbers. Now, the fuzzy total cost can be arrived from equation (1) as follows:

$$\tilde{T}_C = \frac{\tilde{A}}{Q} \oplus \frac{\tilde{S}D}{Q} \oplus \frac{\tilde{H}TQ}{2} \oplus \frac{\tilde{B}a^2T}{2Q} \quad \text{---------------------- (4)}$$

By substituting the pentagonal fuzzy numbers, the above equation becomes

$$\tilde{T}_C = \begin{bmatrix} \dfrac{(a_1,a_2,a_3,a_4,a_5)}{Q} \oplus \dfrac{(\mathrm{s}_1,\mathrm{s}_2,\mathrm{s}_3,\mathrm{s}_4,\mathrm{s}_5)D}{Q} \oplus \\ \dfrac{(\mathrm{h}_1,\mathrm{h}_2,\mathrm{h}_3,\mathrm{h}_4,\mathrm{h}_5)TQ}{2} \oplus \dfrac{(\mathrm{b}_1,\mathrm{b}_2,\mathrm{b}_3,\mathrm{b}_4,\mathrm{b}_5)a^2T}{2Q} \end{bmatrix}$$

By applying the arithmetic operations on pentagonal fuzzy numbers, we get

$$\tilde{T}_C = \begin{bmatrix} \dfrac{2a_1+2s_1D+h_1TQ^2+b_1a^2T}{2Q}, \dfrac{2a_2+2s_2D+h_2TQ^2+b_2a^2T}{2Q}, \dfrac{2a_3+2s_3D+h_3TQ^2+b_3a^2T}{2Q}, \\ \dfrac{2a_4+2s_4D+h_4TQ^2+b_4a^2T}{2Q}, \dfrac{2a_5+2s_5D+h_5TQ^2+b_5a^2T}{2Q}, \end{bmatrix}$$

By applying graded mean integration representation method is used for defuzzification, we get

$$\tilde{T}_C = \frac{1}{16Q}\begin{bmatrix} 2\left(a_1+2a_2+2a_3+2a_4+a_5\right)+2\left(s_1+2s_2+2s_3+2s_4+s_5\right)D+ \\ \left(h_1+2h_2+2h_3+2h_4+h_5\right)TQ^2+\left(b_1+2b_2+2b_3+2b_4+b_5\right)a^2T \end{bmatrix}$$

------- (5)

The above equation gives the fuzzy total cost. To get the fuzzy optimum backorder quantity, we have to differentiate the above equation with respect to Q and equate it to zero.

$$\frac{\partial \tilde{T}_C}{\partial Q} = \begin{bmatrix} \dfrac{-\left(a_1+2a_2+2a_3+2a_4+a_5\right)}{Q^2} - \dfrac{\left(s_1+2s_2+2s_3+2s_4+s_5\right)D}{Q^2} + \\ \dfrac{\left(h_1+2h_2+2h_3+2h_4+h_5\right)T}{2} - \dfrac{\left(b_1+2b_2+2b_3+2b_4+b_5\right)a^2T}{2Q^2} \end{bmatrix}$$

$$\frac{\partial \tilde{T}_C}{\partial Q} = 0 \Rightarrow$$

$$\begin{bmatrix} \dfrac{-\left(a_1+2a_2+2a_3+2a_4+a_5\right)}{Q^2} - \dfrac{\left(s_1+2s_2+2s_3+2s_4+s_5\right)D}{Q^2} + \\ \dfrac{\left(h_1+2h_2+2h_3+2h_4+h_5\right)T}{2} - \dfrac{\left(b_1+2b_2+2b_3+2b_4+b_5\right)a^2T}{2Q^2} \end{bmatrix} = 0$$

After simplifications, we have

$$\tilde{Q}^* = \sqrt{\frac{2\begin{bmatrix}\left(a_1+2a_2+2a_3+2a_4+a_5\right)+ \\ \left(s_1+2s_2+2s_3+2s_4+s_5\right)D\end{bmatrix}+\left(b_1+2b_2+2b_3+2b_4+b_5\right)a^2T}{\left(h_1+2h_2+2h_3+2h_4+h_5\right)T}}$$

-------- (6)

Hence, the equation (6) gives the fuzzy economic backorder quantity and the equation (5) gives the fuzzy total cost.

Numerical Example

- **In Crisp Sense**

Let D = 1000 units / year

A = Rs. 1000
S = Rs. 5 / unit
H = Rs. 300
T = 1 year
B = Rs. 2 / unit
a = 15 units

Then, the optimum backorder quantity is calculated as

$Q^* = 6.44$ units

And the total cost is calculated as

$T_C^* =$ Rs. 1932.61/-

- **In Fuzzy Sense**

Let D = 1000 units / year
T = 1 year
a = 15 units

$$\tilde{A} = (800, 900, 1000, 1100, 1200)$$

$$\tilde{S} = (3, 4, 5, 6, 7)$$

$$\tilde{B} = (1, 1.5, 2, 2.5, 3)$$

$$\tilde{H} = (100, 200, 300, 400, 500)$$

Then, the fuzzy optimum backorder quantity is calculated as

$$\tilde{Q}^* = 6.44 units$$

And the fuzzy total cost is calculated as

$$\tilde{T}_C^* = Rs. 1925.14 / -$$

Conclusion

In this paper, a mathematical model is suggested for a business environment in which shortage and backorder of items are allowed. This model focuses on calculating the economic backorder quantity. Shortage cost and shortage quantity are also taken into consideration for this calculation. For more reliability, the same model is discussed again under fuzzy environment. For fuzzy sense, the inventory quantities are taken as pentagonal fuzzy numbers. To make it as a single value again, graded mean integration representation method is used for defuzzification. The final formulation has been verified using numerical illustrations in both the senses. This paper can be discussed in different angles and can be developed further by future researchers.

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