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APPLICATION OF FUZZY LOGIC APPROACH IN STATISTICAL CONTROL CHARTS

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Abstract. Control charts are widely used in industry as a tool to monitor process characteristics. Deviations from process targets can be detected based on the evidence of statistical significance. The control chart helps to take decisions such as the need for machine or technology replacement to monitor the process with categorical observations. Fuzzy logic is used to cram the uncertainty and vagueness of the data. A new method based on fuzzy logic is proposed for monitoring attribute quality characteristics and also measures the process in the short-run. The short-run α-cut p-control chart technique is developed for fuzzy data in the short-run situation.
Keyword: Fuzzy Logic, Statistical Control Charts, Short-run α -cut p-control chart technique, Short-run data and Fuzzy control charting technique

♣ Note to author: Use 2017 Fuzzy Logic Approach in Statistical Control Charts.

1. Introduction

Improving the productivity and quality of products is a major objective of statistical process control (SPC). SPC is a well-known methodology for improving the quality of the product. It can be used whenever a process produces a product in which some attribute of that product is plagued with undesired variation. Control chart is employed as the most essential tool of SPC that is frequently employed to determine whether the process in a state of statistical control. According to Montgomery, the control chart refers to a graphical display of a quality characteristic that has been measured or computed from a sample versus the sample number or time. The control charts are categorized in to two types, variable control charts are used to monitor continuous characteristics of the product, whereas attribute control charts are applied to monitor the quality characteristics of the product. The control charts are versatile; there must be some disadvantages in it. However, attribute control charts measures the quality characteristics, it results whether the items belongs to confirming or non-conforming item. These charts are not suitable in many of the situations which are considering some of the intermediate levels of the process. In quality-related characteristics there must be some intermediate levels behind in between confirming and non-confirming. These levels are not considered by any of the statistical control charts and also the results may mislead the producer. For example, the quality of the product may classified as good,

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perfect, medium, poor and fair, depending on the products specifications. These types of data are called Categorical data. Categorical observations are handled by multinomial distributions and grouped data approaches, but, it has some of the disadvantages such as, cannot specify if the change in the quality is a result of quality improvement or not, control limits do not depend on sample size. However, the majority of the information is based on only linguistic variables. The vagueness of the data are measurable and estimates by the use of fuzzy set theory.

To overcome the limitations of the binary classification used in control charts for attributes, e.g., the p-chart and the c-chart, Wang and Raz [1990] proposed two approaches for the construction of control charts, namely a probabilistic approach and a membership approach. He (1990) adopted linguistic terms such as perfect, good, medium, poor, and bad to express the intermediate levels of a quality characteristic rather than the traditional choice of conforming or nonconforming. In developing control charts for process outcomes based on linguistic terms, the membership functions of the various linguistic terms are defined on a numerical scale of 01. The data from the producing car spare parts company was used to study about the fuzzy short run α-cut p control chart. We have chosen the car tire production process. For this purpose the data from 30 subgroup size was used.

2. Shewhart Control Chart

One of the primary tools used in the statistical control of a process is the control chart created by Walter Shewhart in 1924. The Shewhart control chart gives a crisp picture of the state of a process by plotting the data produced by a process on a chart bound by upper and lower specification limits [2001]. The main function of a control chart is to monitor a process in order to identify whether or not the process is in control. Incontrol conditions mean that a process is producing parts that are close to the target value with little variation. Out-of-control conditions mean that some type of assignable cause has occurred, and the process is, therefore, yielding products at either an unacceptable distance from the target value, with an unacceptable amount of variation, or both. The control chart consists of three lines: an upper control limit (UCL), a lower control limit (LCL), and a center line (CL) (Figure 1). The upper and lower control limits are the maximum and minimum values for a process characteristic to be considered in-control while the center line is the mean value for the process. For Shewhart charts, 3-sigma (3σ) control limits are used. Three sigma control limits establish bounds on the data that extend above and below the mean of the process by three times the standard deviation of the process statistic being plotted. Data to be plotted on control charts are obtained directly from the process. Data points falling outside the set limits indicate a possible out-of-control condition in the process [2001].
The information plotted on control charts consists of either variable or attribute data. Variable data represent measurable characteristics. Examples of variable data are dimensions such as diameters, volumes, or lengths. Attribute data are data that refer to either a pass or a fail situation. In other words, if a product passes inspection, it is considered a pass, and thus, it conforms to the standards

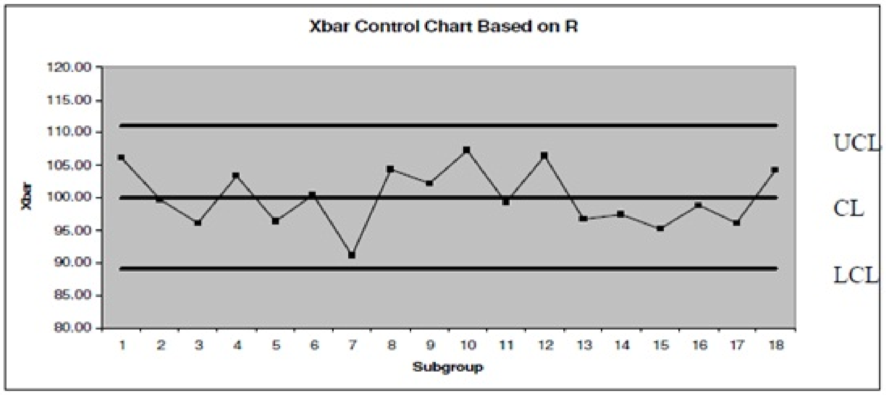

FIGURE 1. x bar control chart

outlined for the product. If a product fails, it is considered nonconforming to the standards outlined for the product.

2.1. The P-Chart. In Statistical Quality Control the p-chart is used to monitor the fraction of nonconforming units for a process. It models the ratio of nonconforming items in relation to the entire population of process data [1990]. Nonconformities, also known as defects, are attributes that are either absent from the product such as a missing hole or switch, or appear on the product when they should not, such as a scratch, indention, or tear [1997]. The CL of the p-chart is the average of the individual sample nonconforming ratios. The CL is given in Equation 1.

$$CL = \bar{p} = \frac{\sum_{i=1}^{m} p_i}{m} \tag{2.1}$$

Where,

$$p_i = \frac{D_i}{n} \tag{2.2}$$

The LCL and UCL of the p chart are then defined as:

$$LCL = \bar{P} - 3\sqrt{\frac{\bar{P}(1-\bar{P})}{N}} \tag{2.3}$$

$$UCL = \bar{P} + 3\sqrt{\frac{\bar{P}(1-\bar{P})}{N}} \tag{2.4}$$

3. Fuzzification of the P - Chart

For a p-chart, the sample mean (Mj) and center line (CL) are defined as:

$$M_j = \frac{\sum_i k_{ij} r_i}{n_j} \qquad j = 1, \ldots, m \tag{3.1}$$

$$CL = \overline{M}_i = \frac{\sum_{j=1} M_j}{m} \tag{3.2}$$

The sample mean, M_j, is essentially a weighted average of the data according to the categories in which the data values are placed. For each of the specified categories, a membership value exists. This membership value serves as the weight for the category in the M_j calculation. In Equation, k_{ij} is the number of data values within the category i for the sample j, ri is the membership value for the particular category i, and n_j is the sample size of sample j. The equation for the center is an average of the sample means. This average results in a fuzzy set where this fuzzy set can be represented by triangular fuzzy numbers (TFNs). Using this representation, the $L_j(2)$ and $R_j(2)$, the left and right plot values, can be defined as in equations.

$$L_j(\alpha) M_j \alpha \tag{3.3}$$

$$R_j(\alpha) 1 - \left[(1 - M_j) \alpha \right] \tag{3.4}$$

The resulting membership function of the mean $\bar{M}$, or CL is given in

$$\mu_{M_j}(x) = \begin{cases} 0, \; if \; x \le 0 \\ \dfrac{x}{\overline{M}}, if \; 0 \le x \le \overline{M} \\ \dfrac{1-x}{1-M}, if \; \overline{M} \le x \le 1 \\ 0, \quad if \; x \ge 1 \end{cases} \tag{3.5}$$

The membership function of the CL is divided into two parts to represent the left and right portions of the TFN. Therefore, a CL, UCL, and LCL exist for each half of the CL. The resulting control limits are given in Equation (7).

$$\text{Control Limits}(\alpha) = \left| \begin{array}{l} CL^L = \overline{M}\alpha \\ LCL^L = \max\left\{ CL^L - 3\sqrt{\dfrac{(CL^L)(1-CL^L)}{n}}, 0 \right\} \\ UCL^L = \min\left\{ CL^L + \sqrt{\dfrac{(CL^L)(1-CL^L)}{n}}, 1 \right\} \\ CL^R = 1 - \left[(1-\overline{M})\alpha \right] \\ LCL^R \max\left\{ CL^R - 3\sqrt{\dfrac{(CL^R)(1-CL^R)}{n}} \right\} \\ UCL^R = \min\left\{ CL^R + \sqrt{\dfrac{(CL^R)(1-CL^R)}{n}}, 1 \right\} \end{array} \right| \quad (7) \tag{3.6}$$

The α cut value can be adjusted according to the inspection tightness desired by the quality controller or management personnel. As the value of α approaches 1, the inspection of the data becomes tighter. Similarly, as the value of α approaches 0, the tightness of the inspection limits loosens. In the former case, as the inspection limits tighten, the probability of a point falling outside the control limits increases; however, in the latter, the inspection limits are loosening and the probability of a point falling outside the control limits decreases. With the fuzzy control chart, the basis for deciding whether or not a process is in control is much the same as with traditional charts. The process is considered in-control if both the left and right plot values plot inside the control limits. If either plot value is found outside its respective control limits, the process is considered out-of-control.

4. Fuzzy Short-Run α -cut p Control Chart

The parameters of the Fuzzy Short-Run α-cut p Control Chart were derived from the principles of the short-run control charts for the fraction nonconforming as well as the reported application of the -cut method for fraction nonconforming charts derived by Gulbay et al. [2004]. The control limits are given in equations 10 and 11. First-Stage

$$\left[\begin{array}{l} \left[\begin{array}{l} CL^{L} = \overline{M}\,\alpha \\ LCL = \max\left\{CL^{L} + k_1\sqrt{\dfrac{(CL^{L})(1-CL^{L})}{n}},0\right\} \\ UCL^{L} = \min\left\{CL^{L} + k_1\sqrt{\dfrac{(CL^{L})(1-CL^{L})}{n}},1\right\} \end{array}\right] \\ \left[\begin{array}{l} CL^{R} = 1-\left[(1-\overline{M})\alpha\right] \\ LCL^{R} = \max\left\{CL^{R} - k_1\sqrt{\dfrac{(CL^{R})(1-CL^{R})}{n}},0\right\} \\ UCL^{R} = \min\left\{CL^{R} + k_1\sqrt{\dfrac{(CL^{R})(1-CL^{R})}{n}},1\right\} \end{array}\right] \end{array}\right] \quad ---\ (10) \tag{4.1}$$

Second-Stage

$$Control\ Limit(\alpha) = \left[\begin{array}{l} \left[\begin{array}{l} CL^{L} = \overline{M}\,\alpha \\ LCL^{L} = \max\left\{CL^{L} + k_1\sqrt{\dfrac{(CL^{L})(1-CL^{L})}{n}},0\right\} \\ UCL^{L} = \min\left\{CL^{L} + k_1\sqrt{\dfrac{(CL^{L})(1-CL^{L})}{n}},1\right\} \end{array}\right] \\ \left[\begin{array}{l} CL^{R} = 1-\left[(1-\overline{M})\alpha\right] \\ LCL^{R}\ \max\left\{CL^{R} - k_1\sqrt{\dfrac{(CL^{R})(1-CL^{R})}{n}},0\right\} \\ UCL^{R} = \min\left\{CL^{R} + k_1\sqrt{\dfrac{(CL^{R})(1-CL^{R})}{n}},1\right\} \end{array}\right] \end{array}\right] \quad ---(11) \tag{4.2}$$

5. Result and Discussions

Using the data from car company components fuzzy short run α-cut p control chart was applied. The quality of the tire production process is tested by using fuzzy short run α-cut p control chart. As with the example of short-run control charts, 10 subgroups were examined initially with 10 additional subgroups added to each succeeding examination of the data. The examination of the first 10 subgroups was conducted, and the chart parameters determined. Since the subgroup size differed over the subgroups, the chart parameters were not constant but varied with each subgroup. The value for inspection tightness was assumed, 0.30.

For the tire company components data for 10 subgroup size Left-Hand Limits:

$$CL^L = 0.039$$
$$LCL^L = 0.083$$
$$UCL^L = 0$$

Right-Hand Limits:

$$CL^R = 0.739$$
$$LCL^R = 0.8391$$
$$UCL^R = 0.639$$

For the tire company components data for 20 subgroup size Left-Hand Limits:

$$CL^L = 0.5$$
$$LCL^L = 0.05207$$
$$UCL^L = 0.0499$$

Right-Hand Limits:

$$CL^R = 0.751$$
$$LCL^R = 0.8379$$
$$UCL^R = 0.6621$$

For the tire company components data for 30 subgroup size Left-Hand Limits:

$$CL^L = 0.048$$
$$LCL^L = 0.0928$$
$$UCL^L = 0.0032$$

Right-Hand Limits:

$$CL^R = 0.748$$
$$LCL^R = 0.8389$$
$$UCL^R = 0.657$$

The resulting chart for the first 10 subgroups is given in Figures 2 and 3. From the Figure 2 we noticed that the point 8 indicate an out-of-control condition, and therefore, the next 10 subgroups are combined with the first 10 for a second evaluation. Therefore, the chart parameters for the second evaluation were based on an average of all 20 subgroups. The results are given in Figures 4 and 5. Again, From the Figure 4 and 5 we noticed that the point 8, 19 and 2 indicate an out-of-control condition, and therefore, the next 10 subgroups are combined with the first 20 for a third evaluation. The results for the examination of all 30 subgroups are given in Figures 6 and 7. From the Figure 7 we noticed that the point 2 and 29 indicate an out-of-control condition. So the parameters are considered here not to be an accurate measure of the condition of the process. So that α value should be minimized and again we have to calculate the UCL, LCL, CL value.

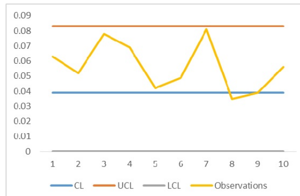

FIGURE 2. Left-Hand Portion (Subgroups 1-10)

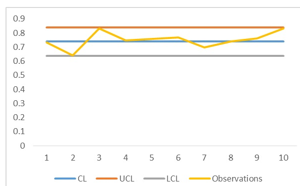

FIGURE 3. Right-Hand Portion (Subgroups 1-10)

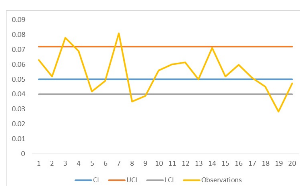

FIGURE 4. Left-Hand Portion (Subgroups 1-20)

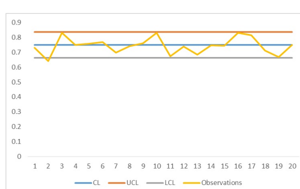

FIGURE 5. Right-Hand Portion (Subgroups 1-20)

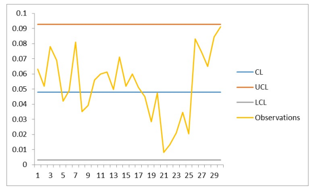

FIGURE 6. Left-Hand Portion (Subgroups 1-30)

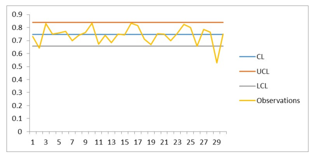

FIGURE 7. Right-Hand Portion (Subgroups 1-30)

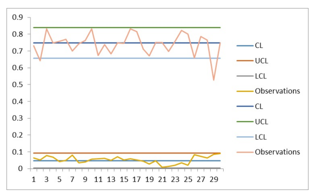

FIGURE 8. Short-Run α-cut p Control Chart (Subgroups 1-30)

6. Conclusion

In this paper we introduced a fuzzy chart for controlling the process for attribute data. The resulting Fuzzy Short-Run α-cut p Control Chart provides a method of monitoring processes that are new, have been recently brought back into control, or contain too little data to monitor by standard methods. With the properties of the α-cut method included, the chart also has the advantage of allowing practitioners to specify differing levels of inspection ranging from 0 to 1, with 0 used for less precise processes, and values closer to 1 used for more intricate processes.

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