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ANALYSIS OF FOUR BAR MECHANISM TO VALIDATE KINEMATIC ANALYSIS USING DH NOTATION APPROACH

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Abstract. Protection and stability control are crucial elements of any four-wheel vehicle, particularly those requiring rollover movements. Various alignment equipment like caster/camber gauge, a set of the turntable, and bubble gauges are used for measurement of front suspension variables such as caster, toe, kingpin inclination and camber. While testing the steering reliability, the steering wheel should be centered. Steering efficiency depends on the orientation of the axis of the kingpin. The direction of the kingpin axis is calculated one at each revolute joint and two at each spherical joint of this four-bar chain. Caster/camber/kingpin / and toe angle are determined, based on the direction of the kingpin axis. Using the Denavit Hartenberg theory, a place of a kingpin axis is decided. The front suspension mechanism contains the coupler link, lower and upper control arm, and the RSSR (Revolute Spherical Spherical Revolute) structure. Therefore, this study concentrates on the comparison of two approaches for kinematic modeling of the front suspension of an automobile. The first method relies on using transformation matrices of the D H Notation at different joints for transnational and rotational relationships. The second method to binary mechanisms for spherical pairs was introduced by Psung Dain Lin and Jung Fa Hsich. The result shows the all the links form a closed loop which confirms its validity.

KEYWORDS: Suspension, Kinematic, Comparison, Joints, Steering geometry.

1. Introduction

Steering geometry plays an important role in controlling the steering behavior of the front suspension. The steering performance parameters such as caster angle, camber angle, kingpin angle are worked out on the position of the kingpin axis [1-2]. The front suspension mechanism consists of a fixed link, upper arm, lower arm and couple link with RSSR mechanism. The front suspension mechanism form the four-bar mechanism with four joints O1, O2, A and B. Joints O1 and O2 is related to a fixed link to which the upper arm and lower arm are fixed with Revolute - Revolute Joints. Joints O1 and A are related to the upper link and Joints O2 and B are related to lower joints [3-5]. Joints A and B are spherical Joints of coupler link with kingpin on which tire is mounted. The front suspension mechanism forms the close loop through which predicts the desired output from the proposed mechanism. Front suspension with joints is shown in figure 1.

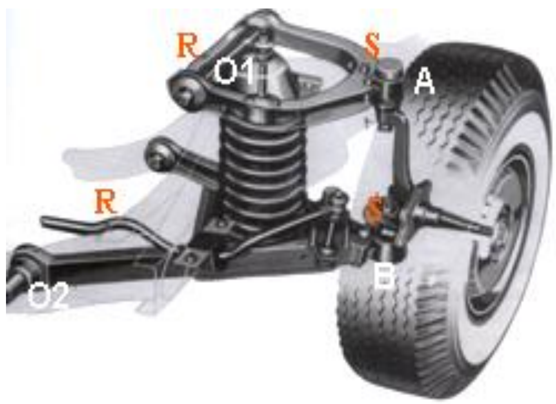

Fig. 1. Front Suspension

The validity of the close-loop is confirmed using the unit matrix formed by multiplying each transformation matrix of the four links of the four-bar mechanism [6]. Kinematic analysis of the front suspension mechanism is done using the D H notation to determine the kingpin axis position which depends on joint A and B. The multiplication of all individual transformation matrices is equal to the identity matrix using the D H notation techniques. To validate the kinematic analysis method suggested by Psang Dain Lin and Jung Hsich is used to confirm the closed-loop [7-9]. The validity is confirmed only if all the links form a closed loop. The concatenation of all the transformation matrices is an identity matrix, forming a closed- loop.

That is $\sum_{i=1}^{n+1} {}^{i-1}\mathbf{A}_i = 1$ Where I is the identity matrix (1-1)

2. Comparison of Kinematic Analysis of Front Suspension

Comparison of two approaches D H Notation and Psung Dain Lin is used for kinematic modeling of the front suspension of an automobile. The First approach is based on D H Notation transformation matrices used for a translational and rotational relationship at various joints. The second approach was suggested by Psung Dain Lin and Jung Fa Hsich for binary mechanisms for spherical pairs [10]. The validity is confirmed only if all the links form a closed loop. The concatenation of all the transformation matrices is an identity matrix, forming a closed loop [11]. The kinematic relationships between the defined angles are found through a series of loop equations. The SRRS mechanism has the kinematic relationship between the joint angles O1, O2, A and B using four parameters S, α, θ and d. This relationship is derived using a loop equation, concerning the position vector O1→ O2→ A→B→ O1 must be equal to the identity matrix [12].

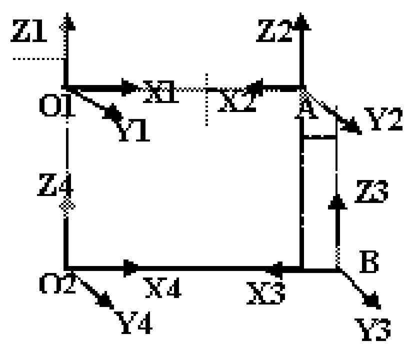

Fig. 2. Denavit Hertenberg coordinate system

Denavit Hertenberg coordinate system for each link is shown in Figure 2. The relationship of each link between two joints is shown below

$$AZ1\begin{bmatrix} a = l1 \\ s = 0 \\ \theta = 180^\circ \\ \alpha = 0^\circ \end{bmatrix} O1Z1 \qquad\qquad BZ1\begin{bmatrix} a = X \\ s = -l2 \\ \theta = 0^\circ \\ \alpha = 0^\circ \end{bmatrix} AZ1$$

Relationship between joints O1 and A   Relationship between joints A and B

$$02Z1\begin{bmatrix} a = -l3 \\ s = 0 \\ \theta = 180^\circ \\ \alpha = 0^\circ \end{bmatrix} BZ1 \qquad 01Z1\begin{bmatrix} a = X \\ s = l2 \\ \theta = 0^\circ \\ \alpha = 0^\circ \end{bmatrix} 02Z1$$

Relationship between joints B and O2 Relationship L4 between joints O1 and O2

The transformation matrix for adjacent coordinate frames I and i-1

$${}^{i-1}A_i = \begin{bmatrix} \cos\theta i & -\cos\alpha i \sin\theta i & \sin\alpha i \sin\theta i & ai\cos\theta i \\ \sin\theta i & \cos\alpha i\cos\theta i & -\sin\alpha i\cos\theta i & ai\sin\theta i \\ 0 & \sin\alpha i & \cos\alpha i & di \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The transformation matrix for link O1A Transformation matrix for link AB

$$\mathbf{{}^{0}A_1} = \begin{bmatrix} 1 & 0 & 0 & l1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^{1}A_2 = \begin{bmatrix} 1 & 0 & 0 & X \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -l2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The transformation matrix for link O2B Transformation matrix for link O1O2

$${}^{2}A_3 = \begin{bmatrix} 1 & 0 & 0 & -l3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^{3}A_0 = \begin{bmatrix} 1 & 0 & 0 & X \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{0}T_1 = \begin{bmatrix} 1 & 0 & 0 & l1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{0}T_2 = {}^{0}T_1 \times {}^{1}T_2 \qquad (2\text{-}1)$$

$$^{0}T_{2} = \begin{bmatrix} 1 & 0 & 0 & l1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} x \begin{bmatrix} 1 & 0 & 0 & X \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -l2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & X+l1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -l2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{0}T_{3} = {}^{0}T_{1} \times {}^{1}T_{2} \times {}^{2}T_{3} \qquad (2\text{-}2)$$

$$^{0}T_{3} = \begin{bmatrix} 1 & 0 & 0 & l1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} x \begin{bmatrix} 1 & 0 & 0 & X \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -l2 \\ 0 & 0 & 0 & 1 \end{bmatrix} x \begin{bmatrix} 1 & 0 & 0 & -l3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & X+l1-l3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -l2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{0}T_{4} = {}^{0}T_{1} \times {}^{1}T_{2} \times {}^{2}T_{3} \times {}^{3}T_{4} \qquad (2\text{-}3)$$

$$^{0}T_{4} = \begin{bmatrix} 1 & 0 & 0 & l1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} x \begin{bmatrix} 1 & 0 & 0 & X \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -l2 \\ 0 & 0 & 0 & 1 \end{bmatrix} x \begin{bmatrix} 1 & 0 & 0 & -l3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} x \begin{bmatrix} 1 & 0 & 0 & X \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{0}T_{4} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{0}T_{4} = I \qquad (2\text{-}4)$$

The concatenation of all the transformation matrices is equal to the identity matrix. Thus validity is confirmed by forming a closed loop.

3. Approach suggested by Psang Dain Lin and Jung-Fa Hsich

The approach recommended by Psang Dain Lin and Jung Fa Hsich in the RSSR mechanism is presented as follows [15].

Analysis of Link O1A (Revolute –Spherical Paired)

The Link O1A is characterized by link length a_i. The representation of link O1A using four parameters S, α, θ, and d is as follows

$$AZ1\begin{bmatrix} a = l1 \\ s = 0 \\ \theta = 180^\circ \\ \alpha = 0^\circ \end{bmatrix} O1Z1$$

The transformation matrix for link O1A

$$^{i-1}A_i = \begin{bmatrix} \cos\theta i & -\cos\alpha i\sin\theta i & \sin\alpha i\sin\theta i & ai\cos\theta i \\ \sin\theta i & \cos\alpha i\cos\theta i & -\sin\alpha i\cos\theta i & ai\sin\theta i \\ 0 & \sin\alpha i & \cos\alpha i & di \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Resultant matrix for Link O1A

$$^{O1}T_A = \begin{bmatrix} 1 & 0 & 0 & l1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Analysis of Link AB (Spherical –Spherical Paired)

The transformation matrix for link AB

$$^{A}T_B = \begin{bmatrix} C\phi iC\psi i & -S\phi i & S\psi iC\phi i & aiC\phi iC\psi i \\ S\phi iC\psi i & C\phi i & S\phi iS\psi i & aiS\phi iC\psi i \\ -S\psi i & 0 & C\psi i & -aiS\psi i + di \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Relational parameters between Joints A and B

a_i = X (Link Length)

d_i = L2 (Offset)

$\theta = 180^0$, $\alpha = 0^0$, $\phi = 0^0$

$\psi = 0^0$

Resultant matrix for Link A and B

$$^{A(SP)}T_{B(SP)} = \begin{bmatrix} 1 & 0 & 0 & X \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Analysis of Link B O2 (Spherical –Revolute Paired)
The transformation matrix for link BO2

$$^{B}T_{O2} = \begin{bmatrix} C\phi iC\psi i & -S\phi i & S\psi iC\phi i & aiC\phi iC\psi i \\ S\phi iC\psi i & C\phi i & S\phi iS\psi i & aiS\phi iC\psi i \\ -S\psi i & 0 & C\psi i & -aiS\psi i + di \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Relational parameters between Joints B and O_2
a_i = - L_3 (Link Length)
d_i = 0 (Offset)
$\theta = 180^0$, $\alpha = 0^0$, $\phi = 0^0$
$\psi = 0^0$

Resultant matrix for Link A and B

$$^{B(SP)}T_{O2(R)} = \begin{bmatrix} 1 & 0 & 0 & -L3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Analysis of Link O2 O1 (Revolute –Revolute Paired)
Relational parameters between Joints O2and O1

$$01Z1 \begin{bmatrix} a = X \\ s = -L2 \\ \theta = 0^\circ \\ \alpha = 0^\circ \end{bmatrix} 02Z1$$

The transformation matrix for link O2O1

$$^{i-1}A_i = \begin{bmatrix} \cos\theta i & -\cos\alpha i \sin\theta i & \sin\alpha i \sin\theta i & ai\cos\theta i \\ \sin\theta i & \cos\alpha i\cos\theta i & -\sin\alpha i\cos\theta i & ai\sin\theta i \\ 0 & \sin\alpha i & \cos\alpha i & di \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{O2}T_{O1} = \begin{bmatrix} 1 & 0 & 0 & X \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -L2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

From equations, overall resultant matrix is

$$^{O1}T_{O1} = {}^{O1}T_A x {}^{A}T_B \text{ x } {}^{B}T_{O2} \text{ x } {}^{O2}T_{O1} \qquad (3\text{-}1)$$

$$^{O1}T_{O1} = \begin{bmatrix} 1 & 0 & 0 & L1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & X \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -L3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & X \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -L2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{O1}T_{O1} = \begin{bmatrix} 1 & 0 & 0 & L1 + X - L3 + X \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L2 - L2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{O1}T_{O1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{O1}T_{O1} = I \qquad (3\text{-}2)$$

4. Result Discussion

Comparison of two approaches D H Notation and Psung Dain Lin is used for kinematic modeling of the front suspension of an automobile. The First approach is based on D H Notation transformation matrices used for a translational and rotational relationship at various joints. The second approach was suggested by Psung Dain Lin and Jung Fa Hsich for binary mechanisms for spherical pairs. The validity is confirmed by the formation of a closed-loop. The concatenation of all the transformation matrices is an identity matrix, forming a closed loop.

Kinematic analysis of front suspension to identify the kingpin axis by determines the coordinate position of point A and point B is validating by both the approaches suggested. The results of both approaches indicate the concatenation of all the transformation matrices is an identity matrix, forming a closed loop. The coordinate of joint A and joint B is used to determine the kingpin axis with results to determine the steering geometry. The ranges of each steering parameter are used to predict the behavior of front suspension. Based on the behavior of front suspension one can decide the condition of bushes on which suspension is mounted and further facilitates the requirement of maintenance.

5. Conclusion

The approach suggested the validity of the analysis and the accuracy of kinematic analysis of front suspension of an automobile. The method adopted for analysis is useful to design the control panel of the front suspension on which all the steering parameters can be displayed to monitor the steering geometry which would help to decide for the servicing of the vehicle.

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