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Comparing the forecast performance of nonlinear models and machine learning process. An empirical evaluation of GARCH family and NAR models in the light of CPEC

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Abstract: **Modeling and forecasting the volatility of financial time series is a strenuous task. An appropriate forecast of volatility provides a better understanding and managing financial market risk, which is helpful for policy-makers, economists and investors. This study aimed to compare the forecast performance of a traditional generalized autoregressive conditional heteroscedasticity (GARCH) family, i.e., GARCH, EGARCH, PARCH, GJR-GARCH, GARCH-M, and IGARCH with the machine learning (ML) process model, namely nonlinear autoregressive neural network (NAR) model. China-Pakistan economic corridor (CPEC) expands one belt road project that is expected to produce enormous economic opportunities in South Asia and has already connected to Europe and beyond. Therefore, daily stock market returns of CPEC linked countries, namely KSE-100 (Pakistan), SSE-100 (China), TADAWUL (Kingdom of Saudi Arabia), KASE (Kazakhstan), KLSE (Malaysia), BIST (Turkey), MOEX (Russia), FTSE (United Kingdom) and CAC40 (France) are used for this purpose from December 1, 2014, to June 8, 2021. The findings of this study revealed that both modeling techniques are capable of forecasting the volatility of stock market returns. However, the ML NAR model outperforms all GARCH models except the GARCH-M model based on six forecast accuracy measure criteria, i.e., root means square error (RMSE), mean absolute error (MAE), mean absolute percentage error (MAPE), Theil-U1, and U2. Furthermore, no significant difference is found in predictive power accuracy between the ML-NAR model and GARCH family models based on Diebold-Mariano (DM) test. It is revealed that the ML-NAR model is a suitable and reliable alternative to traditional GARCH family models for forecasting the volatility of financial time series, especially for CPEC linked stock markets.**

Keywords: Stock market returns, CPEC, GARCH models, machine learning models, forecast performance

INTRODUCTION

China Pakistan economic corridor (CPEC) is known as the game-changer in the Middle East. Countries working on the CPEC or willing to become partners in the future are categorized as developing and developed economies. CPEC is an apple eye for most of the local and global investors. Therefore, the impact of good and bad news related to CPEC also affects the stock markets of these nations. Thus, accurate prediction of stock markets data can play a vital role for investors, economists, and business people to decide to invest in these stock markets. Over the years, the prediction of stock market returns has gained attention in the finance literature. The risk in the financial asset returns and uncertainty modeling forced the continuously developing forecast techniques that can capture the volatility in the short or long term. Investors rely on the predictions of stock returns from statistical modeling techniques usually to gain more profit than average. Malkiel[1] revealed that the impact of any bad news upsets the market securities equilibrium more than any good news. Therefore, past stock market studies used to forecast the volatility and financial information of any specific company, i.e., assets and earnings, are useless for the investors to obtain high profits. GARCH family models are vastly used to model and forecast the volatility in financial markets. ARCH by Engle [2] and GARCH by Bollerslev [3] studied symmetric behavior of the volatility. In contrast, Engle &Bollerslev[4] introduced the integrated GARCH model, Nelson [5]introduced the asymmetric GARCH model, i.e., EGARCH, Glosten, Jagannathan, &R unkle[6] developed the GJR-GARCH model, threshold GARCH model discovered by Zakoian[7] and Schwert[8] introduced and worked on Power ARCH (PARCH) model. The forecast performance of GARCH models has been studied since their development [9]. Similarly, in recent literature, machine learning techniques, i.e., nonlinear neural network models, are the most accurate forecast techniques compared to other statistical linear time series models [10]. The immense popularity of ML techniques, namely the nonlinear autoregressive neural network (NAR) model, is increasing day by day. The primary objective of machine learning models is to recognize the nonlinear and irregular patterns in the time series data, data-driven. ML-NAR model is vastly used to forecast financial and economic indicators [11-18], Fraz & Fatima [10], and Alvarez-Diaz [19]. This paper compares the forecast performances using well-known volatility models, namely the GARCH family and ML nonlinear models. Data of stock returns of nine stock markets related to CPEC are used for this purpose from December 1, 2014, to June 8, 2021. The rest of the paper is arranged as follows: literature review presented in Section 2, data and methodology are discussed in Section 3, Results and discussion are presented in Section 4, while section 5 concludes the study.

LITERATURE REVIEW

Hossainet. al., [20] evaluate the forecast performance of volatility measuring standard GARCH modeling techniques with ML modeling techniques, i.e., back propagation (BP) and neural network (NN) models using monthly data stock market indices, namely Nikkei 225 Hang Seng, FTSE, and DAX. According to their findings, both forecast techniques provided approximately similar forecasting results based on MSE, Normalized mean squared error (NMSE), MAE, Directional Symmetry (DS), and Weighted Directional Symmetry (WDS). Moreover, they also found that both GARCH and the machine learning techniques can assist the stock market trading and develop financial decision support systems. The NAR model outer performs compared to the GARCH-M model in the short-term forecast horizon of Johannesburg securities exchange (JSE) by Bonga-Bongaand Mwaba[16]. In another study, NAR was compared with the GARCH (1,1) model for daily exchange rate data of the Tunisian market by Charefand Ayachi[21]. They found that the forecast ability of the ML model is better than the traditional GARCH (1,1) model based on RMSE. They recommend using techniques instead of the traditional GARCH model due to parameter-free techniques. Mucaj and Sinaj[22] also compared the predictive ability of ML technique with the time series models, namely the autoregressive integrated moving average ARIMA, nonlinear autoregressive neural network model (ANN), and the proposed hybrid method of ARIMA-ANN, respectively. They used USD/ALL exchange rate time series monthly data from 2000 to 2015. They found that the proposed hybrid model is better than ARIMA and ANN models based on RMSE, MAE, MPE, MAPE, U of Theil statistics. Finally, they concluded that the most parsimonious and critical model for stock indices could be GARCH models with SVMs for forecast purposes. Recently, Fraz and Fatima [10] also compared the forecasting performance of nonlinear autoregressive neural network model (NAR) with two well-known linear autoregressive (AR) and autoregressive integrated moving average models (ARIMA) models. They used the yearly data of economic variables from G7 countries, i.e., inflation, exchange rate, and GDP from 1970 to 2015and found that the forecast performance of the ARNN model was better than AR and ARIMA models. In the same year, Alvarez-Diaz [19] compared several forecasting methods, namely the traditional ARIMA model, the ARFIMA and AR model with a nonlinear parametric model, namely GARCH-in-Mean (GARCH-M) model, as well as nonparametric nonlinear autoregressive artificial neural network NAR. He used the weekly Brent oil price growth rate from May 1987 to November, 2018i.e. total 1,643 observations. According to his findings, all methods can accurately predict any directional variation in the Brent oil price. Moreover, he also found no significant forecasting differences among all models based on the DM test. Additionally, he concluded that volatility is an essential feature to improve forecast ability. The mixed results from the literature are not enough to conclude any best forecast model, either nonlinear or machine learning, for stock market returns.[25] studied the comparison of several forecast models namely seasonal autoregressive moving average (SARIMA), self-exciting threshold (SETAR), Holt-Winters, NAR and error trend seasonal (ETS) models. They used data from January 2000 to December 2018. They evaluate the out-of-sample forecast from 2017 to 2018. According to the findings, all models are suitable based on forecast evaluation criteria. Overall, NAR model outperforms other models based on RMSE and MAE while SARIMA is found to best based on MAPE.

DATA AND METHODOLOGY

GARCH models

ARCH and GARCH are prominent tools to estimate the volatility due to feature of capturing the random and nonlinear moments of time series data. The financial economists are always interested in modelling the volatility of stock market returns. Volatility is defined through the conditional mean and variance equations. All the GARCH models of different order of lag variances i.e. p and residual errors i.e. q included in this study are selected on the basis of AIC and BIC information criteria. All GARCH family models with order (1,1) are found to be best among all (p,q) order.

Single regime GARCH model

Generalized Autoregressive Conditional Heteroscedasticity is an extension of ARCH technique which allows the method to support changes in the time dependent volatility. Bollerslev[3] allows the conditional variance to be dependent upon previous own lags. The variance equation is:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \mu_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \quad 1$$

This is a GARCH (1,1) model. Where $\sigma>0$, α_0, α_1 and $\beta_1$are the parameters to be estimated.

$\alpha_1+\beta_1 \leq 1$(stability of process)

3.1.2 Exponential GARCH model

In order to explain the leverage effects in case of financial time series, a commonly used exponential GARCH model purposed by Nelson (1991)[5] is the EGARCH (1, 1) given by:

$$\log(\sigma_t^2) = \alpha_0 + \alpha_1\left|\frac{\mu_{t-1}}{\sigma_{t-1}}\right| + \beta_1 \log(\sigma_{t-1}^2) + \gamma \left|\frac{\mu_{t-1}}{\sigma_{t-1}}\right| \quad 2$$

here, $\alpha_0 > 0, \alpha_1 \geq 0$ and the $\beta_1 \geq 0$, $-1 < \gamma < 1$ are parameters to be estimated.

GJR GARCH model

[6] made modification in GARCH modeling namely Glosten-Jagannathan-Runkle GARCH (GJR-GARCH) model which can be use in presence of asymmetric effect of volatility.

$$\sigma_t = \alpha_0 + [\alpha_1 \gamma (I_{t-1}\varepsilon_{t-1})] + \beta_1 \sigma_{t-1} \quad 3$$

here indicator function is $I(\varepsilon_{t-1} < 0)$.

GARCH-M model

To estimate the conditional mean, GARCH-M model was developed by Engle, Lilien and Robins [23] in 1987.The GARCH-M model is consist of two equations i.e. mean equation (4.1) and variance equation (4.2):

$$R_i = \mu + \beta 1 \sigma_{t-1}^2 + \varepsilon_i \quad 4.1$$

$$\sigma_t^2 = \alpha_0 + \alpha_i \varepsilon_{i-1}^2 + \beta_j \sigma_{t-j}^2 \quad 4.2$$

IGARCH model

Integrated GARCH model is used when the persistent parameters are sum up to 1 and the GARCH process contains a unit root. In IGARCH, the unconditional variance is considered infinite. The model can be written as:

$$\sigma_t^2 = \alpha_0 + \beta_1 \sigma_{t-1}^2 + (1- \beta_1)\alpha_{t-1}^2 \quad 5$$

Where, $\sum_{i=1}^{p} \alpha_i + \sum_{j=1}^{q} \beta_j = 1$

Power ARCH model

The asymmetric power ARCH model was introduced by the Ding et al. [24] to estimate the long memory property in volatility. The PARCH model equation is:

$$\sigma_t^d = \alpha_0 + \sum_{i=1}^{p} \alpha_i (|e_{t-i}| + r_i e_{t-i})^d + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^d \quad 6$$

Where *d* is parameter power term.

Machine learning models

Machine learning modelling techniques are designed to mimic the human brain intelligence and convert it into machine learning process. The basic objective of machine learning models is to understand the irregular patterns in the time series data. It gives generalized output based on its own past information. Due to self-adaptive and data-driven technique it doesn't required no priori assumption for statistical distribution.

NAR model

The nonlinear autoregressive machine learning (NAR) model is a recurrent dynamic network. In NAR model, the current output is based on the values of past output. The NAR model can be written as:

$$\hat{r} = f(r(t-1) + (r(t-2) + (r(t-3) + (r(t-4) + \cdots (r(t-i) + \epsilon(t) \quad 7$$

Where $\widehat{r}$ is the forecast value of $r$time series at time step t, while i is past observations of the time series and $f$is a nonlinear function (unknown), $\epsilon(t)$ is the error of $r$at$t$ time. Neural models are trained under different lags to built the unknown function f.A single hidden layer model is constructed with input and output layers. Neurons are varied in hidden layer from 2 to 8 for the estimation of performance parameters. According to topology of ML technique, NAR model with least MSE is considered as the forecast model. The number of delays is set to be 2. ML neural models are trained under the Levenberg-Marquardt algorithm i.e. LM algorithm. For estimation and forecast purpose, NAR model distribute data into three parts. First part of data is used for training, the remaining parts are used for validation and testing of the time series data. In this study, 70% data is used for training. 15% data is used for validation while 15% data is used for testing. Also, open loop mode is used for model training, validation and testing. The architecture of ML model is presented in Figure 1.

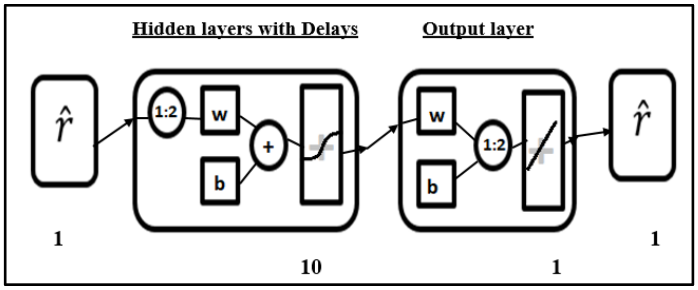

Figure 1. Architecture of ML modelling technique NAR model

To explore the volatility and evaluate the forecast performance of GARCH and NAR models, the stock markets related to CPEC are used. The daily data coverage from December 2014 to June 2021 for each Stock market namely: KSE-100 (Pakistan), SSE-100 (China), TADAWUL (Kingdom of Saudi Arabia), KASE (Kazakhstan), KLSE (Malaysia), BIST (Turkey), MOEX (Russia), FTSE (United Kingdom) and CAC40 (France) are selected. Logarithm difference is applied on all data for each closing price to convert the index returns

$$R_t = 100 \times [\ln(Y_t) - \ln(Y_t\text{-}1)] \quad 8$$

Where Y_t stands for closing price at the period of time t, so that R_t is the percent return for the daily closing price from period t-1 to period t.

After that, the ADF breakpoint unit root test identifies the unit root in the stock market returns depending on two criteria, namely the Akaike info criterion (AIC) and Schwarz info criterion (BIC). ARCH-LM test is also used to identify the presence of an arch effect in the stock market returns. Furthermore, the data is split into two parts. From December 2nd, 2014, to June 8th, 2020, the first part is used for model estimation, while the data from June 9th, 2020, to June 8th, 2021 is used for forecast performance evaluation. Forecast performance is evaluated based on six forecast accuracy criteria; namely, root mean squared error (RMSE), mean absolute error (MAE), mean absolute percentage error (MAPE), symmetric mean absolute percentage error (SMAPE), Theil Inequality Coefficient (Thiel-U1 and U2). Diebold-Mariano test (1995) is also used to check the predictive power accuracy between the GARCH family and the machine learning NAR model.

RESULTS AND DISCUSSION

All the stock returns show the nonlinear behavior, and all indices follow a significantly similar pattern. It can be seen that in the covid-19 pandemic, i.e., 2019 to 2020, all the stock market indices show a decreasing trend, but after 2020, i.e., January 2021, all the indices gradually show an increasing trend. All the stock market returns show high volatility (see Figure 2). According to the Jarque-Bera test, all the stock market returns are not normally distributed. It shows evidence of fat tails in stock returns. All the stock returns are

also serially correlated on level (Table 1). The average return is high in KASE, while FTSE has a minimum average return. Whereas standard deviation is high in SSE-100 and KLSE has a minimum value. Augmented-Dickey fuller (ADF) breakpoint unit root test is used to identify nonstationary stock returns. It is found that all the stock returns contain unit roots in levels. All the stock returns become stationary after 1stdifference depending on Akaike info (AIC) and Schwarz info (BIC) criteria (Table 2). Table 3 indicates that the GARCH-M model gives better forecasts among all GARCH models for BIST stock returns. These findings are based on MAE, MAPE, SMAPE, Theil-U1, and Theil U2. In contrast, the NAR model outperforms the GARCH-M model based on RMSE only. Likewise, based on all forecast evaluation criteria, the GARCH-M model gives the best forecast performance among all GARCH models and the NAR model for CAC40, FTSE, and KASE stock returns

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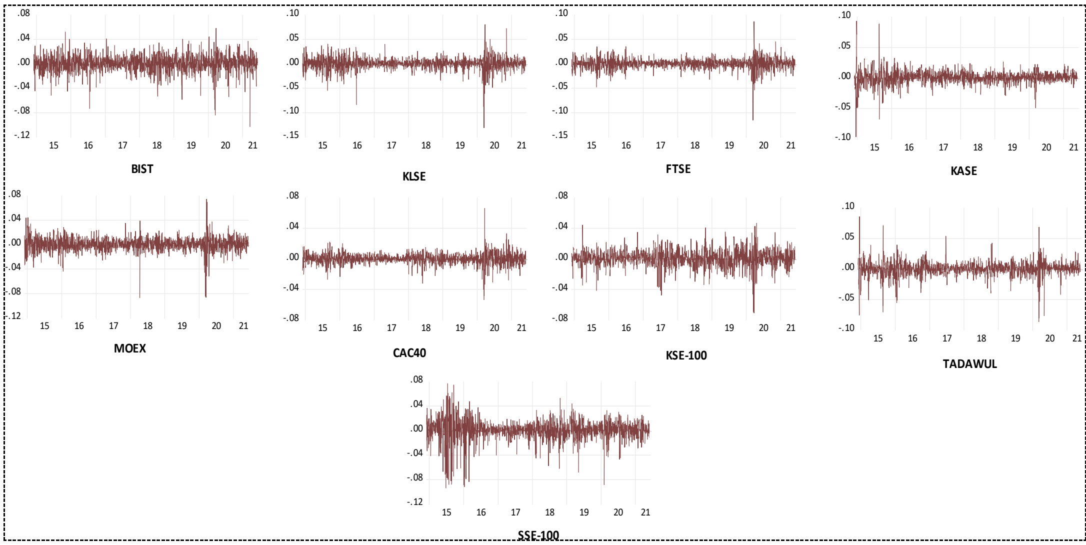

FIGURE2.
CPEC related stock market returns

The forecast performance of the GARCH-M model is also found to be better than the NAR model for CAC40, FTSE, and KASE stock returns based on all forecast evaluation criteria. Whereas, forecast performance of the machine learning NAR model is found to be the best among all GARCH models in KLSE and MOEX stock market returns. It is quite surprising that for KLSE and MOEX, the NAR model captures the volatility better than any GARCH model. This finding shows that the NAR model can be used for volatility forecast instead of GARCH models, especially for these two stock markets.

TABLE 1.
Descriptive statistics and ARCH-LM test

Indices	Mean	Std. Dev.	Skew.	Kurt.	Jarque-Bera	ARCH-LM
BIST	0.00031	0.014	-0.858	8.069	2034.912*	0.494*
CAC40	0.00024	0.012	-1.071	16.080	12480.710*	0.279*
FTSE	0.00003	0.011	-0.929	16.951	14072.110*	0.032*
KASE	0.00072	0.010	0.014	19.345	18979.480*	0.127*
KLSE	0.00007	0.007	-0.288	13.170	7371.126*	1.763*
KSE-100	0.00025	0.011	-0.622	7.801	1747.501*	0.050*
MOEX	0.00052	0.011	-0.703	12.303	6288.193*	0.213*
SSE-100	0.00020	0.017	-1.179	9.550	3442.704*	1.186*
TADAWUL	0.00012	0.012	-0.808	13.841	8535.024*	0.004*

*Author's estimation Note:*Significant at 1%

TABLE 2.
Break point unit root evidence

	AIC-Criterion		Schwarz- Criterion	
Indices	At level	1st difference	At level	1st difference
	t-Statistic	t-Statistic	t-Statistic	t-Statistic
BIST	-3.898	-42.583*	-4.004	-40.921*
CAC40	-3.325	-41.052*	-3.654	-39.872*
FTSE	-4.374	-41.949*	-4.436	-40.718*
KASE	-2.924	-41.281*	-2.872	-40.958*
KLSE	-3.669	-41.125*	-3.876	-40.440*
KSE100	-2.893	-36.383*	-3.043	-36.177*
MOEX	-4.686	-42.482*	-5.156	-41.586*
SSE100	-4.099	-38.288*	-4.252	-37.063*
TADAWUL	-3.718	-37.383*	-4.001	-37.383*

*Author's estimation Note: *Significant at 1% **Significant at 5% *** Significant at 10%

The IGARCH (1,1) model gives the best forecast performance among all GARCH models for the TADAWUL and KSE-100 stock returns. Also, the forecast performance of IGARCH (1,1) outperforms the NAR model for TADAWUL and KSE-100 stock returns. These results are based on all forecast evaluation criteria except for MAE. According to MAE criteria, GARCH-M (1,1) is found to be a better forecast model for KSE-100 only. Lastly, the forecast performance of the EGARCH (1,1) model gives the best among all GARCH models for the SSE-100. According to the MAE criteria, GARCH (1,1) has the best forecasting power than other studied GARCH models. Overall, it is found that the GARCH models are best to capture the volatility of stock returns for all the stock markets. The forecast performance of the GARCH-M model is better compared to other GARCH models and the NAR model for four stock market returns. Also, the NAR model is the best forecast model for two stock market returns.

TABLE 3.
Forecast comparison criteria's for all CPEC stock market indices

Forecast Models	RMSE	MAE	MAPE	SMAPE	Theil U1	Theil U2
			BIST			
EGARCH	19.283	12.834	0.986	0.984	0.007	0.999
GARCH	19.268	12.796	0.983	0.981	0.007	0.998
GARCH-M	19.256	12.791	0.983	0.980	0.007	0.996
GJR-GARCH	19.285	12.840	0.987	0.984	0.007	0.999
IGARCH	19.271	12.806	0.984	0.981	0.007	0.998
PARCH	19.281	12.830	0.986	0.983	0.007	0.999
NAR	19.036	13.093	1.001	0.999	0.007	0.998
			CAC40			
EGARCH	59.483	41.920	0.788	0.789	0.005	0.999
GARCH	59.347	41.822	0.787	0.787	0.005	0.996
GARCH-M	59.247	41.741	0.786	0.786	0.005	0.994
GJR-GARCH	59.438	41.880	0.788	0.788	0.005	0.998
IGARCH	59.376	41.837	0.787	0.788	0.005	0.997
PARCH	59.494	41.931	0.788	0.789	0.005	0.999
NAR	59.656	42.248	0.793	0.794	0.005	1.001
			FTSE			

EGARCH	68.094	49.166	0.777	0.777	0.005	1.001
GARCH	68.02	49.054	0.776	0.775	0.005	0.999
GARCH-M	67.851	48.865	0.773	0.772	0.005	0.995
GJR-GARCH	68.071	49.119	0.776	0.777	0.005	1.001
IGARCH	68.027	49.070	0.776	0.776	0.005	0.999
PARCH	68.094	49.166	0.777	0.777	0.005	1.001
NAR	68.005	49.727	0.786	0.785	0.005	0.997
			KASE			
EGARCH	16.32	12.271	0.449	0.45	0.003	0.976
GARCH	16.279	12.271	0.449	0.45	0.003	0.974
GARCH-M	16.272	12.27	0.449	0.45	0.003	0.973
GJR-GARCH	16.303	12.271	0.449	0.45	0.003	0.975
IGARCH	16.275	12.272	0.449	0.45	0.003	0.973
PARCH	16.309	12.271	0.449	0.45	0.003	0.975
NAR	16.314	12.383	0.452	0.452	0.003	0.976
			KLSE			
EGARCH	13.316	9.975	0.634	0.634	0.004	1.000
GARCH	13.312	9.961	0.633	0.633	0.004	1.000
GARCH-M	13.275	9.962	0.634	0.633	0.004	0.997
GJR-GARCH	13.314	9.971	0.634	0.634	0.004	1.001
IGARCH	13.312	9.962	0.633	0.634	0.004	1.001
PARCH	13.314	9.971	0.634	0.634	0.004	1.001
NAR	13.180	9.894	0.629	0.629	0.004	0.990
			KSE-100			
EGARCH	392.675	296.200	0.704	0.705	0.005	0.996
GARCH	391.838	295.559	0.703	0.703	0.005	0.993
GARCH-M	391.676	295.286	0.702	0.702	0.005	0.993
GJR-GARCH	392.753	296.251	0.704	0.705	0.005	0.996
IGARCH	391.579	295.325	0.702	0.702	0.005	0.993
PARCH	392.703	296.218	0.704	0.705	0.005	0.996
NAR	397.830	303.721	0.721	0.722	0.005	1.006
			MOEX			
EGARCH	30.117	23.505	0.741	0.741	0.005	0.994
GARCH	30.102	23.495	0.740	0.741	0.005	0.994
GARCH-M	30.079	23.478	0.740	0.740	0.005	0.993
GJR-GARCH	30.133	23.521	0.741	0.741	0.005	0.995
IGARCH	30.084	23.483	0.740	0.740	0.005	0.993
PARCH	30.134	23.522	0.741	0.742	0.005	0.995
NAR	30.052	23.315	0.735	0.736	0.005	0.993
			SSE-100			
EGARCH	80.135	59.445	0.861	0.86	0.006	0.998

GARCH	80.166	59.423	0.861	0.86	0.006	0.999
GARCH-M	80.168	59.427	0.861	0.86	0.006	0.999
GJR-GARCH	80.181	59.425	0.861	0.86	0.006	0.999
IGARCH	80.23	59.437	0.861	0.861	0.006	1.001
PARCH	80.182	59.425	0.861	0.86	0.006	0.999
NAR	80.881	61.323	0.888	0.887	0.006	1.006
TADAWUL						
EGARCH	64.282	45.058	0.516	0.517	0.004	1.009
GARCH	63.442	44.217	0.507	0.507	0.004	0.996
GARCH-M	63.543	44.355	0.508	0.509	0.004	0.997
GJR-GARCH	63.948	44.665	0.512	0.512	0.004	1.004
IGARCH	63.374	44.172	0.506	0.507	0.004	0.995
PARCH	64.017	44.745	0.513	0.513	0.004	1.005
NAR	63.972	44.929	0.515	0.516	0.004	1.002

*Author's estimation. All GARCH family models are estimated at order (1,1).

Since the forecast performance of all the GARCH type models is very close to the NAR model, the Diebold-Mariano test compares the forecast performance of the machine learning NAR model with all GARCH models in this study. According to the findings in Table 7, the predictive accuracy is equal for the machine learning NAR model and all GARCH models.

TABLE 4.
Diebold-Mariano test for NAR model with GARCH family

Models	Loss fn	Stock market indices								
		BIST	**CAC40**	**FTSE**	**KLSE**	**KASE**	**KSE-100**	**MOEX**	**SSE-100**	**TADAWUL**
GARCH vs NAR	Abs Error	-0.893	-0.867	-1.459	1.073	-0.518	-1.628	0.674	-3.273	-0.810
	Sq Error	0.371	-0.604	0.030	1.480	-0.150	-1.041	0.167	-1.261	-0.607
EGARCH vs NAR	Abs Error	19.283	-0.824	-1.086	1.228	-0.517	-1.506	0.706	-3.412	0.139
	Sq Error	19.036	-0.423	0.158	1.465	0.025	-0.911	0.212	-1.368	0.349
GARCH-M vs NAR	Abs Error	-0.910	-0.978	-1.684	1.086	-0.522	-1.688	-1.223	-3.182	-0.660
	Sq Error	0.352	-0.740	-0.283	1.158	-0.180	-1.065	-1.537	-1.194	-0.487
GJR-GARCH vs NAR	Abs Error	-0.763	-0.885	-1.209	1.177	-0.521	-1.495	0.755	-3.217	-0.290
	Sq Error	0.409	-0.508	0.122	1.471	-0.044	-0.898	0.261	-1.215	-0.028
IGARCH vs NAR	Abs Error	-0.863	-0.901	-1.444	1.087	-0.516	1.663	0.641	-3.048	-0.865
	Sq Error	0.378	-0.591	0.046	1.483	-0.167	1.074	0.109	-1.079	-0.684
PARCH vs NAR	Abs Error	0.792	0.803	1.087	-1.182	0.521	1.502	-0.756	3.214	0.201
	Sq Error	-0.400	0.400	-0.158	-1.470	0.018	0.906	-0.262	1.213	-0.052

*Author's estimation Note: *Significant at 1%, **Significant at 5%, ***Significant at 10%

Therefore, the difference in forecast errors between NAR and all GARCH models is not statistically significant at a 5% significance level. These results are based on two loss functions, namely absolute errors and squared errors loss functions.

5. CONCLUSION

This study aims to review and compare the forecast performance of the nonlinear statistical and machine learning models for the volatility of stock market returns from CPEC related countries. The multidimensional China-Pakistan economic corridor is the game-changer in central Asia. It is expected that the CPEC will bring more development, growth, and prosperity for both nations. It will also be highly beneficial for those nations' members of CPEC. Modeling the volatility of financial markets is necessary for future decision-making, especially for

investors, economists, and policy-makers. Daily data from December 2014 to July 2021 is used in this study. GARCH family models, namely GARCH, EGARCH, IGARCH, GARCH-M, GJR-GARCH, and PARCH models, are evaluated and compared. Furthermore, one machine learning technique, namely the neural network autoregressive model (NAR), is used and compared the forecast ability with all GARCH type models. GARCH type models are considered the best time series models to capture volatility. In contrast, machine learning techniques are known to be the best artificial intelligence techniques that can be utilized in every situation for any time series data set. Based on the ADF KLSE and MOEX.ML model is the best forecast model for these two stock markets among all GARCH models based on all forecast evaluation criteria. Interestingly, the two nations are geographically far away from each other. Additionally, the forecast performance of the NAR model is found to be better compared to mostly GARCH models in other stock markets. Moreover, the IGARCH model gives the best forecast performance among other GARCH and NAR models for KSE-100 and TADAWUL stock returns. Lastly, the EGARCH model best forecasts the SSE-100 stock returns based on all forecast evaluation criteria.
Overall, these empirical findings indicate that all GARCH models can significantly explain the volatility in the stock market indices. Also, the NAR model appears to outperform the GARCH type models in two cases. The out-of-sample forecast performance of the machine learning NAR model is remarkable and very close to the traditional GARCH models. Therefore, Diebold-Mariano test is used. There is no significant difference found between the predictive accuracy between the GARCH family models and the NAR model. This evidence shows that the assumptions-free machine learning NAR model can forecast the volatility as an alternate of GARCH models. These findings will help policy-makers and national and international investors to use the best forecast model to gain better profits. These findings are similar to Alvarez- Diaz (2020) [19], in which he concludes that there is no significant difference between the GARCH-M and ML-NAR models through forecasting. Also, these results are pretty similar to Hossain et al. (2009) [20], in which they concluded that the GARCH models are better for forecasting financial time series data. Further studies can be conducted on the stock returns volatility to explore the forecast performance of the GARCH family and ML techniques in the COVID-19 pandemic situation.

breakpoint unit root test, all the stock market returns are stationary at first difference. Furthermore, the ARCH LM test confirms the serial correlation in levels. After that, the data split into two parts. Data from December 2014 is used for estimation, while the remaining is used for the out-of-sample forecast. According to the empirical findings, the GARCH-M model gives the best forecast performance among other GARCH type models based on MAE, MAPE, SMAPE, Theil U1, and Theil U2 for the BIST, CAC40, FTSE, and KASE. In contrast, the ML NAR models capture the volatility better than any GARCH type model for two stock returns indices, namely

This research paper contributes to the existing literature by investigating the forecast performance of GARCH family models and machine learning neural network models for stock market returns for CPEC linked countries. The empirical findings could be used by the local/ international economists, investors and policy-makers to use and forecast from reliable forecast model for better investments and profits especially for CPEC linked countries. It is also a chapter of PhD research work.

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