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A COMPARISON OF CONTROL CHARTS BASED ON SIX SIGMA

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Abstract. Control chart performs a essential position in decreasing the method variability and subsequently presents cohesive realistic framework for exceptional development. Even aleven though the bigger shifts withinside the method are detected with the aid of using Shewharts manipulate chart(1931), the development of six sigma manipulate charts effects in lesser range of defects withinside the method which is recommended and ensured with the aid of using Motorola. Six sigma idea became brought with the aid of using an American engineer Bill Smith at the same time as running at Motorola in 1986. A six sigma method is one wherein 99.99966% of all possibilities to supply a few function of a component are statistically predicted to be freed from defects. In the development of parametric manipulate chart commonly the figure distribution is thought to be regular. In this paper, we propose a six sigma manipulate chart for suggest the usage of variety with the aid of using assuming the figure population follows Moderate distribution which identifies the defects successfully and indicates for the removal of the ones defects , in the end it'll reduces the waste.

1. Introduction

Construction of manipulate charts are the maximum vital approach to display the manufacturing method and acquire excessive exceptional of products. To look at the adjustments withinside the method the graphical illustration of manipulate chart is used. A manipulate chart is drawn with manipulate strains that one represents a crucial line (CL) for the common, the alternative one represents higher line (UCL) for the higher manipulate lmit and the 0.33 one represents decrease line (LCL) for decrease manipulate limits. The method is said as out of manipulate (Montgomery, 2009), whilst the manipulate statistic is plotted past the LCL or UCL limits. Attribute and variable charts are the kinds of manipulate charts. The non-conforming gadgets are separated out from the conforming gadgets the usage of characteristic manipulate charts whilst the exceptional traits aren't measurable at the same time as tracking the method. When the exceptional traits are measurable, variable manipulate charts are used to display the method.

In production and carrier processes, regular distribution is carried out whilst the effects are measurable. Quality is without delay proportional to variant due to the fact the nearer the method end result adheres to set a preferred, the higher the exceptional. The exceptional decreases whilst variant increases. The method is in

Key words and phrases. Process manipulate, Six Sigma and Moderate distribution.

statistical manipulate most effective whilst that method has its personal preferred and degree of exceptional this is achieved. If that preferred isnt properly enough, the method must be advanced with the aid of using making suitable adjustments. When the method preferred is acceptable, the method ought to be managed in order that different supply of variant doesnt creep in. The form of the distribution will extrade to a few non regular sample whilst assignable purpose of variant is present, however now and again loads of measurements are had to locate a real non regular sample.

The idea of Six Sigma became brought with the aid of using an American engineer Bill Smith at the same time as running at Motorola and afterward it became evolved with the aid of using the engineer M.Harry who analyzed versions in effects of the companys inner approaches and found out that with the aid of using measuring versions it'll be viable to enhance the running of the system. The system became geared toward taking movement to enhance the general overall performance. The corporations, which can be working towards Six Sigma, are predicted to supply three.four or much less range of defects according to million possibilities. Six sigma idea became utilized by Radhakrishnan and Sivakumaran (2008) withinside the production of sampling plans together with unmarried, double and repetitive institution sampling plans with the aid of using indexing thru Six Sigma Quality Levels (SSQLs) wherein Poisson distribution is the bottom line distribution. Radhakrishnan (2009) recommended unmarried sampling plan listed thru Six Sigma exceptional levels (SSQLs) primarily based totally on Intervened Random Effect Poisson Distribution and Weighted Poisson Distribution because the final analysis distributions. Radhakrishnan and Balamurugan (2011) constructed manipulate charts primarily based totally on six sigma tasks for the range of defects and common range of defects according to unit. W.A. Shewhart (1931) originated manipulate chart primarily based totally on three sigma manipulate limits. If the identical W.A. Shewhart charts are used for the goods of the corporations which undertake six sigma tasks withinside the method, then no pattern factor will fall out of doors the manipulate limits seeing that development withinside the exceptional is observed.. In this paper an strive is made to assemble a six sigma primarily based totally manipulate chart for suggest the usage of variety below Moderate distribution.

2. Moderate distribution

Desai (2011) has brought slight distribution as an opportunity to regular distribution. The parameters of slight distribution are suggest μ and suggest deviation δ . The chance density feature of a random variable X which follows slight distribution is described as,

$$f(x) = \frac{1}{\pi\delta} e^{-\frac{1}{\pi}\left(\frac{X-\mu}{\delta}\right)^2}, \ -\infty < X < \infty, \ \delta > 0$$

When, $X \sim M(\mu.\delta)$, then the variable Z is defined as, $Z = \frac{X-\mu}{\delta}$ Which has the probability density function defined as,

$$g(Z) = \frac{1}{\pi} e^{-\frac{1}{\pi}Z^2}, \ -\infty < Z < \infty$$

And this variable Z is called standard moderate variate. The corresponding distribution is called as standard moderate distribution.

3. Methods of materials

To determine the process standard deviation ($\sigma_{MD:6\sigma}$). the tolerance level (TL) and process capability (C_P) are fixed. Apply the value of $\sigma_{MD:6\sigma}$ in the control limits $\bar{\bar{X}} + \frac{A_{MD:6\sigma}}{\sqrt{n}}\sigma_{MD:6\sigma}$ to get the six sigma based control limits for mean using range under Moderate distribution, The value of $A_{MD:6\sigma}$ is obtained using $P(Z < z_\sigma) = 1 - \frac{\alpha_1}{2}, \alpha_1 = 3.4 \times 10^{-6}$ and Z is a standard moderate variate. For a specified TL and C_p of the process, the value of σ (termed as $\sigma_{MD:6\sigma}$) is calculated from $c_p = \frac{TL}{6\sigma}$ using a JAVA script for various combinations of TL and C_p.
The six sigma based control limits for mean using process capability under Moderate distribution are

$$\text{UCL}_{MD:6\sigma} = \bar{\bar{X}} + \frac{A_{MD:6\sigma}}{\sqrt{n}}\sigma_{MD:6\sigma}$$

$$\text{CL}_{MD:6\sigma} = \bar{\bar{X}}$$

$$\text{LCL}_{MD:6\sigma} = \bar{\bar{X}} - \frac{A_{MD:6\sigma}}{\sqrt{n}}\sigma_{MD:6\sigma}$$

4. Illustration

The data from Table-1 shows the measurement of rivets for 25 subgroups of 5 items each.

$$\bar{\bar{X}} = 20.8 \text{ and } \bar{R} = 2.16$$

4.1. Three Sigma control limits for mean using range. The 3-Sigma control limits suggested by Shewharts (1931) are

$$\bar{\bar{X}} \pm \frac{3}{\sqrt{n}}\left(\frac{\bar{R}}{d_2}\right)$$

The process is out of control, since the sample numbers 15 and 23 lie outside the upper control limit and the sample numbers 6, 7, 13, 17 and 22 lie outside the lower control limit.

$$UCL_{3\sigma} = \bar{\bar{X}} + \frac{3}{\sqrt{n}}\left(\frac{\bar{R}}{d_2}\right) = 20.8 + \left\{\frac{3}{\sqrt{5}}\left(\frac{2.16}{2.326}\right)\right\} = 22.1$$

$$\text{Central line } CL_{3\sigma} = \bar{\bar{X}} = 20.8$$

$$LCL_{3\sigma} = \bar{\bar{X}} - \frac{3}{\sqrt{n}}\left(\frac{\bar{R}}{d_2}\right) = 20.8 - \left\{\frac{3}{\sqrt{5}}\left(\frac{2.16}{2.326}\right)\right\} = 19.6$$

(From the statistical Table-2, $d_2 = 2.326$ for sample size n=5)

TABLE 1. **Measurement of rivets**

Sample No	Observation					Mean	Range
1	20	21	22	22	23	21.6	2
2	18	19	20	20	22	19.8	2
3	17	18	20	22	25	20.4	5
4	20	21	21	21	22	21.0	1
5	19	20	22	23	24	21.6	4
6	18	18	19	20	22	19.4	2
7	18	18	19	20	20	19.0	2
8	18	20	20	21	23	20.4	3
9	20	21	22	23	24	22.0	3
10	19	20	20	20	21	20.0	1
11	20	20	20	22	23	21.0	2
12	20	21	22	22	23	21.6	2
13	18	19	19	19	22	19.4	1
14	20	21	21	22	22	21.2	2
15	20	23	23	24	24	22.8	4
16	20	20	21	21	14	21.2	1
17	18	18	20	20	20	19.2	2
18	20	22	23	23	24	22.4	3
19	19	19	20	20	23	20.2	1
20	21	21	22	22	24	22.0	1
21	20	22	22	22	23	21.8	2
22	17	18	18	19	21	18.6	2
23	21	23	23	24	24	23.0	3
24	20	20	21	21	22	20.8	1
25	19	20	21	21	22	20.6	2

TABLE 2. **Statistical quality control constant based on 3-Sigma**

sample Size (n)	d_2
2	1.128
3	1.693
4	2.059
5	2.326
6	2.534
7	2.704
8	2.847
9	2.970
10	3.078

4.2. Six sigma based control limits for mean using range under Moderate distribution. For a given TL=4 and C_p=0.93, the value of $\sigma_{MD:6\sigma}$ can be obtained as 1.26 and the value of $Z_{6\sigma}$ obtained from the Moderate distribution as 5.64.

The six sigma based mean using range chart under Moderate distribution for a specified TL, $A_{MD:6\sigma}$and n is

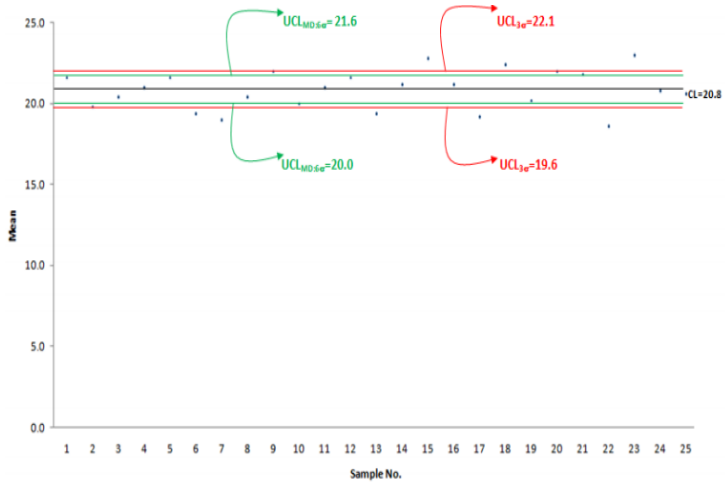

FIGURE 1. **Comparison of the process: 3 sigma limits and six sigma based control limits under Moderate distribution**

$$UCL_{MD:6\sigma} = \bar{\bar{X}} + \frac{A_{MD:6\sigma}}{\sqrt{n}}\sigma_{MD:6\sigma} = 20.8 + \frac{4.5}{\sqrt{5}}(0.53) = 21.7$$

$$\text{Central line } CL_{MD:6\sigma} = \bar{\bar{X}} = 20.8$$

$$LCL_{MD:6\sigma} = \bar{\bar{X}} - \frac{A_{MD:6\sigma}}{\sqrt{n}}\sigma_{MD:6\sigma} = 20.8 - \frac{4.5}{\sqrt{5}}(0.53) = 20.0$$

It is observed from the Figure-1 that many sample points have fallen outside the control limits, the process is out of control under moderate distribution compared with existing control chart.

5. Conclusion

Usually, Shewhart (1931) suggest the usage of variety manipulate chart for tracking the method variability is primarily based totally on a few assumptions with preferred deviation (σ).We presented the six sigma primarily based totally manipulate chart on slight distribution for suggest the usage of variety. The final results of numerical instance suggests that the proposed approach leads higher to the overall performance withinside the presence of normality, together with many factors fall out of doors the manipulate limits than the prevailing manipulate charts and the manipulate limits c programming language of proposed manipulate chart is smaller than the manipulate limits c programming language of Shewhart. It is apparent that the product/carrier isn't in properly exceptional as predicted, consequently a change and development is wanted withinside the method/system.

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