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NORMALIZED LAPLACIAN ENERGY OF COMPLETE, COMPLETE BIPARTITE AND COMPLETE TRIPARTITE GRAPHS

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Abstract. Let G be a finite, undirected and simple graph. The normalized laplacian energy of the graphs are found on the basic of the energy concepts in graphs. The normalized laplacian matrix of a graph, NL(G) (with no isolated vertices) is defined as 1, if i=j and deg(v_i) ≠0, $\frac{-1}{\sqrt{d_i d_j}}$, if i=j and they are adjacent and 0 otherwise. Here the normalized laplacian energy of the complete with bipartite and tripartite graphs are explained..

KEYWORDS: Normalized laplacian matrix , laplacian energy, bipartite and tripartite graphs.

1. Introduction

Zadeh [28] have initiated fuzzy sets [30] [31] [32]. Parvathi and Karunambigai[13] have initiated the idea of Intuitionistic Fuzzy Graphs (IFGs). Gani and Begum [5] talked about the extension of fuzzy graphs. Products in IFGs were discussed by Sahoo & Pal [17].Sahoo and Pal [18,19] studied some types of fuzzy graphs. Sahoo et al [21] initatied new ideas in intuitionistic fuzzy graphs. Kalaiarasi and Mahalakshmi have also expressed fuzzy strong graphs [8].Shanmugavadivu and Gopinath, suggested non homogeneous ternary five degrees equation [24]. Shanmugavadivu and Gopinath, have also expressed on the homogeneous five degree equation [25], Bozhenyuk et al[2] has talked about dominating set and Mapreduce Methodology for Elliptical Curve Discrete Logarithmic Problems [29].

Ore and Berge introduced the concept of domination in 1962. Cockayne and Hedetniemi have further studied about domination in graphs[6]. Somasundaram and Somasundaram have initiated domination in fuzzy graphs by making use of effective edges[23]. Xavior et al. [27] has talked about domination in fuzzy graphs but differently. Dharmalingam and Nithya have also expressed domination parameters for fuzzy graphs[3]. Equitable domination number for fuzzy graphs was introduced by Revathi and Harinarayaman in [16]. Sarala and Kavitha have also expressed (1,2)-domination for fuzzy graphs[22]. Gani and Chandrasekaran have talked about strong arcs[12]. Sunitha and Manjusha have also expressed strong domination [26]. Kalaiarasi and Mahalakshmi have also expressedfuzzy inventory EOQ optimization mathematical model [9]. Kalaiarasi and Gopinath suggested fuzzy inventory order EOQ model with machine learning [10]. Fuzzy Incidence Graphs (FIGS) discussed by Dinesh [4]. Mordeson talked about incidence cuts in FIGS [11].Priyadharshini et al.[18] have also expressed a fuzzy MCDM approach for measuring the business impact of employee selection [15].

The design of this articlein section 2 provides some preliminary results which are required to understand the remaining part of the article. In section 3 CIFIG is defined. In section 4 conveys meaning domination in CIFIG. In section 5 we examine Strong Intuitionistic Fuzzy Incidence Dominating Set (SIFIDS) and SIFIDN and Weak Intuitionistic Fuzzy Incidence Dominating Set (WIFIDS) and WIFIDN. In section 6 application of intuitionistic fuzzy incidence domination number is given.

The concept of energy originated in chemistry. The energy of G was first defined by Gutman in 1978 as the sum of the absolute values of the eigen values of A[G]. According to R.B. Bapat and Sakanta Pati, if the energy of a graph is rational then it must be an even integer. Shanmugavadivu & R.Gopinath have also expressed on the homogeneous five degree equation. Kalaiarasi & R.Gopinath analysed and introduced Fuzzy inventory and arc sequences in different graphs and explained the join product in mixed split IFGs. Priyadharshini & R.Gopinath [18]have also expressed a fuzzy MCDM approach for measuring the business impact of employee selection. An important result is that the energy of the graph is greater than the number of vertices of the graph. Here the normalized laplacian energy of complete graphs with bipartite and tripartite are explained.

2.Normalized Laplacian Energy of the Complete, Complete Bipartite and Complete Tripartite Graphs

Normalized Laplacian Matrix

The ***normalized laplacian matrix*** of a graph(with no isolated vertices), is defined as

$$\mathrm{NL(G)} = \begin{cases} 1, if\ i = j\ and\ \deg(v_i) \neq 0 \\ \frac{-1}{\sqrt{d_i d_j}}, if\ i = j\ and\ they\ are\ adjacent \\ 0\ , otherwise \end{cases}$$

Normalized Laplacian Energy

Let $\lambda_1, \lambda_2, \ldots \lambda_n$ be the eigen values of $L^+(G)$,then the ***normalized laplacian energy*** is defined as $NLE(G) = \sum_{i=1}^{n} |\lambda_i(L) - 1|$.

Theorem 1

The normalized laplacian energy of the complete graphs K_n of order $n \geq 3$ is n-1

Proof

Let K_n be a complete graph of order $n \geq 3$.

To Prove: The normalized laplacian energy of K_n is n-1. To calculate the normalized laplacian energy, we have to construct the normalized laplacian matrix.

By the definition,

The normalized laplacian matrix of a graph(with no isolated vertices), is defined as

$$\mathrm{NL(G)} = \begin{cases} 1, if\ i = j\ and\ \deg(v_i) \neq 0 \\ \frac{-1}{\sqrt{d_i d_j}}, if\ i = j\ and\ they\ are\ adjacent \\ 0\ , otherwise \end{cases}$$

The normaliized laplacian matrix of K_n is n×n matrix of the form

$$NL_{ij} = A_{ij} = \begin{cases} 1, if\ i = j \\ \dfrac{-1}{n-1}, if\ i \neq j \end{cases}$$

For this matrix, the eigen values are (n-1) rationals and a zero whose sum is equal to n.

By the definition

The normalized laplacian energy is defined as

$NLE(G) = \sum_{i=1}^{n} |\lambda_i(L) - 1|$ or $NLE(G) = \sum_{i=1}^{n} |\lambda_i(I - L)|$

Therefore,

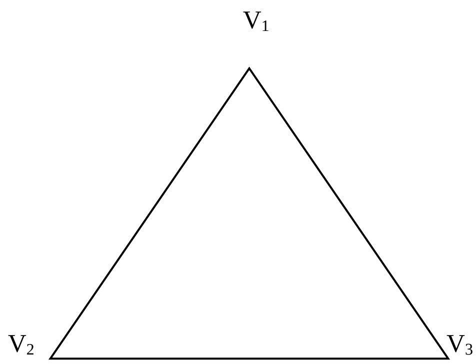
NLE(G)=$|(0 + n) - 1|$

=$|n - 1|$

NLE(G)=n-1

Hence it is proved.

Example: K_3.



The eigen values are = 0,1.5,1.5

The sum of the eigen values is 3

The normalized laplacian energy is 2.

3. Energy Relations

Theorem 3.1

The normalized laplacian energy of the complete bipartite graphs $K_{n,n}$, n≥2, is 2n-1.

Proof

Let $K_{n,n}$ be a complete bipartite graph of order n≥2.

To Prove: The normalized laplacian energy of $K_{n,n}$ is 2n-1

To calculate the normalized laplacian energy, we have to construct the normalized laplacian matrix.

By the definition, the normalized laplacian matrix of a graph(with no isolated vertices), is defined as $NL(G) = \begin{cases} 1, if\ i = j\ and\ \deg(v_i) \neq 0 \\ \frac{-1}{\sqrt{d_i d_j}}, if\ i = j\ and\ they\ are\ adjacent \\ 0\ , otherwise \end{cases}$

The normaliized laplacian matrix of $K_{n,n}$ is 2n×2n matrix of the form

$$NL_{ij} = A_{ij} = \begin{cases} 1, if\ i = j \\ \dfrac{-1}{2n}, if\ i \neq j \end{cases}$$

For this matrix, the eigen values are n rationals whose sum is equal to 2n.

By the definition, The normalized laplacian energy is defined as

$NLE(G) = \sum_{i=1}^{n} |\lambda_i(L) - 1|$ or $NLE(G) = \sum_{i=1}^{n} |\lambda_i(I - L)|$

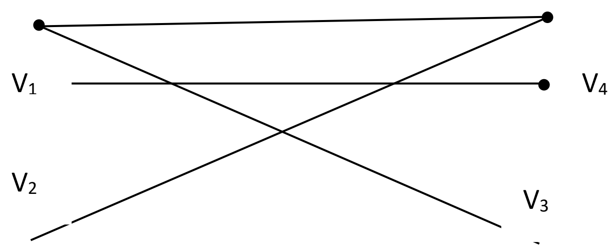
Therefore,

NLE(G)=$|(2n) - 1|$

=$|2n - 1|$

NLE(G)=2n-1

Hence it is proved.



The eigen values are =

4. Energy Relations

Theorem 4.1

The normalized laplacian energy of the complete tripartite graphs $K_{n,n,n}$, n≥2 is 3n-1.

Name of the Graph	Energy of $K_{n,n}$
$K_{2,2}$	3
$K_{3,3}$	5
$K_{4,4}$	7
$K_{5,5}$	9
$K_{6,6}$	11

Proof

Let $K_{n,n,n}$ be the complete tripartite graph with n≥2.

To Prove: The normalized laplacian energy of the complete tripartite graphs $K_{n,n,n}$, n≥2 is 3n-1.

To calculate the normalized laplacian energy, we have to construct the normalized laplacian matrix.

By the definition,the normalized laplacian matrix of a graph(with no isolated vertices),

is defined as $\mathrm{NL(G)} = \begin{cases} 1, if\ i = j\ and\ \deg(v_i) \neq 0 \\ \frac{-1}{\sqrt{d_i d_j}}, if\ i = j\ and\ they\ are\ adjacent \\ 0\ , otherwise \end{cases}$

The normaliized laplacian matrix of $K_{n,n,n}$ is 3n×3n matrix of the form

$$NL_{ij} = A_{ij} = \begin{cases} 1, if\ i = j \\ \frac{-1}{3n}, if\ i \neq j \end{cases}$$

For this matrix, the eigen values are n rationals whose sum is equal to 3n.

By the definition

The normalized laplacian energy is defined as

$NLE(G) = \sum_{i=1}^{n} |\lambda_i(L) - 1|$ or $NLE(G) = \sum_{i=1}^{n} |\lambda_i(I - L)|$

Therefore,

NLE(G)=$|(3n) - 1|$

=$|3n - 1|$

NLE(G)=3n-1

Hence it is proved.

5. Energy Relations

Here, incorporates an everyday life model. Assume there are five energy relations clinics are working (24 hours) in a city for giving crisis treatment to individuals. Here in our energy relations e are not referencing the original names of these clinics in this manner think about the clinics h_{11}, h_{22}, h_{33}, h_{44} and h_{55}. Energy relations the vertices show the clinics and edges show the contract conditions between the clinics to share the facilities. The incidence pairs show the transferring of patients from one clinic energy relation to another because of the lack of resources.

Name of the Graph	Energy of $K_{n,n}$
$K_{2,2,2}$	5
$K_{3,3,3}$	8
$K_{4,4,4}$	11
$K_{5,5,5}$	14
$K_{6,6,6}$	17

6. Conclusion

The idea of domination in CIFIGs is imperative from religious just as an applications perspective. In this paper, the possibility of complete intuitionistic fuzzy incidence graph, strong and weak intuitionistic fuzzy incidence dominating set and strong and weak intuitionistic fuzzy incidence domination number is talked about. Further work on these thoughts will be accounted for in impending papers.

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