

BOUNDARY LAYER FLOW OVER A MOVING IN A NANOFUID WITH VISCOUS DISSIPATION IN A SATURATED POROUS MEDIA

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Abstract In the present analysis, Boundary layer flow over a moving plate in a nanofluid with Viscous Dissipation in saturated porous medium is analyzed. Using proper similarity transformations, the governing nonlinear PDEs were transformed into a system of nonlinear ODEs. Governing boundary value problem is numerically solved using 4th order Runge-Kutta shooting technique. The impact of different parameters like Porous medium parameter (A), Prandtl number (Pr), Eckert number (Ec), Lewis number (Le), Brownian motion (Nb) and thermophoresis (Nt) with their different values for velocity, temperature, and nanoparticle volume fraction profile are existing as numerically and graphically. It is established that Nb and Nt are the most well-known parameters responsible for different results of velocity, temperature and nanoparticle volume fraction.

Keywords Moving plate, Nanofluid, Saturated porous medium, Viscous dissipation.

INTRODUCTION

Over the last few decades, flow through porous media has been the focus of numerous studies. Food processing and storage, geophysical systems, and other chemical and mechanical engineering applications use thermally induced flows in porous media. Also, electrochemistry, and fibrous insulation, underground discarding of nuclear or non-nuclear waste, micro-electronics cooling, metallurgy, and the design of the nuclear reactor, has sparked interest in this subject. Heat transfer because of a linearly stretching surface via porous medium is one of the important phenomena. The role of convection boundary layer flow is always important and plays a huge role in the engineering and industrial field. For example, engine cooling systems and electronic devices, rubber sheets, glass fiber production, etc. are such important applications. The fluid containing nanometer-sized particles is referred to as nanofluid. The term nanofluid was created by [1]. The ultra-fine particles are used to changes in liquid thermal conductivity and viscosity [2]. As a result of transpiration, they reported self-similar boundary layer flow across moving surfaces [3]. The spectrum of known dual solutions for zero transpiration has been proven to expand with suction and contract with blowing. They analyzed convective heat transfer of nanofluids: A critical review [4]. It has been discovered that nanoparticles have a significant potential for improving heat transmission. Suspended nanoparticles serve an important function in improving fluid thermal conductivity and chaotic movement of the ultra-fine particles increases the oscillation, which speeds up the energy exchange process.

Transpiration effects on permeable moving surface with nanofluid flow and heat transfer with parallel stream over a moving surface are noticed [5]-[6]. Dual solution exists for sheet and stream flow in reverse directions numerically. Suction increases the coefficient of skin friction and rate of heat transfer at the surface, as well as delaying boundary layer separation, when compared to injection. Dual solutions are revealed that the plate and free stream flow go in reverse directions. On a permeable stretched sheet, many authors exhibit nanofluid boundary layer

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flow with varying fluid characteristics and slip condition [7]-[9]. Each dimensionless number is a decreasing function of the Nur for Le , Nb , and Nt , and Shr is an increase for higher Pr and decay for lower Pr . As the thermal conductivity parameter enlarges, velocities and temperature profiles also rises. As the thermal conductivity value rise, so does the wall shear stress.

Brinkman Forchheimer Model on boundary layer flow in a saturated porous medium past a stretching surface due to Nanofluid [10]. It can be shown that as Le increases, so does solutal boundary layer thickness reduces. Physically, concentration rises due to decay in Brownian diffusion. In addition, On a reduced nusselt number, Nb and Nt have opposing effects. For a fixed value of porous medium parameter trend of Nb and Nt on reduced nusselt and sherwood number is similar. For lower levels of Nb and Nt , the vary in the decreased Shr is modest, but it becomes more prominent as Nb and Nt grow.

Influence of partial slip with a constant wall temperature past a stretching sheet in heat transfer of nanofluid is discussed [11]. On a stretching sheet, the slip parameter has a considerable impact on flow velocity and shear stress at surface. Thickness of hydrodynamic and thermal boundary layer thicknesses reduced and rise, respectively, as velocity parameter was raised. The reduced nusselt and Sherwood number follows opposite trend for velocity slip.

The role of MHD buoyancy stagnation point flow and heat transfer of a nanofluid across a stretching/shrinking sheet with convectively heated is investigated [12]. Shear stress rate and rate of mass transfer reduced as buoyancy force strength increased, whereas local nusselt number increased. They reported on MHD boundary layer nanofluid flow heat transfer past a permeable stretching sheet through slip boundary conditions [13]. As both Nb and Nt parameter grow up the heat transfer rate fall down. In another way, local mass transfer rate follows reverse trend. With the rise in Ec , the surface temperature rises as well.

[14] investigated MHD nanofluid stagnation point flow past a stretching/shrinking sheet along slip effects and heat generation/absorption along convective boundary conditions. As a result, heat source parameter enhanced temperature, there is no unique solution exists for shrinking sheet. In an external homogeneous ambient stream, Buongiorno's Model is used to analyze unsteady nanofluid boundary layer flow across a moving surface. [15]. The thicknesses of related boundary layers as momentum, thermal, and volume friction expand as upper branch solution gets thicker. The bottom branch solution, on the other hand, shows the opposite trend.

They observed flow and heat transfer of MHD nanofluid with viscous dissipation and moving plate [16]-[17]. Skin friction coefficient magnitude is highly influenced by the physical parameters like M , Re , and Kr . Nusselt number fall down as Kr , M and Ec rises. Thermal boundary layer thickness is heavily influenced by these variables. In the incidence of viscous dissipation, pure conduction is practically to take place as the plate velocity parameter decreases the nusselt number.

Being inspired by the above fruitful literature, in the present study we consider saturated porous medium for boundary layer flow in a nanofluid with viscous dissipation in a moving plate which constitutes the novelty of the problem. All above discussions considered the nanofluid flow with various physical properties, but porous medium also plays a vital role in flow properties in various problems; hence the present study fulfill the research gap in a nanofluid. There is no investigation available of different parameters like porous medium parameter (A), Prandtl number (Pr), Eckert number (Ec), Lewis number (Le), Brownian motion (Nb) and thermo-phoresis (Nt) with their diverse values for nanofluid flow problem. Hence, this paper aimed to study the effect of above

mentioned different parameters with the saturated porous medium on moving plate. Nb and Nt are the most noticeable parameters responsible for varied solutions of the investigated problem, according to the numerical and graphical data.

MATHEMATICAL FORMULATION

In this paper, two-dimensional steady laminar boundary layer flow nanofluid problem over moving plate with viscous dissipation in saturated porous medium is considered. Where T and T_w is temperature within the boundary layer and wall temperature respectively. T_∞ is ambient temperature. Also, free stream velocity is given by U_∞ and $u_w(x) = \epsilon U_\infty$ the plate velocity, with the plate velocity parameter ϵ [15]. Nano-particle volume fraction is C , nano-particle volume fraction at the surface C_w and the ambient nanoparticle volume fraction C_∞ . Under boundary layer approximations, and governing equations of momentum, energy, and nano-particle volume fraction can be expressed as follows.

$$\frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} \tag{1}$$

$$\left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \nu \frac{\partial^2 u}{\partial y^2} - \frac{u^2}{k} \tag{2}$$

$$\left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y^2} + \tau \left[D_B \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \right] + \frac{\mu}{\rho C_p} \left(\frac{\partial u}{\partial y} \right)^2 \tag{3}$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2} \tag{4}$$

with boundary conditions

$$u = u_w(x) = \epsilon U_\infty, v = -v_0, T = T_w, C = C_w \text{ for } y = 0 \tag{5}$$

$$u \rightarrow U_\infty, T \rightarrow T_\infty, C \rightarrow C_\infty \text{ for } y \rightarrow \infty \tag{6}$$

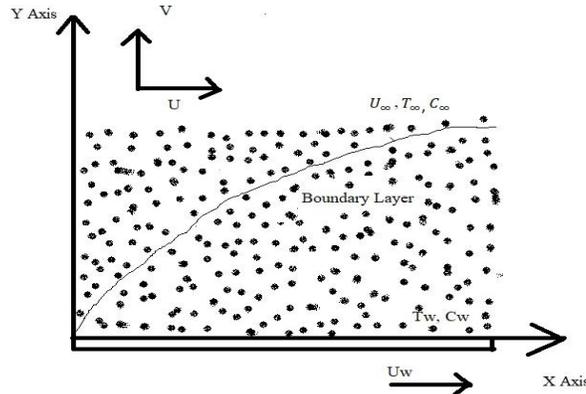


Fig. 1 Geometrical interpretation of Problem

where (u, v) are velocities components in the direction of x and y -axis, thermal conductivity is k , fluid density is ρ , coefficient of viscosity is μ , specific heat a constant pressure is C_p , coefficient of Brownian diffusion D_B , coefficient of thermo-phoresis diffusion D_T , ratio of effective heat capacity of the nanoparticle material and ordinary fluid denoted by τ . The mathematical analysis of the problem is simplifying via similarity transforms. To transform the equations (1)-(4) along boundary conditions (5)-(6), then following similarity transformations were introduced [3, 6]. The geometrical interpretation are given as below in figure 1

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$$\eta = y \sqrt{\frac{U_\infty}{2\vartheta x}}, \psi = \sqrt{2U_\infty \vartheta x} f(\eta), \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \quad (7)$$

The equations of continuity is holds good by stream function $\psi(x, y)$ such that.

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \text{ and } u = U_\infty f'(\eta), v = \sqrt{\frac{U_\infty \vartheta}{2x}} f(\eta) + \frac{U_\infty y}{2x} f'(\eta) \quad (8)$$

By using transformation equation (7) and (8) into (2)- (4), the converted coupled nonlinear ordinary differential equation is given as follows:

$$f''' + f f'' - 2\Lambda f' = 0 \quad (9)$$

$$\frac{1}{Pr} \theta'' + f \theta' + Nb \theta' \phi' + Nt \theta'^2 + Ec f'^2 = 0 \quad (10)$$

$$\phi'' + \frac{Nt}{Nb} \theta'' + Le f \phi' = 0 \quad (11)$$

with the corresponding boundary conditions

$$f = S, f' = \varepsilon, \theta = 1, \phi = 1 \text{ for } \eta \rightarrow 0.$$

$$f'(\eta) \rightarrow 1, \theta(\eta) \rightarrow 0, \phi(\eta) \rightarrow 0 \text{ as } \eta \rightarrow \infty.$$

Where prime denotes derivative of variable w.r.t. η . Where η is function of two variables x, y . The governing parameters are presented as follows:

$$Pr = \frac{\vartheta}{\alpha} \text{ (Prandtl Number)}$$

$$Nb = \frac{D_B(C_w - C_\infty)}{\vartheta} \text{ (Brownian motion Parameter)}$$

$$Nt = \frac{D_T(T_w - T_\infty)}{\vartheta} \text{ (Thermophoresis Parameter)}$$

$$Ec = \frac{u^2}{c_p \Delta T} \text{ (Eckert Number)}$$

$$Le = \frac{\vartheta}{D_B} \text{ (Lewis Number)}$$

$$\Lambda = \frac{\varepsilon^2 \vartheta}{ck} \text{ (Porous medium parameter)}$$

Notice that $\varepsilon > 0$ for downstream movement of the plate [3]. The engineering interest quantities physically known as shear stress rate, heat and mass transfer rate and denoted as C_f , Nu_x and Sh_x respectively.

$$C_f = \frac{2\tau_w}{\rho u_e^2}, Nu_x = \frac{x q_w}{k(T_w - T_\infty)}, Sh_x = \frac{x j_w}{k(C_w - C_\infty)}$$

The surface shear stress τ_w , surface heat flux q_w , and surface mass flux j_w are given as follows.

$$\text{where } \tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0}, \quad q_w = -k \left(\frac{\partial T}{\partial y} \right)_{y=0}, \quad j_w = -D_B \left(\frac{\partial C}{\partial y} \right)_{y=0}$$

where $\mu = \vartheta \rho$ and k is the dynamic viscosity and thermal conductivity respectively. By by means of appropriate similarity variables in (7), which gives the following results.

$$C_f (Re_x)^{\frac{1}{2}} = f''(0), Nu_x \left(\frac{Re_x}{2} \right)^{-\frac{1}{2}} = -\theta'(0), Sh_x \left(\frac{Re_x}{2} \right)^{-\frac{1}{2}} = -\phi'(0)$$

Where $Re_x = \frac{U_\infty x}{\nu}$ is local Reynolds number.

NUMERICAL APPROACH

In the present study boundary layer nanofluid over a moving plate by means of viscous dissipation in the presence of saturated porous medium were investigated in this study using a proficient 4th Runge-Kutta method and shooting method ODE45 solver for equations (9)-(11) with various parameters $Pr, Ec, Le, Nb, Nt, \Lambda$. The system of first-order seven differential equations is used to transform non-linear governing odes. Momentum, energy, and concentration (Nanoparticle volume fraction) equations in f are third-order, θ and ϕ second-order respectively. To solve the system of equations, this technique required seven boundary conditions, but only four are available in this case: two initial conditions for f' and a single condition for both θ and ϕ respectively. The values at ambient conditions have already been provided, i.e. $\eta \rightarrow \infty$. When adopting the shooting method, the ambient position plays a critical role in generating the unknown initial $\eta = 0$. The boundary value problem is solved by using equations (9)-(11) to obtain $f''(0), \theta'(0), \phi'(0)$. Solution technique is repetitive with a larger value of until just the requisite significant digit differences between two succeeding values of $f''(0), \theta'(0), \phi'(0)$. Inside boundary layer, the last value is regarded a finite number for the various physical parameters for f', θ and ϕ . Then, to compress a coupled third-order boundary layer value issue in f', θ and ϕ second order a system of seven simultaneous first-order differential equations with seven unknowns was adopted.

$$f''' = -ff'' + 2\Lambda f' \quad (12)$$

$$\theta'' = -Pr(f\theta' + Nb\theta'\phi' + Nt\theta'^2 + Ec f''^2) \quad (13)$$

$$\phi'' = -\frac{Nt}{Nb} \theta'' - Le\phi' \quad (14)$$

Defined new variables as follows.

$$y_1 = f, y_2 = f', y_3 = f'', y_4 = \theta, y_5 = \theta', y_6 = \phi, y_7 = \phi'. \quad (15)$$

The highly non-linear coupled ODEs with appropriate transformed into system of seven first order ODEs in following manner.

$$y_1' = y_2$$

$$y_2' = y_3$$

$$y_3' = -y_1 y_3 + 2\Lambda y_2$$

$$y_4' = y_5$$

$$y_5' = -Pr(y_1 y_5 + Nb y_5 y_7 + Nt y_5^2 + Ec y_3^2)$$

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$$y_6' = y_7$$

$$y_7' = \frac{Nt}{Nb} Pr(y_1 y_5 + Nb y_5 y_7 + Nt y_5^2 + Ecy_3^2) - Le y_7$$

Prime sign indicates differentiation with respect to η , and appropriate boundary conditions written in that manner.

$y_1(0) = S, y_2(0) = \varepsilon, y_3(0) = A, y_4(0) = 1, y_5(0) = B, y_6(0) = 1, y_7(0) = C$. Where are A, B, C assumed value to get the solution. This approach converts a boundary value problem into an initial value problem (IVP). Then IVP is resolved by assuming omitted beginning value for parameters via an efficient shooting technique. The results are obtained represented by tables and graphically. Also, main features are analyzed and discussed.

RESULT AND DISCUSSION

The obtained numerical and graphical results are discussed in this section as follows.

Table 1 indicates comparison of $\frac{-\theta'(0)}{\sqrt{2}}$ with the earlier results [17] for unlike values of Pr with porous medium parameter (Λ). With the values of $0.1 \leq Nb, Nt \leq 0.5$ and $1 \leq Le \leq 40$. It is noticed that the present obtained solution is better than existing literature results.

Table 1. Comparison values of $-\theta'(0)/\sqrt{2}$ with porous medium parameter for alike values of Pr with $\varepsilon = Nb = Nt = Ec = Le = 0$ and $\Lambda = 0.5$.

Pr	[17]	Present study
0.7	0.292680	0.2084
0.8	0.306917	0.2179
1	0.332057	0.2347
5	0.576689	0.4014
10	0.728141	0.5057

The values of C_f, Nur and Shr for various values of ε presented in Table 2. It analyzed that with the rise in plate velocity parameter ε results to increase in C_f , but a decrease in Nur and Shr , respectively.

Table 2. Values of Nur, Shr and C_f with porous medium parameter for diverse values of ε with $Pr = 7, Nb = Nt = Ec = 0.1, Le = 10$ and $\Lambda = 0.5$.

ε	Nur	Shr	C_f
0	-0.0236	0.0160	0.4647
0.1	-0.0293	0.0111	0.5313
0.5	-0.0436	0.0018	0.7455
1	-0.0505	-0.0016	0.9377
2	-0.0492	-0.0009	1.1540

Table 3 indicated analysis of reduced nusselt number Nur and Sherwood number Shr with diverse values of Nb and Nt . It is concluded that as Nb and Nt increases results into decrease in Nur while Shr decreases and increases with increasing in Nb and Nt , respectively.

Table 3. Values of Nur , Shr with porous medium parameter for diverse values of Nb and Nt with $Pr = 7$, $Ec = 0.1$, $Le = 10$, $\varepsilon = 0.5$ and $\Lambda = 0.5$.

Nb	Nt	Nur	Shr
0.1	0.1	-0.0436	0.0018
0.2	0.1	-0.0437	0.0009
0.3	0.1	-0.0437	0.0006
0.4	0.1	-0.0438	0.0005
0.5	0.1	-0.0438	0.0004
0.1	0.2	-0.0438	0.0032
0.1	0.3	-0.0440	0.0044
0.1	0.4	-0.0442	0.0053
0.1	0.5	-0.0445	0.0061

From fig. 2. impact of porous medium parameter (Λ) on velocity distribution is represented. From this figure, With an enhancement in Λ , the velocity profile appears to grow. Because increasing porosity lowers the amplitude of the Darcian body force and raises fluid mobility in the boundary layer, this is the case.

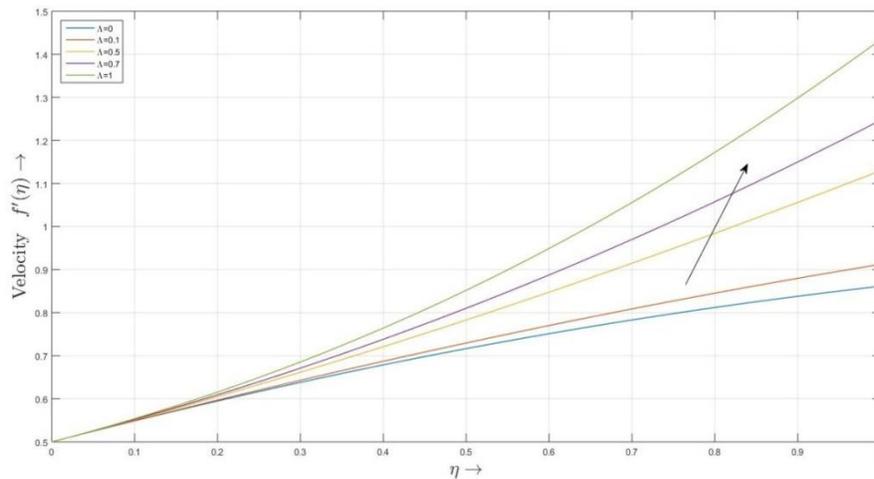


Fig. 2 $f'(\eta)$ Vs η for different Λ

Fig 3 denotes the role of porous medium parameter (Λ) on temperature distribution. As can be seen that, temperature distribution decreases as increases. Due to this reason the density of the base fluid is inversely proportional to permeability parameter, as porosity increases, density decreases and temperature decreases.

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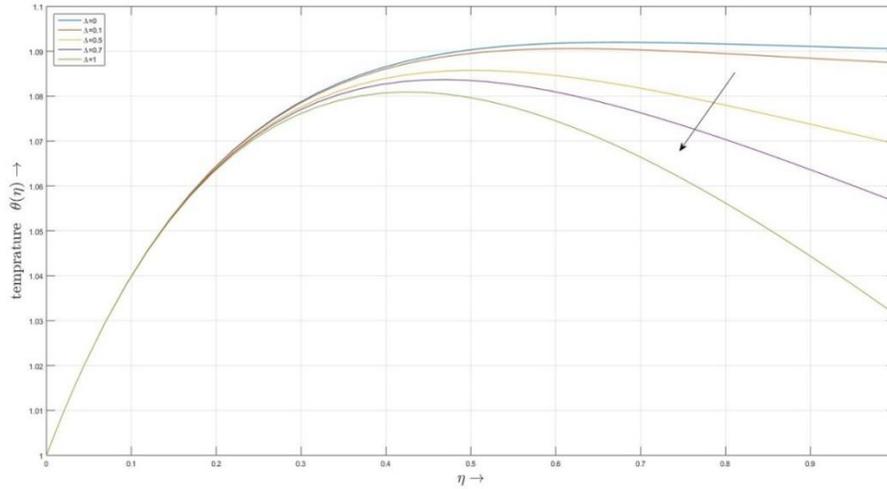


Fig. 3 $\theta(\eta)$ Vs η for different Λ

Fig. 4. signify the influence of porous medium parameter (Λ) on Nanoparticle Volume friction. It is noticed that as Λ increases, concentration (nano particle Volume friction) profile is also enhanced.

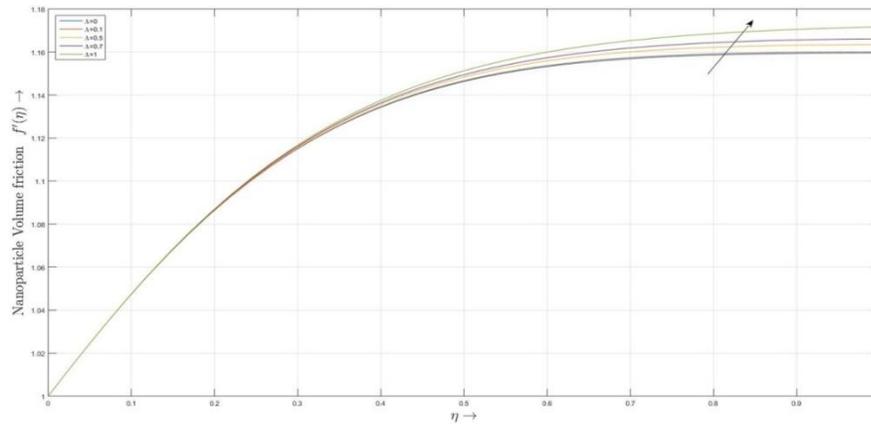


Fig. 4 $\phi(\eta)$ Vs η for different Λ

Fig 5 indicates the role of plate velocity parameter (ϵ) on temperature profile. As the value of ϵ grows up, the thickness of corresponding boundary layer decline. The temperature finally begins to drop. The difference in velocity between the plate and the fluid, in a physical sense, permits the fluid to move farther away from the plate region more hastily. Finally, thermal boundary layer is becoming thinner.

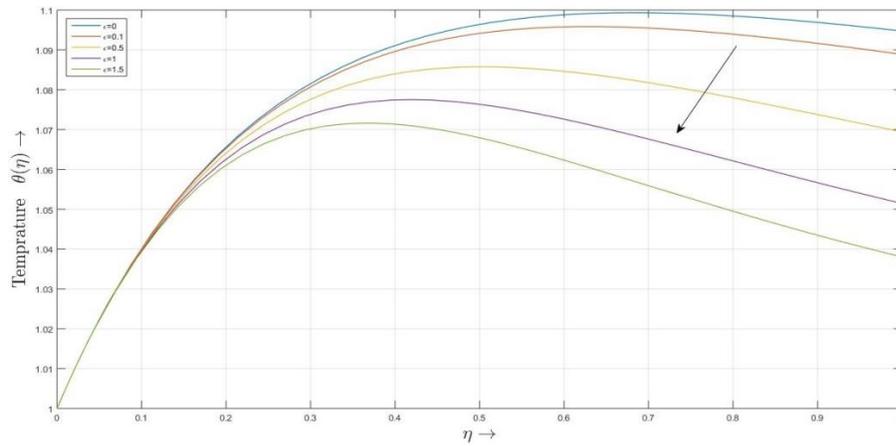


Fig. 5 $\phi(\eta)$ Vs η for different ε

Fig. 6. observed the influence of plate velocity parameter (ε) on reduced skin friction coefficient C_f with $f'(0) = \varepsilon$ and $f' \rightarrow 1$ as $\eta \rightarrow \infty$. It is analyzed that C_f is negative for $\varepsilon > 0$ and $C_f \rightarrow 0$ as ε reaches unity. Therefore C_f decreases for increasing value of ε .

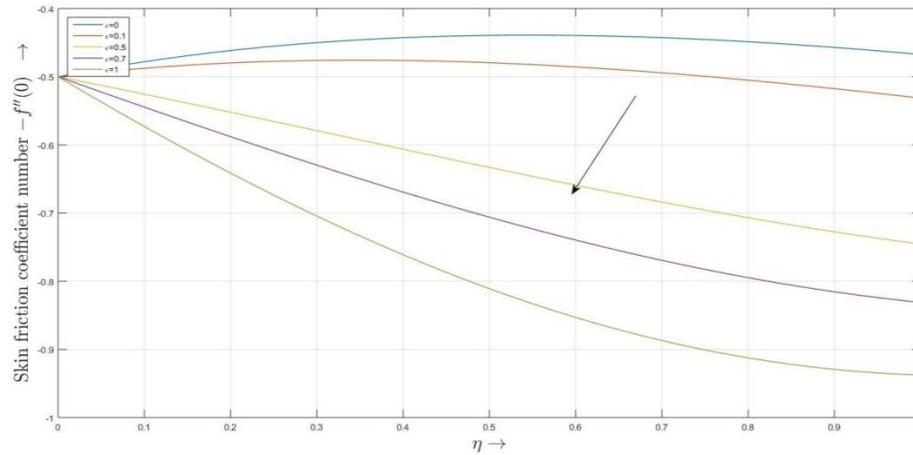


Fig. 6 $f''(\eta)$ Vs η for different ε

Fig. 7. and 8. indicates the impact on reduced nusselt number Nur and sherwood Shr with ε for unlike values of Ec in the presence of Λ respectively. From the figures it is analyzed that with the viscous dissipation term Ec changes the behavior of Nur . As Ec increases, reduced nusselt number Nur also enhanced. For the case $Nur = 0$, there is no convection takes place i.e., pure conduction condition holds. Also, variation of reduced Sherwood number Shr diminishes when viscous dissipation term Ec increased. But after some particular value of ε , it behaves like a constant.

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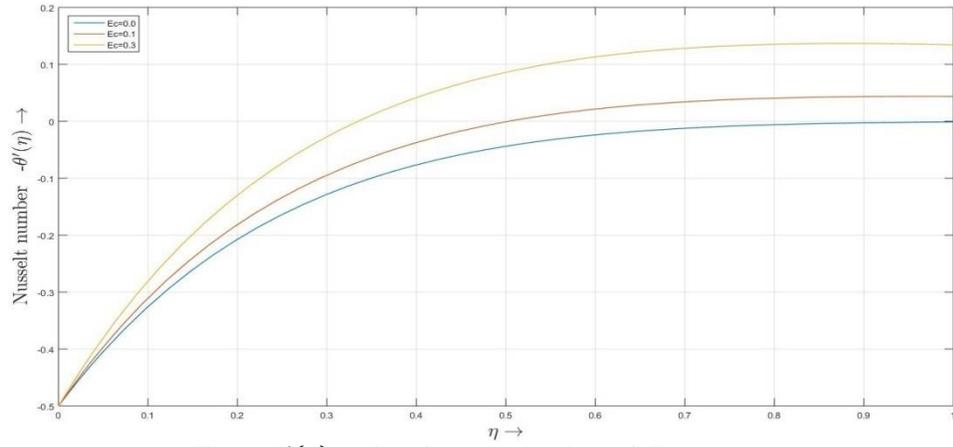


Fig. 7 - $\theta'(\eta)$ with ε for various values of Ec

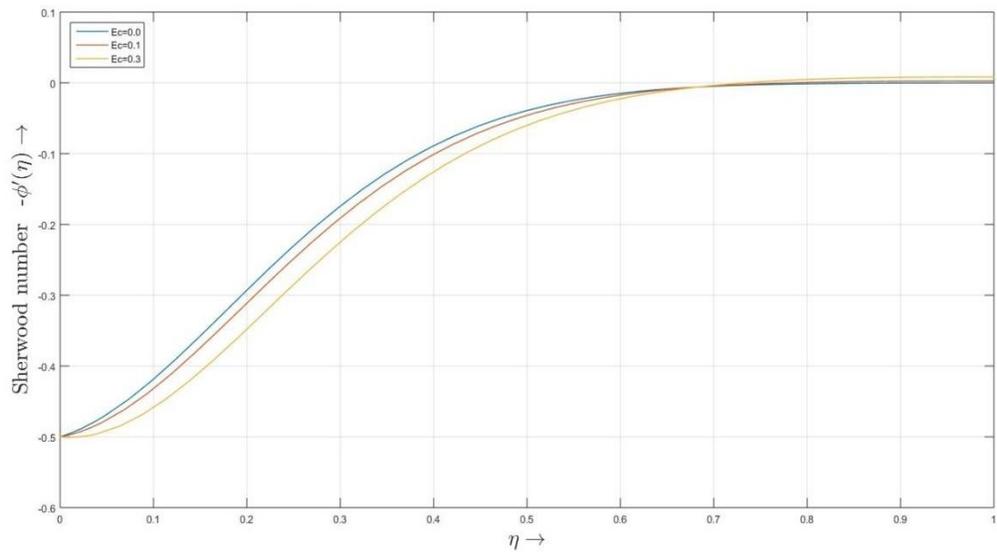


Fig. 8 - $\phi'(\eta)$ with ε for various values of Ec

Fig. 9. and 10. shows the deviation of reduced nusselt number Nur and Sherwood Shr with Pr for diverse values of Le in the incidence of Λ . From these figures, it is noticed that Nur decline and Shr increases with Pr for the increase in values of Le respectively.

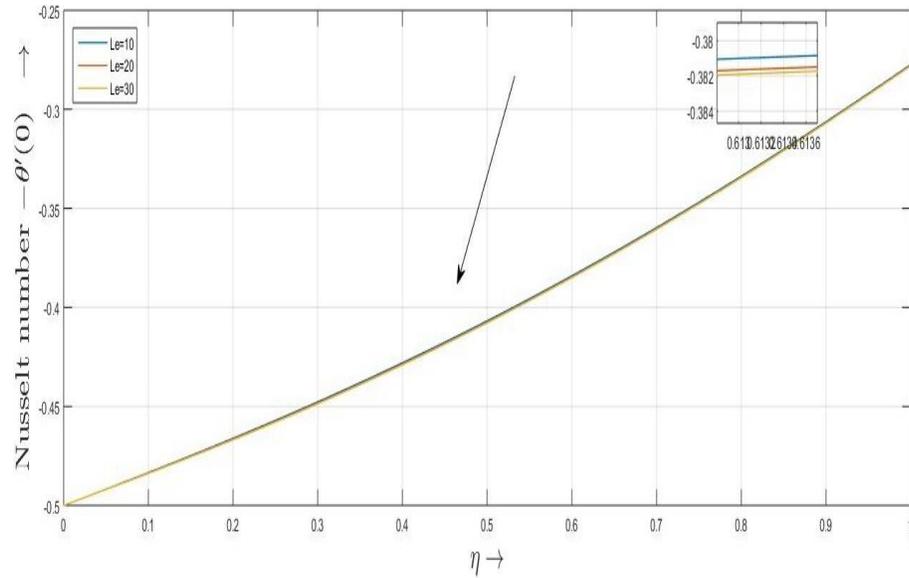


Fig. 9 - $\theta'(\eta)$ with Pr for various values of Le

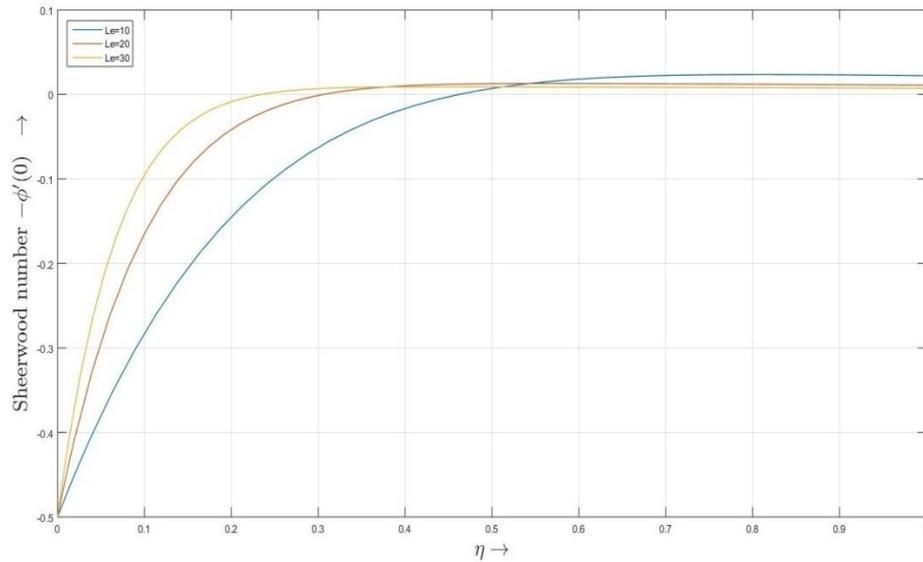


Fig. 10- $\phi'(\eta)$ with Pr for various values of Le

Fig. 11. analyzed the effect of various values of Ec and Pr on nanoparticle volume friction $\phi(\eta)$. It analyzes that as Ec rises nanoparticle volume friction falls down in the presence of constant prandtl number Pr . Also as Pr increases, the nanoparticle volume friction is reduced with a constant value of Ec .

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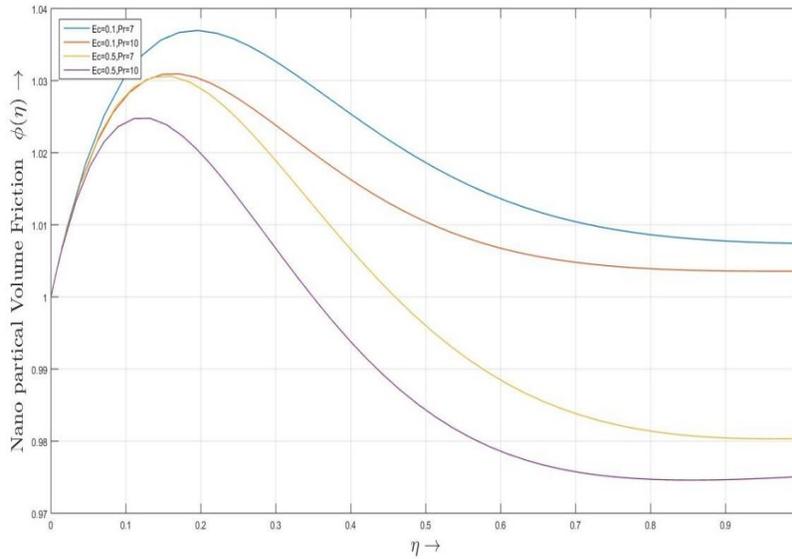


Fig. 11 $\phi(\eta)$ for various values of Ec and Pr

Fig. 12. indicates deviation of nanoparticle volume friction for Nb and ϵ . It is noticed that nanoparticle volume friction decrease as Nb and ϵ increases. The moving effects reduced the nanoparticle volume friction profile.

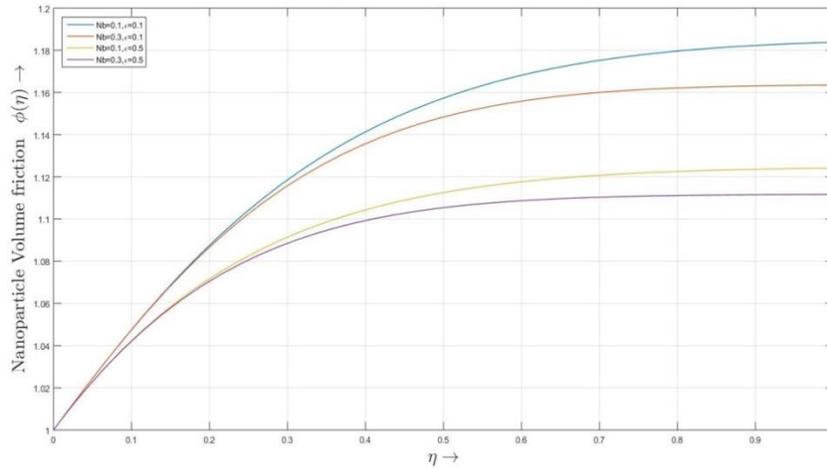


Fig. 12 $\phi(\eta)$ for various values of Nb and ϵ

The effect of Nt and Le on nanoparticle volume friction illustrated in fig. 13. From the figure it is analyze that nanoparticle volume friction is enhanced with rising in Nt while opposite trend is followed by nanoparticle volume friction for Le , i.e. as Le increase the nanoparticle volume friction diminishes.

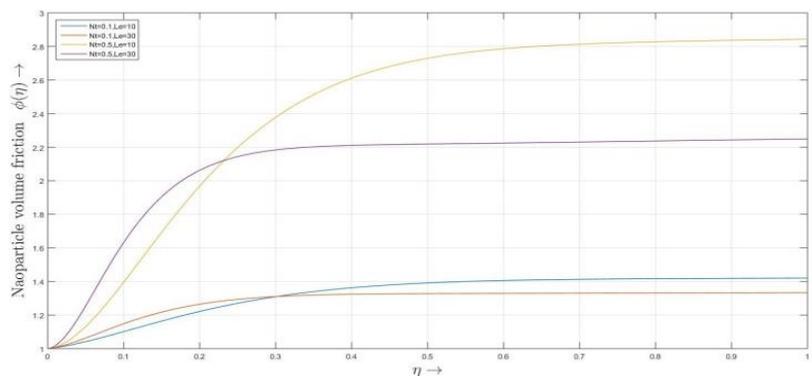


Fig. 13 $\phi(\eta)$ for various values of Nt and Le

The role of Le and Ec on temperature profiles is noticed in fig. 14. The thickness of thermal boundary layer decline with the rise in Le . In occurrence of porous medium parameter Λ as Ec increase, the temperature profile is reduced.

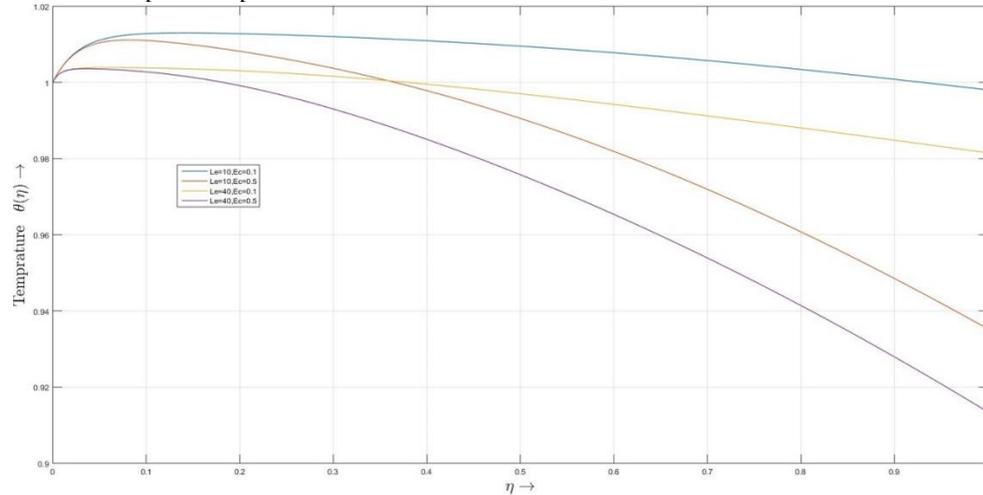


Fig.14. $\theta(\eta)$ for various values of Le and Ec

The behavior of Nb and Nt parameters on temperature is presented in fig. 15. It is notice that as Nb and Nt parameters increases results to decreases in temperature physically there is decay in thickness of thermal boundary layer thickness with Λ .

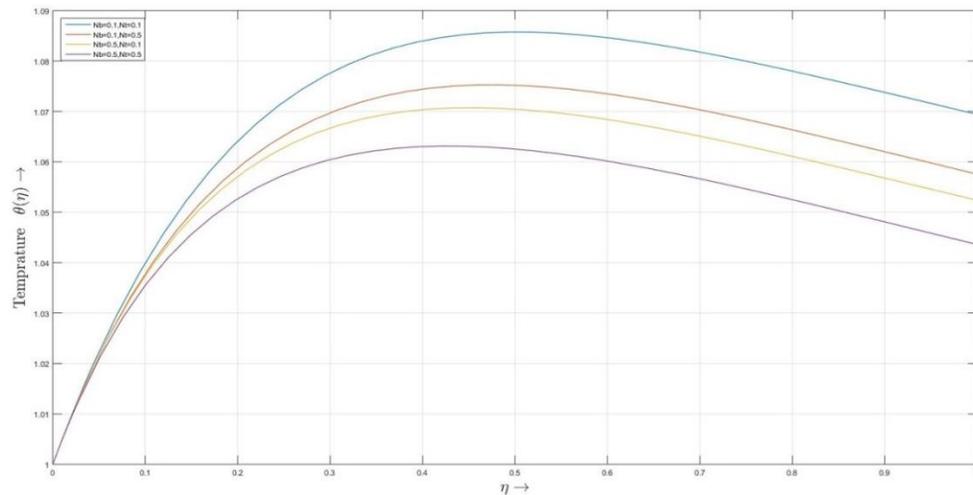


Fig. 15 $\theta(\eta)$ different values of Nb and Nt

CONCLUSION

In the present study boundary layer nanofluid flow over a moving plate with viscous dissipation in the presence of saturated porous medium is considered. The similarity transformations were used to derive the dimensionless boundary value problem. The boundary layer fluid flow problem are governing by various parameters like Brownian motion (Nb), thermophoresis (Nt), lewis number (Le), Prandtl number (Pr), Eckert number (Ec), plate velocity parameter (ϵ), porous medium parameter (Λ). Numerical simulation was given by the 4th order

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Runge-Kutta shooting technique with aid of ODE45 solver. The key outcomes are listed below.

- i. The velocity and concentration profile elevates while the temperature profile decreases with increasing A .
- ii. Skin friction and temperature distribution both are diminish as plate velocity parameter ε increases.
- iii. Nur decreases and Shr increases for increasing in Le with constant value of Pr .
- iv. Nanoparticle volume friction decreases as Ec increases with a constant value of Pr .
- v. Nanoparticle volume friction decreases with increasing in Brownian and velocity ratio parameters.
- vi. Temperature profile follows the reverse trend with Le and Ec .

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