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FUZZY SOFT TRI-PARTITE GRAPHS AND ITS COMPLEMENT

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Abstract. Fuzzy soft graphs are efficient numerical tools for simulating the uncertainty of the real world. A fuzzy soft graph is a perfect fusion of the fuzzy soft set and the graph model that is widely used in a variety of fields. This paper discusses a few unique notions of Fuzzy Soft Tri-partite Graphs (FSTG), as well as the concepts of Complement of Fuzzy Soft Tri-partite Graphs (CFSTG). Because soft sets are most useful in real-world applications, the newly developed concepts of soft tripartite fuzzy graphs will lead to many theoretical applications by adding extra fuzziness in analyzing. The authors look at some of their properties and come up with a few results that are related to these concepts.Furthermore, the authors investigated some fundamental theorems and illustrated an application of size of fuzzy soft tripartite graphs in employee selection for an institution using the fuzzy soft tripartite graph.

KEYWORDS: Fuzzy Soft Tri-partite Graphs (FSTG), Complement of Fuzzy Soft Tri-partite Graphs(CFSTG).

1. Introduction

Zadeh's [31] [33] [34] [35] fuzzy set theory, established in 1965, is the best explanation for interacting with sources of uncertainty. Kotzig *et al.*[9] defined the properties of magic graphs in 1970. Vertices represent the elements, while edges express the relationships. Rosenfield [11,12,24] created the notion of fuzzy graphs in 1975, providing an overview of fuzzy sets to graph theory.In 1986, Honda *et al.*[8] have discussed some fuzzy set concepts and applications. A graph is a simple way to communicate data and the relationship between various types of substances.In 1987, Bhattacharya[5,13,14] examined fuzzy graphs and made some remarkable observations. So many researchers introduced numerous maintainable and remarkable concepts in fuzzy graphs. In 1994, Moderson introduced the concepts of complement of fuzzy graphs.

In 1999, Molodtsov[19] created a fuzzy soft theory to rectify inexact technological difficulties in social science, entrepreneurship, medicine, and the surroundings.He also applied this theory to a variation of many other areas, including crispness of function, game theory.In recent years, soft set research has been very active, with researchers from all over the world participating. In 2001, Maji *et al.* [15,16,17,18] proposed the concept of fuzzy soft sets, which are a perfect combination of a fuzzy set and a soft set. In 2009, Aygünolu *et al.* [1,10] have given some ideas in Mathematics with Applications.In 2013, Ghosh *et al.* [7,9] proposed some ideas on the operations of intuitionistic fuzzy soft sets. In 2014, Rajesh *et al.* [23] proposed the notion of soft graphs and also they investigated some of their characteristics.In the year 2015, Akram *et al.* [2,5] proposed fuzzy soft graphs developed FSG independently. They present the concept of FSG as well as a few properties related to them in their paper.All the concepts of FSG(Strong, Complete, Regular) have been introduced by Akram *et al.*[2,6,7].In 2016, Akram *et al.*, [3,8] presented the concepts of FSG, and its applications in social networks and road networks. They also explained into different types of arcs in FSG. Al-Masarwah *et al.*[4] introduced some new notions in CFSG. T.K.Mathew Varkey *et al.*[16] discussed some notes on Bipartite and Balanced Fuzzy Graph Structures in 2017. In 2018, N. Sarala *et al.* [30,31] proposed some applications of Fuzzy soft Bi-partite graphs. In 2019, Khan MJ *et al.* [15] have described picture fuzzy soft sets and their implementation in software applications. Sarala *et al.* [28, 29] gave some ideas in Bipolar and intuitionistic Fuzzy Soft Digraph in 2019. Sarala.N *et al.*[27], Fuzzy Soft Digraph Implementation in Genetic Eye and Hair Color in 2020. In the same year Shanmugavadivu & Gopinath gave some numerical ideas of degree of equations. Kalaiarasi & Geethanjali analysed and introduced Fuzzy inventory and Kalaiarasi & Gopinath discussed arc sequences in different graphs and explained the join product in mixed split IFGs.In 2020 Priyadharshini *et al.*[22] explained some ideas in fuzzy MCDM approach for measuring the business.The Fuzzy Soft Tri-Partite Graphs and its Complement are combined in this study to form a unique mathematical model and Mapreduce Methodology for Elliptical Curve Discrete Logarithmic Problems [32].

Only the principles and applications of fuzzy soft bipartite graphs were discussed previously. Later, discovered a novel size concept that was only used for fuzzy soft tri-partite graphs. Following that, the concepts were applied to the complement of fuzzy soft tri-partite graphs. Fuzzy soft-tripartite graphs produce better results than fuzzy soft bipartite graphs. The value of the fuzzy soft tripartite graph is larger than that of the fuzzy soft bipartite graph, which gives exact maximum values.

A few main ideas of FSTG and Its Complement are also discussed briefly. The following is how the paper is positioned out: Section 2 discusses the fundamental concepts of fuzzy graph theory. Section 3 introduces the notion of FSTGs. Section 4 describes the concept of CFSTGs. Finally, Section 5 illustrates a model for implementing these FSTG. Section 6 contains the conclusion.

2. Preliminaries

Definition 2.1 [2]

A fuzzy graph is an ordered triple $G_F(V_F, \sigma_F, \mu_F)$ where V_F is a set of vertices $\{u_{F_1}, u_{F_2}, \ldots\ldots, u_{F_n}\}$ and σ_F is a fuzzy subset of V_F that is $\sigma_F: V_F \to [0,1]$ and is denoted by $\sigma_F = \{(u_{F_1}, \sigma_F(u_{F_1})), (u_{F_2}, \sigma(u_{F_2})), \ldots\ldots, (u_{F_n}, \sigma(u_{Fn}))\}$ and μ_F is a fuzzy relation on σ_F.

Definition 2.2[12]

Let $V = \{x_1, x_2, \ldots\ldots x_n\}$ non empty set. E (Parameters set) and $A \subseteq E$ also let

(i)$\rho: A \to F(V)$ (collection of all fuzzy subsets in V) $e \to \rho(e) = \rho_e$ (say)and $\rho_e: V \to [0,1], X: \to \rho_e(X_i)(A, \rho)$: *Fuzzy soft vertex.*

(ii)$\mu: A \to F(V \times V)$ (collection of all fuzzy subsets in $(V \times V) e \to \mu(e) = \mu_e$ (say)

And $\mu_e: v \times v \to [0,1](x_i, x_j) \to \mu_e(x_i, x_j)(A, \mu)$: *Fuzzy soft edge*. Then $((A, \rho), (A, \mu))$ is called a *fuzzy soft graph* if f$\mu_e(x_i, x_j) \leq \rho_e(x_i) \wedge \rho_e(x_j)$ for all $e \in A$ and for all $i, j = 1,2, \ldots, n$ and this fuzzy soft graph is denoted by $G_{A,V}$.

Definition 2.3[18]

A fuzzy soft graph $G_{A,V} = ((A, \rho), (A, \mu))$ is said be a fuzzy soft Bi-partite graph. If the vertex set $V$is partition into two disjoint vertex pair and$\mu_e(x_i, x_j) = \rho_e(x_i) \wedge \rho_e(x_j)$ for all $x_i \in v_i$and$y_j \in v_j$

Definition 2.4[18]

If a fuzzy soft graph $G_{A,V} = ((A, \rho), (A, \mu))$ is said be a fuzzy soft Bi-partite graph, then Size of Fuzzy soft bipartite graphis

$S(G_{A,V \cup v_{j_i}})$ =$\sum_{e \in A}(\sum_{x_i y_j \in V_i \cup v_j} \mu_e(x_i, x_j))$.

3. Fuzzy Soft Tri-Partite Graphs

Definition 3.1. A FSG$G_{A,V} = \big((A, \wp), (A, \Im)\big)$ is said to be a FSTG. If the vertices can be partitioned into 3 disjoint vertex pair and

$$\Im_e(x_{ii}, y_{jj}) \leq min\{\wp_e(x_{ii}) + \wp_e(y_{jj})\}$$

$$\Im_e(y_{jj}, z_{kk}) \leq min\{\wp_e(y_{jj}), \wp_e(Z_{kk})\}$$

$$\Im_e(z_{kk}, x_{ii}) \leq min\{\wp_e(z_{kk}), \wp_e(x_{ii})\}$$

It is denoted by $\left(G_{A,V_i \cup V_j \cup V_K}\right)$

Definition 3.2. If a FSG $G_{A,V} = \big((A, \wp), (A, \Im)\big)$ is said to be a FSTG, then size of FSTG is

$$S\left(G_{A,V_i \cup V_j \cup V_K}\right) = \sum_{e \in A} \sum_{a_i b_j c_K \in V_i U V_j U V_k} \Im_e(a_i, b_j, c_k)$$

Where $S[F(e_1)] = \sum_{a_i b_j c_K \in V_i U V_j U V_k} \Im_{e_1}(a_i, b_j, c_k)$

$$S[F(e_2)] = \sum_{a_i b_j c_K \in V_i U V_j U V_k} \Im_{e_2}(a_i, b_j, c_k)$$

$$S[F(e_3)] = \sum_{a_i b_j c_K \in V_i U V_j U V_k} \Im_{e_3}(a_i, b_j, c_k)$$

Definition 3.3. If a FSG$G_{A,V} = \big((A, \wp), (A, \Im)\big)$ is said to be a FSTG, then degree of FSTG is,$d(a_i) = \sum_{e \in A} \sum_{b_j c_k \in v_j \cup v_k} \Im_e(b_j, c_k)$

$$d(b_j) = \sum_{e \in A} \sum_{a_j c_k \in v_j \cup v_k} \Im_e(a_i, c_k)$$

$$d(c_k) = \sum_{e \in A} \sum_{a_i b_j \in v_i \cup v_j} \Im_e(a_i, b_j)$$

Definition 3.4. If a FSG$G_{A,V} = \big((A, \wp), (A, \Im)\big)$ is said to be a FSTG, then order of FSTG is, $O\left(G_{A,V_i \cup V_j \cup V_k}\right) = \sum_{e \in A} \sum_{a_i b_j c_K \in V_i U V_j U V_k} \wp_e(a_i, b_j, c_k)$

Where

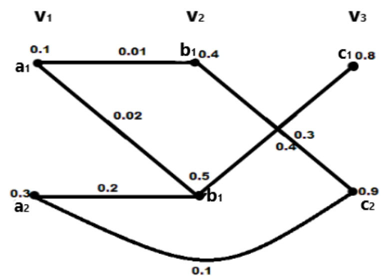
$$O[F(e_1)] = \sum_{a_i b_j c_K \in V_i U V_j U V_k} \wp_{e_1}(a_i, b_j, c_k)$$

$$O[F(e_2)] = \sum_{a_i b_j c_K \in V_i U V_j U V_k} \wp_{e_2}(a_i, b_j, c_k)$$

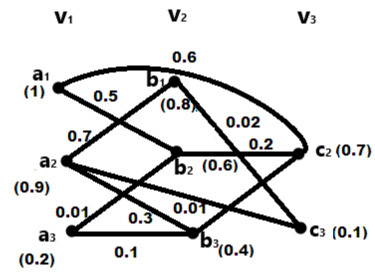
$$O[F(e_3)] = \sum_{a_i b_j c_K \in V_i U V_j U V_k} \wp_{e_3}(a_i, b_j, c_k)$$

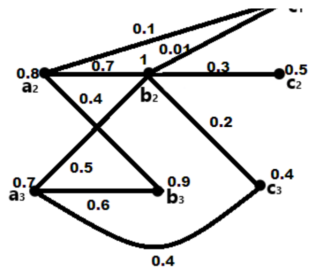
Example 3.1

$F(e_1)$



$F(e_2)$





$F(e_3)$

ρ	a_1	a_2	a_3	b_1	b_2	b_3	c_1	c_2	c_3
e_1	0.1	0.3	0	0.4	0.5	0	0.8	0.9	0
e_2	1	0.9	0.2	0.8	0.6	0.4	0	0.7	0.1
e_3	0	0.8	0.7	0	1	0.9	0.2	0.5	0.4

μ	e_1	e_2	e_3
a_1b_1	0.01	0	0
a_1b_2	0.02	0.5	0
a_1c_2	0	0.6	0
a_2b_1	0	0.7	0
a_2b_2	0.2	0	0.7
a_2b_3	0	0.3	0.4
a_2c_1	0	0	0.1
a_2c_2	0.1	0	0
a_2c_3	0	0.01	0
a_3b_2	0	0.3	0.5
a_3b_3	0	0.1	0.6
a_3c_3	0	0	0.4
b_1c_2	0.3	0	0
b_1c_3	0	0.02	0
b_2c_1	0.4	0	0.01

b_2c_2	0	0.02	0.3
b_2c_3	0	0	0.2
b_3c_2	0	0.03	0

Size of FSTG

$$S[F(e_1)] = 1.03 S[F(e_2)] = 2.47 S[F(e_3)] = 3.21$$

$$\therefore S\left(G_{A,V_i \cup V_j \cup V_k}\right) = 6.71$$

Order of FSTG

$$O[F(e_1)] = 3 \qquad O[F(e_2)] = 4.7 O[F(e_3)] = 4.5$$

$$\therefore O\left(G_{A,V_i \cup V_j \cup V_k}\right) = 12.2$$

Degree of FSTG

$$d(a_1) = 1.13 d(a_2) = 2.51 d(a_3) = 1.53$$

$$d(b_1) = 0.32 d(b_2) = 0.93 d(b_3) = 0.03$$

4. Complement of fuzzy soft tri-partite graph

Definition 4.1. A FSTG $G_{A,V_i \cup V_j \cup V_k}$ is known as a strong fuzzy soft tri-partite graph (SFSTG) if

$$\Im_e(x_{ii}, y_{jj}) = min\{\wp_e(x_{ii}) + \wp_e(y_{jj})\}$$

$$\Im_e(y_{jj}, z_{kk}) = min\{\wp_e(y_{jj}), \wp_e(Z_{kk})\}$$

$$\Im_e(z_{kk}, x_{ii}) = min\{\wp_e(z_{kk}), \wp_e(x_{ii})\} \forall e \in A$$

and is complete FSTG if

$$\Im_e(x_{ii}, y_{jj}) = min\{\wp_e(x_{ii}) + \wp_e(y_{jj})\}$$

$$\Im_e(y_{jj}, z_{kk}) = min\{\wp_e(y_{jj}), \wp_e(Z_{kk})\}$$

$$\Im_e(z_{kk}, x_{ii}) = min\{\wp_e(z_{kk}), \wp_e(x_{ii})\} \forall e \in A$$

Definition 4.2. Let $G_{A,V_i \cup V_j \cup V_k}$ be a FSTG. Then $G_{A,V_i \cup V_j \cup V_k}$ is called isolated fuzzy soft tri-partite graph (IFSTG)if

$\Im_e(x_{ii}, y_{jj}) = 0,$ $\Im_e(y_{jj}, z_{kk}) = 0 \;\; and$ $\Im_e(z_{kk}, x_{ii}) = 0 \rightleftarrows\rightleftarrows \forall x_{ii}, y_{jj}, z_{kk} \in V \times V and e \in A$

Definition 4.3. Let $G_{A,V_i \cup V_j \cup V_K}$ be a FSTG. The CFSTG is defined as $\overline{G}_{A,V_i \cup V_j \cup V_K}$ where

$$\overline{\Im_e}(x_{ii}, y_{jj}) = min\{\wp_e(x_{ii}), \wp_e(y_{jj})\} - \Im_e(x_{ii}, y_{jj})$$

$$\overline{\Im_e}(y_{jj}, z_{kk}) = min\{\wp_e(y_{jj}), \wp_e(z_{kk})\} - \Im_e(y_{jj}, z_{kk})$$

$$\overline{\Im_e}(z_{kk}, x_{ii}) = min\{\wp_e(z_{kk}), \wp_e(x_{ii})\} - \Im_e(z_{kk}, x_{ii})$$

Definition 4.4.

A CFSTG $\overline{G}_{A,V_i \cup V_j \cup V_K}$ is called SFSTG if

$$\overline{\Im_e}(x_{ii}, y_{jj}) = min\{\wp_e(x_{ii}), \wp_e(y_{jj})\}$$

$$\overline{\Im_e}(y_{jj}, z_{kk}) = min\{\wp_e(y_{jj}), \wp_e(z_{kk})\}$$

$$\overline{\Im_e}(z_{kk}, x_{ii}) = min\{\wp_e(z_{kk}), \wp_e(x_{ii})\} \forall e \in A$$

and is complete FSTG if

$$\overline{\Im_e}(x_{ii}, y_{jj}) = min\{\wp_e(x_{ii}), \wp_e(y_{jj})\}$$
$$\overline{\Im_e}(y_{jj}, z_{kk}) = min\{\wp_e(y_{jj}), \wp_e(z_{kk})\}$$
$$\overline{\Im_e}(z_{kk}, x_{ii}) = min\{\wp_e(z_{kk}), \wp_e(x_{ii})\} \forall e \in A$$

Proposition 4.1

$$S\left(\overline{G}_{A,V_i \cup V_j}\right) + S\left(G_{A,V_i \cup V_j}\right) \leq 2\sum_{e\in A} \sum_{x_{ii}\neq y_{jj}} \wp_e(x_{ii}) \wedge \wp_e(y_{jj})$$
$$S\left(\overline{G}_{A,V_j \cup V_k}\right) + S\left(G_{A,V_j \cup V_k}\right) \leq 2\sum_{e\in A} \sum_{y_{jj}\neq z_{kk}} \wp_e(y_{jj}) \wedge \wp_e(z_{kk})$$
$$S\left(\overline{G}_{A,V_k \cup V_i}\right) + S\left(G_{A,V_k \cup V_i}\right) \leq 2\sum_{e\in A} \sum_{x_{ii}\neq z_{kk}} \wp_e(z_{kk}) \wedge \wp_e(x_{ii})$$

Proof.

Since $\Im_e(x_{ii}, y_{jj}) \leq \wp_e(x_{ii}) \wedge \wp_e(y_{jj})$________________(1)

$$\overline{\Im_e}(x_{ii}, y_{jj}) = \wp_e(x_{ii}) \wedge \wp_e(y_{jj}) - \Im_e(x_{ii}, y_{jj})$$

Also, $\overline{\Im_e}(x_{ii}, y_{jj}) \leq \wp_e(x_{ii}) \wedge \wp_e(y_{jj})$________________(2)

From (1) and (2) we've

$$\Im_e(x_{ii}, y_{jj}) + \overline{\Im_e}(x_{ii}, y_{jj}) \leq 2[\wp_e(x_{ii}) \wedge \wp_e(y_{jj})]$$

Now $\sum_{e\in A} \sum_{x_{ii}\neq y_{jj}} \left(\Im_e(x_{ii}, y_{jj}) + \overline{\Im_e}(x_{ii}, y_{jj})\right) \leq \sum_{e\in A} \sum_{x_{ii}\neq y_{jj}} 2[\wp_e(x_{ii}) \wedge \wp_e(y_{jj})]$

$$\sum_{e\in A} \sum_{x_{ii}\neq y_{jj}} \Im_e(x_{ii}, y_{jj}) + \sum_{e\in A} \sum_{x_{ii}\neq y_{jj}} \overline{\Im_e}(x_{ii}, y_{jj}) \leq 2\sum_{e\in A} \sum_{x_{ii}\neq y_{jj}} \wp_e(x_{ii}) \wedge \wp_e(y_{jj})$$

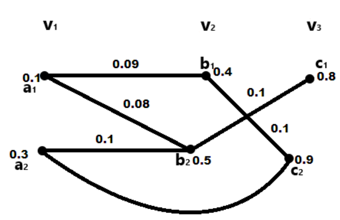
Hence

$$S\left(G_{A,V_i \cup V_j}\right) + S\left(\overline{G}_{A,V_i \cup V_j}\right) \leq 2\sum_{e\in A} \sum_{x_{ii}\neq y_{jj}} [\wp_e(x_{ii}) \wedge \wp_e(y_{jj})]$$

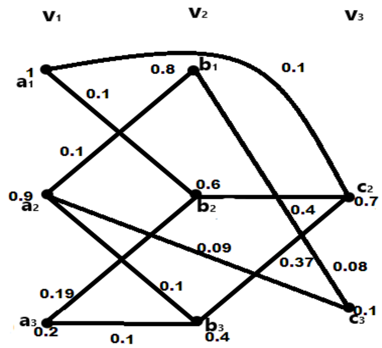
Similarly

$$S\left(G_{A,V_j \cup V_K}\right) + S\left(\overline{G}_{A,V_j \cup V_K}\right) \leq 2\sum_{e\in A} \sum_{y_{jj}\neq z_{kk}} [\wp_e(y_{jj}) \wedge \wp_e(z_{kk})]$$
$$S\left(G_{A,V_K \cup V_i}\right) + S\left(\overline{G}_{A,V_K \cup V_i}\right) \leq 2\sum_{e\in A} \sum_{x_{ii}\neq z_{kk}} [\wp_e(x_{ii}) \wedge \wp_e(z_{kk})]$$

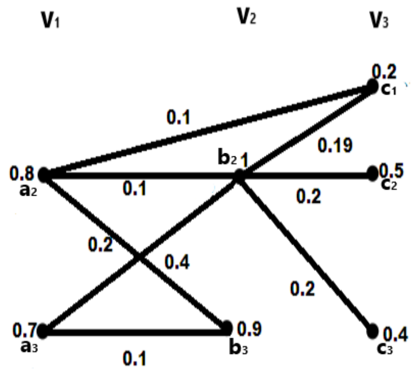
$F(\overline{e_1})$



$F(\overline{e_2})$



$F(\overline{e_3})$



$$S\left(\overline{G}_{A,V_i\cup V_j}\right) = 1.66 S\left(\overline{G}_{A,V_j\cup V_K}\right) = 1.64 S\left(\overline{G}_{A,V_K\cup V_i}\right) = 0.49$$

$$S\left(G_{A,V_i\cup V_j}\right) = 4.04 S\left(G_{A,V_j\cup V_K}\right) = 1.46 S\left(G_{A,V_K\cup V_i}\right) = 1.21$$

$$S\left(\overline{G}_{A,V_i\cup V_j}\right) = 5.7 S\left(\overline{G}_{A,V_j\cup V_K}\right) = 3.1 S\left(\overline{G}_{A,V_K\cup V_i}\right) = 1.7$$

$$\sum \wp_e(x_{ii}) \wedge \wp_e(y_{jj}) = 5.7 \sum \wp_e(y_{jj}) \wedge \wp_e(z_{kk}) = 3.1 \sum \wp_e(z_{kk}) \wedge \wp_e(x_{ii}) = 1.7$$

$$\therefore S\left(\overline{G}_{A,V_i\cup V_j}\right) + S\left(G_{A,V_i\cup V_j}\right) \leq 2 \Sigma_{e\in A} \quad \Sigma_{x_{ii}\neq y_{jj}} \quad [\wp_e(x_{ii}) \wedge \wp_e(y_{jj})]$$

Properties of complement of fuzzy soft tri-partite graph

1. Theorder of $\overline{G}_{A,V_i\cup V_j\cup V_K}$ is equal to the order of $G_{A,V_i\cup V_j\cup V_K}$

$O[F(e_1)] = 3 \quad O[F(e_2)] = 4.7 O[F(e_3)] = 4.5$

$$O\left(G_{A,V_i\cup V_j\cup V_K}\right) = 12.2$$

$O[F(\overline{e_1})] = 3 \quad O[F(\overline{e_2})] = 4.7 O[F(\overline{e_3})] = 4.5$

$$O\left(\overline{G}_{A,V_i\cup V_j\cup V_K}\right) = 12.2$$

$$\therefore O\left(\overline{G}_{A,V_i\cup V_j\cup V_K}\right) = O\left(G_{A,V_i\cup V_j\cup V_K}\right) = 12.2$$

2. The proportion of edge set elements of $\overline{G}_{A,V_i\cup V_j\cup V_K}$ is not exactly or equivalent to

the proportion of edge set elements of $G_{A,V_i\cup V_j\cup V_K}$.

3. Node set of $\overline{G}_{A,V_i \cup V_j \cup V_K}$ is same as the node set of $G_{A,V_i \cup V_j \cup V_K}$.

Number of vertices in the complement of fuzzy soft tripartite graph is equal to Number of vertices in the fuzzy soft tripartite graph

4. $S\left(\overline{G}_{A,V_i \cup V_j}\right) + S\left(G_{A,V_i \cup V_j}\right) = \sum_{e \in A} \sum_{x_{ii}, y_{jj} \in \Im} \wp_e(x_{ii}) \wedge \wp_e(y_{jj})$

$$S\left(\overline{G}_{A,V_j \cup V_K}\right) + S\left(G_{A,V_j \cup V_K}\right) = \sum_{e \in A} \sum_{y_{jj}, z_{kk} \in \Im} \wp_e(y_{jj}) \wedge \wp_e(z_{kk})$$

$$S\left(\overline{G}_{A,V_k \cup V_i}\right) + S\left(G_{A,V_k \cup V_i}\right) = \sum_{e \in A} \sum_{x_{ii}, z_{kk} \in \Im} \wp_e(z_{kk}) \wedge \wp_e(x_{ii})$$

$$S\left(\overline{G}_{A,V_i \cup V_j}\right) = 1.66 S\left(\overline{G}_{A,V_j \cup V_K}\right) = 1.64 S\left(\overline{G}_{A,V_K \cup V_i}\right) = 0.49$$

$$S\left(G_{A,V_i \cup V_j}\right) = 4.04 S\left(G_{A,V_j \cup V_K}\right) = 1.46 S\left(G_{A,V_K \cup V_i}\right) = 1.21$$

$S\left(\overline{G}_{A,V_i \cup V_j}\right) = 5.7 S\left(\overline{G}_{A,V_j \cup V_K}\right) = 3.1 S\left(\overline{G}_{A,V_K \cup V_i}\right) = 1.7 \sum \wp_e(x_{ii}) \wedge \wp_e(y_{jj}) = 5.7 \sum \wp_e(y_{jj}) \wedge \wp_e(z_{kk}) = 3.1 \sum \wp_e(z_{kk}) \wedge \wp_e(x_{ii}) = 1.7$

Theorem 4.1

The complement of a SFSTG is also SFSTG.

Proof.

Let $G_{A,V_i \cup V_j \cup V_K}$ be a SFSTG.

(i) $\overline{\Im_e}(x_{ii}, y_{jj}) = min\{\wp_e(x_{ii}), \wp_e(y_{jj})\}$

$$\overline{\Im_e}(y_{jj}, z_{kk}) = min\{\wp_e(y_{jj}), \wp_e(z_{kk})\}$$

$$\overline{\Im_e}(z_{kk}, x_{ii}) = min\{\wp_e(z_{kk}), \wp_e(x_{ii})\} \forall e \in A$$

By the definition of $\overline{G}_{A,V_i \cup V_j \cup V_K}$ is defined as,

$$\overline{\Im_e}(x_{ii}, y_{jj}) = min\{\wp_e(x_{ii}), \wp_e(y_{jj})\} - \Im_e(x_{ii}, y_{jj})$$

$$\overline{\Im_e}(y_{jj}, z_{kk}) = min\{\wp_e(y_{jj}), \wp_e(z_{kk})\} - \Im_e(y_{jj}, z_{kk})$$

$$\overline{\Im_e}(z_{kk}, x_{ii}) = min\{\wp_e(z_{kk}), \wp_e(x_{ii})\} - \Im_e(z_{kk}, x_{ii})$$

Now consider,

$$\overline{\Im_e}(x_{ii}, y_{jj}) = min\{\wp_e(x_{ii}), \wp_e(y_{jj})\} - \Im_e(x_{ii}, y_{jj})$$

$$= min\{\wp_e(x_{ii}), \wp_e(y_{jj})\} - min\{\wp_e(x_{ii}), \wp_e(y_{jj})\} \forall x_{ii}, y_{jj} \in V, e \in A$$

$$= \begin{cases} 0, \Im_e(x_{ii}, y_{jj}) > 0 \\ min\{\wp_e(x_{ii}), \wp_e(y_{jj})\} \Im_e(x_{ii}, y_{jj}) = 0 \end{cases}$$

$$\therefore \overline{\Im_e}(x_{ii}, y_{jj}) = 0,$$

$$\overline{\Im_e}(x_{ii}, y_{jj}) = min\{\wp_e(x_{ii}), \wp_e(y_{jj})\} \forall x_{ii}, y_{jj} \in V$$

Similarly

$$\overline{\Im_e}(y_{jj}, z_{kk}) = \begin{cases} 0 \\ min\left(\wp_e(y_{jj}), \wp_e(z_{kk})\right) \end{cases}$$

$$\overline{\Im_e}(z_{kk}, x_{ii}) = \begin{cases} 0 \\ min\left(\wp_e(z_{kk}), \wp_e(x_{ii})\right) \end{cases}$$

∴The complement of a SFSTG is also a SFSTG.

Theorem 4.2

The complement of a complete FSTG is also complete FSTG.

Proof.

Let $G_{A,V_i \cup V_j \cup V_K}$ be a compete FSTG.

$$\overline{\Im_e}(x_{ii}, y_{jj}) = min\{\wp_e(x_{ii}), \wp_e(y_{jj})\}$$

$$\overline{\Im_e}(y_{jj}, z_{kk}) = min\{\wp_e(y_{jj}), \wp_e(z_{kk})\}$$

$$\overline{\Im_e}(z_{kk}, x_{ii}) = min\{\wp_e(z_{kk}), \wp_e(x_{ii})\} \forall e \in A$$

By the definition of$\overline{G}_{A,V_i \cup V_j \cup V_K}$is defined as,

$$\overline{\Im_e}(x_{ii}, y_{jj}) = min\{\wp_e(x_{ii}), \wp_e(y_{jj})\} - \Im_e(x_{ii}, y_{jj})$$

$$\overline{\Im_e}(y_{jj}, z_{kk}) = min\{\wp_e(y_{jj}), \wp_e(z_{kk})\} - \Im_e(y_{jj}, z_{kk})$$

$$\overline{\Im_e}(z_{kk}, x_{ii}) = min\{\wp_e(z_{kk}), \wp_e(x_{ii})\} - \Im_e(z_{kk}, x_{ii})$$

Now consider,

$$\overline{\Im_e}(x_{ii}, y_{jj}) = min\{\wp_e(x_{ii}), \wp_e(y_{jj})\} - \Im_e(x_{ii}, y_{jj})$$

$$= min\{\wp_e(x_{ii}), \wp_e(y_{jj})\} -$$

$min\{\wp_e(x_{ii}), \wp_e(y_{jj})\} \forall x_{ii}, y_{jj} \in V, e \in A$

$$= \begin{cases} 0, \Im_e(x_{ii}, y_{jj}) > 0 \\ min\{\wp_e(x_{ii}), \wp_e(y_{jj})\} \Im_e(x_{ii}, y_{jj}) = 0 \end{cases}$$

$$\therefore \overline{\Im_e}(x_{ii}, y_{jj}) = 0,$$

$$\overline{\Im_e}(x_{ii}, y_{jj}) = min\{\wp_e(x_{ii}), \wp_e(y_{jj})\} \forall x_{ii}, y_{jj} \in V$$

Similarly

$$\overline{\Im_e}(y_{jj}, z_{kk}) = \begin{cases} 0 \\ min\left(\wp_e(y_{jj}), \wp_e(z_{kk})\right) \end{cases}$$ and$\overline{\Im_e}(z_{kk}, x_{ii}) =$

$\begin{cases} 0 \\ min(\wp_e(z_{kk}), \wp_e(x_{ii})) \end{cases}$

$\therefore$ The complement of a complete FSTG is also a complete FSTG.

Theorem 4.3

If $G_{A,V_i \cup V_j \cup V_K} = ((A, \wp), (A, \Im))$is a FSTG. Then$G_{A,V_i \cup V_j \cup V_K}$ is an IFSTG if f CFSTG is a complete FSTG.

Proof

Given $G_{A,V_i \cup V_j \cup V_K}$ is an IFSTG. Now consider $G_{A,V_i \cup V_j}$is an IFSTG.

Then $\Im_e(x_{ii}, y_{jj}) = 0 \rightarrow (1)$

We know that,

$$\overline{\Im_e}(x_{ii}, y_{jj}) = min\{\wp_e(x_{ii}), \wp_e(y_{jj})\} - \Im_e(x_{ii}, y_{jj}) \rightarrow (2)$$

Substitute (1) in (2)

$$\overline{\Im_e}(x_{ii}, y_{jj}) = min\{\wp_e(x_{ii}), \wp_e(y_{jj})\}$$

Similarly$\overline{\Im_e}(y_{jj}, z_{kk}) = min\{\wp_e(y_{jj}), \wp_e(z_{kk})\} \& \overline{\Im_e}(z_{kk}, x_{ii}) =$
$min\{\wp_e(z_{kk}), \wp_e(x_{ii})\}$

Hence the CFSTG is a complete FSTG. Conversely, Given the CFSTG is a complete FSTG.

(i.e) $\overline{\Im_e}(x_{ii}, y_{jj}) = min\{\wp_e(x_{ii}), \wp_e(y_{jj})\} \rightarrow (3)$

To prove,

$G_{A,V_i \cup V_j \cup V_k}$ is an IFSTG.

We know that,

$$\overline{\Im_e}(x_{ii}, y_{jj}) = min\{\wp_e(x_{ii}), \wp_e(y_{jj})\} - \Im_e(x_{ii}, y_{jj})$$

$$\Rightarrow \overline{\Im_e}(x_{ii}, y_{jj}) = min\{\wp_e(x_{ii}), \wp_e(y_{jj})\} - \Im_e(x_{ii}, y_{jj})$$

$$= min\{\wp_e(x_{ii}), \wp_e(y_{jj})\} - min\{\wp_e(x_{ii}), \wp_e(y_{jj})\} By (3)$$

$$\Im_e(x_{ii}, y_{jj}) = 0$$

Similarly

$$\overline{\Im_e}(y_{jj}, z_{kk}) = 0 \& \overline{\Im_e}(z_{kk}, x_{ii}) = 0$$

Hence $G_{A,V_i \cup V_j \cup V_K}$ is an IFSTG.

Theorem 4.4

If $G_{A,V_i \cup V_j \cup V_K} = ((A, \wp), (A, \Im))$ is a FSTG. Then $G_{A,V_i \cup V_j \cup V_K}$ is an IFSTG iff CFSTG is a SFSTG.

Proof

Given $G_{A,V_i \cup V_j \cup V_K}$ is an IFSTG. Now consider $G_{A,V_i \cup V_j \cup V_K}$ is an IFSTG.

Then $\Im_e(x_{ii}, y_{jj}) = 0 \rightarrow (1)$

We know that,

$$\overline{\Im_e}(x_{ii}, y_{jj}) = min\{\wp_e(x_{ii}), \wp_e(y_{jj})\} - \Im_e(x_{ii}, y_{jj}) \rightarrow (2)$$

Substitute (1) in (2)

$$\overline{\Im_e}(x_{ii}, y_{jj}) = min\{\wp_e(x_{ii}), \wp_e(y_{jj})\}$$

Similarly

$\overline{\Im_e}(y_{jj}, z_{kk}) = min\{\wp_e(y_{jj}), \wp_e(z_{kk})\} \& \overline{\Im_e}(z_{kk}, x_{ii}) = min\{\wp_e(z_{kk}), \wp_e(x_{ii})\}$ Hence the CFSTG is a SFSTG. Conversely, Given the CFSTG is a SFSTG.

(i.e) $\overline{\Im_e}(x_{ii}, y_{jj}) = min\{\wp_e(x_{ii}), \wp_e(y_{jj})\} \rightarrow (3)$

To prove,

$G_{A,V_i \cup V_j \cup V_K}$ is an IFSTG.

We know that,

$$\overline{\Im_e}(x_{ii}, y_{jj}) = min\{\wp_e(x_{ii}), \wp_e(y_{jj})\} - \Im_e(x_{ii}, y_{jj})$$

$$\Rightarrow \overline{\Im_e}(x_{ii}, y_{jj}) = min\{\wp_e(x_{ii}), \wp_e(y_{jj})\} - \Im_e(x_{ii}, y_{jj})$$

$$= min\{\wp_e(x_{ii}), \wp_e(y_{jj})\} - min\{\wp_e(x_{ii}), \wp_e(y_{jj})\} By (3)$$

$$\Im_e(x_{ii}, y_{jj}) = 0$$

Similarly

$$\overline{\Im_e}(y_{jj}, z_{kk}) = 0 \& \overline{\Im_e}(z_{kk}, x_{ii}) = 0$$

Hence $G_{A,V_i \cup V_j \cup V_k}$ is an IFSTG

5. Application

To accomplish objectives in their day-to-day existence, they need the best school, the understudies assume a significant part in professional openings in any organization. Recruiting the best school and the best understudy and they pick the best programming organization is a challenging process. A study is carried out here using FSTG. The study's goal is to find the best match between *College*s, students, and software companies.

Consider *College*s, students, and software companies to be three sets of disjoint veretx sets, with matching qualities as parameters and preferences for *College*s, studentsand software companies as edges. Let $V = v_i \cup v_j \cup v_k = \{v_i: (a_1, a_2, a_3)\}, \{v_j: (b_1, b_2, b_3)\}, \{v_k: (c_1, c_2, c_3)\}$ are set of all three disjoint vertices and $A = \{e_1, e_2, e_3, e_4\}$ are parameter set.

Identified qualities of *College*s are given below,

a_1 =Good communication and a comfortable environment

a_2 = Professional Advancement and On-Site Opportunity

a_3 = On-Site Opportunity and a comfortable environment

Identified qualities of Students are given below,

b_1 =Self-discipline and Responsible

b_2 =Professional skills and Responsible

b_3 =Confident and Professional skills

Identified qualities of Software Companies are given below,

c_1 =Good salary and Friendly work environment

c_2 =Transparency and Good salary

c_3 =Professional Advancement and Friendly work environment

and the parameters are,

e_1 ={*College*- a comfortable environment, Student- Confident, Company- Good salary}

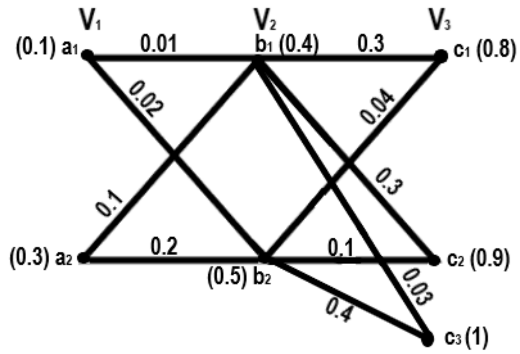
e_2 ={*College*- Professional Advancement, Student- Responsible, Company- Friendly work environment}

e_3 ={*College*-On-Site Opportunity, Student- Professional skills, Company- Transparency}

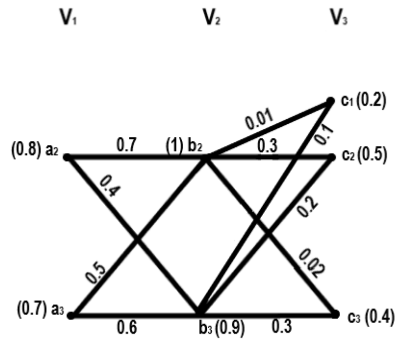
e_4 ={*College*-Good communication, Student- Self-discipline, company- Professional Advancement}

Example 5.1

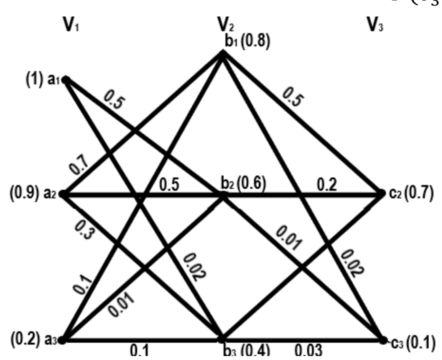
$$F(e_1)$$



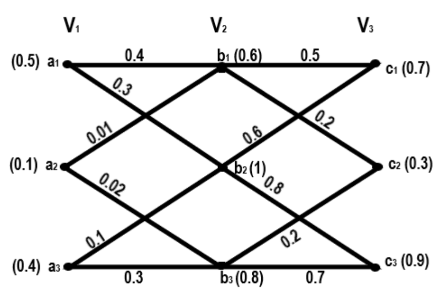
$F(e_2)$



$F(e_3)$



$F(e_4)$



ρ	a_1	a_2	a_3	b_1	b_2	b_3	c_1	c_2	c_3

e_1	0.1	0.3	0	0.4	0.5	0	0.8	0.9	1
e_2	1	0.9	0.2	0.8	0.6	0.4	0	0.7	0.1
e_3	0	0.8	0.7	0	1	0.9	0.2	0.5	0.4
e_4	0.5	0.1	0.4	0.6	1	0.8	0.7	0.3	0.9

μ	e_1	e_2	e_3	e_4
a_1b_1	0.01	0	0	0.4
a_1b_2	0.02	0.5	0	0.3
a_1b_3	0	0.02	0	0
a_2b_1	0.1	0.7	0	0.01
a_2b_2	0.2	0.5	0.7	0
a_2b_3	0	0.3	0.4	0.02
a_3b_1	0	0.1	0	0
a_3b_2	0	0.01	0.5	0.1
a_3b_3	0	0.1	0.6	0.3
b_1c_1	0.3	0	0	0.5
b_1c_2	0.3	0.5	0	0.2
b_1c_3	0.03	0.02	0	0
b_2c_1	0.04	0	0.01	0.6
b_2c_2	0.1	0.2	0.3	0
b_2c_3	0.4	0.01	0.02	0.8
b_3c_1	0	0	0.1	0
b_3c_2	0	0.3	0.2	0.2
b_3c_3	0	0.3	0.3	0.7

$$S[F(e_1)] = 1.5S[F(e_2)] = 3.29S[F(e_3)] = 3.13S[F(e_4)] = 4.13$$

Most of the best *Colleges* taught good communication skills, software companies hired self-disciplined students, and students preferred software companies with good Professional Advancement. According to the above discussion, "the most favourable matching occurs between good communication in *College*, self discipline student, and Professional Advancement in company.".

6. Comparison Analysis

Only the principles and applications of fuzzy soft bipartite graphs were discussed previously. Later, discovered a novel size concept that was only used for fuzzy soft tri-partite graphs. Following that, the concepts were applied to the complement of fuzzy soft tri-partite graphs. Fuzzy soft-tripartite graphs produce better results than fuzzy soft bipartite graphs. The value of the fuzzy soft tripartite graph is larger than that of the fuzzy soft bipartite graph, which gives exact maximum values.

7. Conclusion

FSTG and its complements are a modern phenomenon based on the combination of fuzzy graphs. The introduction of these FSTG is a novel idea that can be expanded into a variety of graph hypothetical concepts. This FSTG and its complement, explored their properties, and established related theorems to contribute to the theoretical aspect of fuzzy graph theory.The FSTG and its complement, as well as several of its fundamental properties, have been defined. The order, size, and degree of FSTG have been defined with appropriate examples.FSTG have used this FSTG in a practical application. This work may be expanded in the future to include concepts such as intuitionistic. It is possible to investigate FSTG, bipolar FSTG, and m-polar FSTG. Furthermore, many real-world applications can be investigated.

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