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SOLUTIONS TO ABC-FRACTIONAL ORDER NEUTRAL MIXED INTEGRO DELAY DIFFERENTIAL EQUATIONS WITH IMPULSE

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Abstract. This work establishes the existence and uniqueness (EUS) of solutions for impulsive neutral mixed integro fractional delay differential equations involving recently explored ABC-fractional derivatives. Fixed-point techniques are applied to confirm EUS of solutions of ABC- fractional order integrodifferential system with impulse.

1. Introduction

Fractional calculus is a generalised classical calculus with order of real or complex values. In recent decades, it has developed tremendously and has turned into one of the most powerful tools to describe dynamical systems in numerous scientific disciplines. Unlike the ordinary differential operator, fractional derivatives are global in nature and produce concise results by using minimum parameters than that of the classical model. For the fundamental developments in this field, one can refer [41, 44]. Because of these astonishing results and more flexibility, some of the researchers not only exploited fractional operators in modelling and understanding the complex system, but also enriched the calculus with various fractional operators. Some of the fractional operators contain singular kernel. To prevail over this, Caputo and Fabrizio defined an operator involving an exponential function which is non-singular[17]. Many researchers have worked on developing the theory of FDE using CF derivative [1, 3, 11, 26, 35, 37, 47, 48].

Motivated by CF derivative, Atangana and Baleanu (AB) put forward a novel nonsingular fractional derivative possessing the Mittag-Leffler function and it influences new attributes [12]. Many researchers have tried to replace the ordinary differential operator with AB fractional derivative in physical systems, as it is more applicable and yields effective results [4, 5, 6, 7, 13, 23, 24, 28, 29, 30, 31, 33, 34, 38, 39, 40, 42, 43, 50, 51]. Alkahtani et al.[10] did numerical research on dynamics of Chau's loop. Gomez et al. [22] obtained analytical solutions for the electric loop with AB derivatives. Chang et al. [54] applied AB derivatives to solve variational Herglotz problems. Kamal Shah et al. [46] analyzed the qualitative behaviour of the mathematical model of SARS - COV -19 under fractional non-singular derivative.

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Some dynamical problems subjected to short-term perturbations, which allow discontinuities in the evolution phenomena, are studied using impulsive fractional differential equations.[15, 16, 18, 19, 25, 36]. we refer the monographs [14, 32, 45]. Mathematical models involving the fractional differential operators with delay, depend on duration of certain hidden processes as past history [2, 8, 21].Integro-differential equations are generalizations of FDE, Fredholm and Volterra integral equations and are approximation to partial differential equations, which represent much of the continum phenomena. These kinds of equations are employed in modeling many problems in mathematical physics, biophysics, rheology such as heat conduction in materials with memory.[9, 20, 27, 52, 53, 55].

Hasib Khan et al.[28] ensured EU and data dependence of the solution of impulsive FDE with ABC-derivative using Schauder fixed point theorem. Recently, Ravichandran et al.[43] proved the EUS results of neutral impulsive FIDE's in the ABC sense of the form:

$$\begin{aligned} &{}_0^{ABC}D^{\gamma}[v(t) - \Theta(t, v_t)] = f\Big(t, v_t, \int_0^t k(t, s, v_s)ds\Big), \qquad 0 < \gamma < 1 \\ &\Delta v(t_k) = v(t_k^+) - v(t_k^-) = I_k^* v(t_k), \\ &v|_{[-\tau^*, 0]} = v_0, \end{aligned}$$

where $v(t) \in C[0, 1], f : \mathcal{J} \times PC^1 \times \mathcal{J} \to \mathbb{R}^n$ is continuous. $I_k^* : \mathbb{R}^n \to \mathbb{R}^n, v(t_s) = v(t+s)$ for $-\tau^* \leq s \leq 0$ where $v(t_k^+) = \lim_{\delta \to 0^+} v(t_k + \delta)$ and $v(t_k^-) = \lim_{\delta \to 0^-} v(t_k - \delta)$.

In papers [25], [38] and other, the existence of non linear FDE is proved using Schuader and Krasnoselskii fixed point theorem, whereas in this manuscript, the existence of Neutral FIDE of mixed type with impulses and delay is discussed using Schaefer's fixed point theorem and the obtained results are verified with an example. Inspired by the above works [28] and [43], we have presented the EU of the solution for the following impulsive ABC- fractional neutral mixed type integrodifferential equation using Schaefer's fixed point theorem,

$$ {}_0^{ABC}D^{v}\Big[x(t) - \sigma(t, x_t)\Big] = \mathcal{B}\Big(t, x_t, \int_0^t \eta(t, s, x_s)ds, \int_0^T \xi(t, s, x_s)ds\Big) \tag{1.1}$$

$$t \in \mathcal{J}^c = \mathcal{J} - \{t_1, t_2.......t_m\}, \qquad \mathcal{J} = [0, T], \qquad x \neq t_k$$

$$\Delta x(t_k) = x(t_k^+) - x(t_k^-) = I_k^* x(t_k), \tag{1.2}$$

$$x|_{[-\tau^*, 0]} = \zeta + \mathcal{F}^*.$$

where $v \in (0, 1)$, $\mathcal{F}^*(x) = \sum_{i=1}^m \lambda_i x_i(t)$, $x_i \in PC^1$, $\sum_{i=1}^m \lambda_i < 1$ for $i = 1, 2, ..., m$, $\mathcal{B}\Big(t, x_t, \int_0^t \eta(t, s, x_s)ds, \int_0^T \xi(t, s, x_s)ds\Big)$ and $\sigma(t, x_t)$ are Lebesgue measurable functions such that $\mathcal{B} \in C\Big(\mathcal{J} \times X \times X \times X \to X\Big)$ where X is a Banach space X, $\sigma : \mathcal{J} \times PC^1 \to \mathbb{R}^n$ are piecewise continuous and $I_k^* : \mathbb{R}^n \to \mathbb{R}^n$ are continuous on PC^1 where $PC^1 = PC^1\Big([-\tau^*, 0], \mathbb{R}^n\Big)$ is the space of piecewise continuous function with norm $\|x\|_\infty = \sup_{t \in [-\tau^*, T]} \|x(t)\|$ and $\zeta : [-\tau^*, 0] \to \mathbb{R}^n$. $\eta, \xi : \mathcal{D} \times X \to X$ are continuous, where $\mathcal{D} = \{(x, s) \in \mathcal{J} \times \mathcal{J} : t \geq s\}$. Then $x_t(s) = x(t+s)$ for $-\tau^* < s \leq 0$, $x(t_k^+) = \lim_{\delta \to 0^+} x(t_k + \delta)$ and $x(t_k^-) = \lim_{\delta \to 0^-} x(t_k - \delta)$.

Consider $Px(t) = \int_0^t \eta(t,s,x_s)ds$ and $Qx(t) = \int_0^T \xi(t,s,x_s)ds$. Then (1.1) becomes

$$
\begin{aligned}
&{}_0^{ABC}D^v\Big[x(t)-\sigma(t,x_t)\Big] = \mathcal{B}\Big(t,x_t,Px(t),Qx(t)\Big), \qquad (1.3)\\
&t\in\mathcal{J}^c = \mathcal{J}-\{t_1,t_2,.......,t_m\}, \quad \mathcal{J}=[0,T], \quad x\neq t_k,\\
&\Delta x(t_k) = x(t_k^+)-x(t_k^-) = I_k^* x(t_k),\\
&x|_{[-\tau^*,0]} = \zeta+\mathcal{F}^*.
\end{aligned}
$$

The aim of this paper is to obtain EUS of the above IVP of ABC-fractional order impulsive neutral mixed integrodifferential equation with delay using Schaefer's fixed point theorem.

The rest of the paper is sorted out as follows: In Section 2, definitions and lemmas of ABC-fractional derivatives are recalled. In Section 3, integral form of ABC-derivative of the above-mentioned problem is proved. In Section 4, the main result - EUS of the proposed ABC-fractional order impulsive neutral differential equation is constructed. In the last section, the obtained results are verified with an example.

2. Prerequisite

In this segment, we elicit definitions and tools on fractional operators with the non-singular kernel[44, 41, 17, 12, 35].
For $0<\alpha<1$, the Caputo fractional derivative of order α starting at a is given as

$$
{}_0^{C}D^\alpha y(t) = \frac{1}{\Gamma(1-\alpha)}\int_a^t (t-s)^{-\alpha}y'(s)ds.
$$

Let $\psi\in H^*(a,b), a<b$, v in [0,1]. The ABC - fractional derivative of ψ of order v is defined by

$$
{}_0^{ABC}D^v\psi(\tau^*) = \frac{B^*(v)}{1-v}\int_0^{\tau^*}\psi'(s)E_v\Big[\frac{-v(\tau^*-s)^v}{1-v}\Big]ds
$$

where $B^*(0)=B^*(1)=1$.
The associated AB fractional -integral of a function ψ is defined by

$$
{}_0^{AB}I_t^v\psi(\tau^*) = \frac{1-v}{B^*(v)}\psi(\tau^*) + \frac{v}{B^*(v)\Gamma(v)}\int_0^{\tau^*}\psi(s)(\tau^*-s)^{v-1}ds.
$$

Lemma 2.1. *Newton -Leibntiz formula holds for the ABC-fractional order differential and integral operators.*

$$
{}_0^{AB}I_t^v\Big({}_0^{ABC}D_{\tau^*}^v(\theta)\Big) = \psi(\tau^*)-\psi(0)
$$

For $\psi(t)$ defined on $[e,f]$ and $v\in(m,m+1)$ for some $m\in N_0$,

$$
\begin{aligned}
&{}_e^{ABR}D^v\Big({}_e^{AB}I^v\psi(t)\Big) = \psi(t)\\
&{}_e^{AB}I^v\Big({}_e^{ABR}D^v\psi(t)\Big) = \psi(t)-\sum_{k=0}^{m-1}\psi^{(k)}(e)\frac{(t-e)^k}{\Gamma(k-1)}.
\end{aligned}
$$

Theorem 2.2. *(Schaefer's fixed point theorem)* [49]*: A continuous and compact operator ϕ has a fixed point, if the set $\{u \in X; u = \lambda\phi u$ for $0 \leq \lambda \leq 1\}$ is bounded.*

3. Integral form of ABC - FIDE

Here, we analyse the impulsive neutral ABC - FIDE and its integral form.

Theorem 3.1. *For $v \in (0,1)$ and $\mathcal{B}, \sigma \in PC^1[0,T]$ such that $\sigma(0,x_0) = 0$ and $\mathcal{B}\Big(0, x_0, 0, \int_0^T \xi(0,s,x_s)ds\Big) = 0$, $x(t)$ is a solution of (1.3) provided that*

$$x(t) = \begin{cases} \zeta + \mathcal{F}^*, \quad t \in [-\tau^*, 0] \\ \zeta(0) + \mathcal{F}^*(0) + \sigma(t, x_t) +_0^{AB} I^v \mathcal{B}\Big(t, x_t, Px(t), Qx(t)\Big), \quad t \in [0, t_1] \\ \zeta(0) + \mathcal{F}^*(0) + \sigma(t, x_t) +_0^{AB} I^v \mathcal{B}\Big(t, x_t, Px(t), Qx(t)\Big) \\ +I_1^*\Big(x(t_1^-)\Big), \quad t \in [t_1, t_2] \\ . \\ . \\ \zeta(0) + \mathcal{F}^*(0) + \sigma(t, x_t) +_0^{AB} I^v \mathcal{B}\Big(t, x_t, Px(t), Qx(t)\Big) \\ + \sum_{k=1}^m I_k^*\Big(x(t_k^-)\Big). \quad t \in [t_m, T] \end{cases}$$

Proof. The solution $x(t)$ of the problem (1.3), we have for $t \in [0, t_1]$,

$$\begin{aligned} {}_0^{AB}I^v\Big[{}_0^{ABC}D^v\Big\{x(t) - \sigma(t,x_t)\Big\}\Big] &= {}_0^{AB}I^v\Big[\mathcal{B}\Big(t, x_t, Px(t), Qx(t)\Big)\Big] \\ x(t) - \sigma\Big(t, x_t\Big) &= {}_0^{AB}I^v\Big[\mathcal{B}\Big(t, x_t, Px(t), Qx(t)\Big)\Big] + c_0 \\ x(t) = \zeta(0) + \mathcal{F}^*(0) + \sigma(t, x_t) &+_0^{AB} I^v\Big[\mathcal{B}\Big(t, x_t, Px(t), Qx(t)\Big)\Big] \end{aligned}$$

and

$$x(t_1^-) = \zeta(0) + \mathcal{F}^*(0) + \sigma(t_1, x_{t_1}) +_0^{AB} I^v\Big[\mathcal{B}\Big(t_1, x_{t_1}, Px(t_1), Qx(t_1)\Big)\Big].$$

For $t \in (t_1, t_2]$, $\quad \Delta x(t_k) = I_k^* x(t_k)$, we have $x(t_1^+) - x(t_1^-) = I_1^* x(t_1)$

$$\begin{aligned} x(t) &= x(t_1^+) - x(t_1) + x(t) \\ &= x(t_1^-) + I_1^*\Big(x(t_1^-)\Big) - \sigma(t_1, x_{t_1}) - \zeta(0) - \mathcal{F}^*(0) - \\ &\quad {}_0^{AB}I^v\Big[\mathcal{B}\Big(t_1, x_{t_1}, Px(t_1), Qx(t_1)\Big] + \zeta(0) + \mathcal{F}^*(0) + \sigma(t, x_t) \\ &\quad +_0^{AB} I^v\Big[\mathcal{B}\Big(t, x_t, Px(t), Qx(t)\Big)\Big] \end{aligned}$$

Using $x(t_1^-)$ in the above equation, we get

$$x(t) = \zeta(0) + \mathcal{F}^*(0) + \sigma(t, x_t) + I_1^*\Big(x(t_1^-)\Big) +_0^{AB} I^v\Big[\mathcal{B}\Big(t, x_t, Px(t), Qx(t)\Big)\Big].$$

For $t \in (t_2, t_3]$, we have

$$\begin{aligned} x(t) &= x(t_2^+) - x(t_2) + x(t) \\ &= x(t_2^-) + I_2^*\Big(x(t_1^-)\Big) - \sigma(t_2, x_{t_2}) - \zeta(0) - \mathcal{F}^*(0) \\ &\quad -{}_0^{AB} I^v\Big[\mathcal{B}\Big(t_2, x_{t_2}, Px(t_2), Qx(t_2)\Big] + \zeta(0) + \mathcal{F}^*(0) + \sigma(t, x_t) \\ &\quad +{}_0^{AB} I^v\Big[\mathcal{B}\Big(t, x_t, Px(t), Qx(t)\Big)\Big]. \end{aligned}$$

Using $x(t_2^-)$ in the above equation, we get

$$x(t) = \zeta(0) + \mathcal{F}^*(0) + \sigma(t, x_t) + \sum_{k=1}^{2} I_k^*\Big(x(t_k^-)\Big) +{}_0^{AB} I^v\Big[\mathcal{B}\Big(t, x_t, Px(t), Qx(t)\Big)\Big].$$

As we proceed, it leads to the case when $t \in (t_m, T]$ and we get

$$x(t) = \zeta(0) + \mathcal{F}^*(0) + \sigma(t, x_t) + \sum_{k=1}^{m} I_k^*\Big(x(t_k^-)\Big) +{}_0^{AB} I^v\Big[\mathcal{B}\Big(t, x_t, Px(t), Qx(t)\Big)\Big].$$

□

4. Theorems for Main Results

Let $h : \mathcal{J} \to \mathbb{R}$ is measurable in $\mathcal{L}^\omega(\mathcal{J})$ with the norm:

$$\|h\|_{\mathcal{L}}^{\omega} = \begin{cases} \Big(\int_{\mathcal{J}} |g(t, x_t)|^{\omega} dt\Big)^{\frac{1}{\omega}}, 1 \le \omega < \infty \\ \inf_{\mu(\bar{J})} \Big(sup_{t \in J - \bar{J}} |g(t, x_t)|\Big), \omega = \infty \end{cases}$$

where $\mu(\bar{J})$ is the Lebesgue measure on $\bar{J}$ and $\|h\|_{\mathcal{L}}^{\omega} < \infty$. Now we define an operator Ω as

$$\Omega x(t) = \begin{cases} \zeta + \mathcal{F}^*, \quad t \in [-\tau^*, 0] \\ \zeta(0) + \mathcal{F}^*(0) + \sigma(t, x_t) +{}_0^{AB} I^v \mathcal{B}\Big(t, x_t, Px(t), Qx(t)\Big), \quad t \in [0, t_1] \\ \zeta(0) + \mathcal{F}^*(0) + \sigma(t, x_t) +{}_0^{AB} I^v \mathcal{B}\Big(t, x_t, Px(t), Qx(t)\Big) \\ +I_1^*\Big(x(t_1^-)\Big), \quad t \in [t_1, t_2] \\ . \\ . \\ . \\ \zeta(0) + \mathcal{F}^*(0) + \sigma(t, x_t) +{}_0^{AB} I^v \mathcal{B}\Big(t, x_t, Px(t), Qx(t)\Big) \\ + \sum_{k=1}^{m} I_k^*\Big(x(t_k^-)\Big). \quad t \in [t_m, T] \end{cases}$$

For the proof of our main result, we need the following assumptions

(A1) Let $x \in \mathcal{PC}^1[0, T]$ and suppose that $\mathcal{B} \in C\Big(\mathcal{J} \times X \times X \times X, X\Big)$ is piecewise continuous and there exists positive constants $\mathcal{L}_1, \mathcal{L}_2$ and $\mathcal{L}$ such that $\|\mathcal{B}(t, x_1, u_1, z_1) - \mathcal{B}(t, x_2, u_2, z_2)\| \le \mathcal{L}_1\Big(\|x_1 - x_2\| + \|u_1 - u_2\| + \|z_1 - z_2\|\Big)$ for all x, u, z in Y, where Y=$C[J, X]$ be the set of continuous functions on J with values in the Banach space X, $\mathcal{L}_2 = \max_{t\in\mathcal{J}} \|\mathcal{B}(t,0,0,0)\|$ and $\mathcal{L}=\max\{\mathcal{L}_1,\mathcal{L}_2\}$.

(A2) There exists poistive constants $\mathbb{N}_1$, $\mathbb{N}_2$ and $\mathbb{N}$ such that
$\|P(t,s,x_1)-P(t,s,x_2)\| \le \mathbb{N}_1(\|x_1-x_2\|)$
for all x_1,x_2 in Y, $\mathbb{N}_2=\max_{(t,s)\in\mathcal{D}} \|P(t,s,0)\|$ and $\mathbb{N}=\max\{\mathbb{N}_1,\mathbb{N}_2\}$.

(A3) There exists positive constants $\mathbb{C}_1$, $\mathbb{C}_2$ and $\mathbb{C}$ such that
$\|Q(t,s,x_1)-Q(t,s,x_2)\| \le \mathbb{C}_1(\|x_1-x_2\|)$
for all x_1,x_2 in Y, $\mathbb{C}_2=\max_{(t,s)\in\mathcal{D}} \|Q(t,s,0)\|$ and $\mathbb{C}=\max\{\mathbb{C}_1,\mathbb{C}_2\}$.

(A4) Let the impulses $I_k^* \in \mathcal{C}(\mathcal{R}^n,\mathcal{R}^n)$ be bounded and for a $\vartheta^*>0$, we have $\mu(t)\in L^{\frac{1}{\delta}}(J)$ with $\left\|I_k^*\Big(x(t_k^-)\Big)-I_k^*\Big(u(t_k^-)\Big)\right\| \le \vartheta^*(t)\|x-u\|_\infty$
for each $x,x\in PC^1\Big([-\tau^*,T]\Big)(\zeta)$, and for each $r\in I^+$, $k=1,2,\cdots,m$
then $\mathcal{N}^*=\max\Big\{\left\|I_k^*\Big(x(t_k^-)\Big)\right\| : \|x\|\le r\Big\}$.

(A5) Let for a $\delta\in(0,h)$ and there exists function $\mu(t)$ in $\mathcal{L}^{\frac{1}{\delta}}(\mathcal{J})$ such that
$\|\sigma(t,x_t)-\sigma(t,u_t)\| \le \mu(t)\|x_t-u_t\|$, for $t\in\mathcal{J}$ and $x_t,u_t\in\mathcal{PC}^1$.

Lemma 4.1. *If $(A1)-(A3)$ are satisfied, then for any $t\in\mathcal{J}$ and $x_1,x_2\in Y$,* $\|Px_1(t)-Px_2(t)\| \le \mathbb{N}t\|x_1-x_2\|$, $\|Qx_1(t)-Qx_2(t)\| \le \mathbb{C}T\|x_1-x_2\|$, $\|Px(t)\| \le t\Big(\mathbb{N}_1\|x\|+\mathbb{N}_2\Big)$, $\|Qx(t)\| \le T\Big(\mathbb{C}_1\|x\|+\mathbb{C}_2\Big)$.

Theorem 4.2. *: If the conditions $(A1)-(A5)$ hold, and $\Big(\mathbb{L}\|x_{t_2}-x_{t_1}\|+\mathbb{N}t\|x_{t_2}-x_{t_1}\|+\mathbb{C}T\|x_{t_2}-x_{t_1}\|\Big)=q(t_2-t_1)$ and $q\in\mathcal{L}^1(\mathcal{J},\mathcal{R}_+)$ and if the inequality*

$$\Big[\|\zeta(0)+\mathcal{F}^*(0)\|+\|\sigma(t,x_t)\|+m\mathcal{N}^*+\Big(\frac{1-v}{B^*(v)}+\frac{t^v}{B^*(v)\Gamma v}\Big)p\|x_t\|\Big] \le \varrho^* \quad (4.1)$$

holds, then the problem (1.3) has a solution where $p\|x_t\|=\Big(\mathcal{L}\|x_t\|+\mathbb{N}t\|x_t\|+\mathbb{C}T\|x_t\|\Big)$ and for each positive r, $\mathbb{B}_r\in\{x\in Y:\|x\|\le r\}$, $k=1,2,\cdots,m$.

Proof. We divide the proof in the following parts.
Part 1: Ω is continuous
Let $\{x_n\}$ be a sequence such that $x_n\to x$ in $PC^1\Big([-\tau^*,T]\Big)$.
Since $\mathcal{B}\Big(t,x_t,Px(t),Qx(t)\Big)$, $\sigma(t,x_t)$ and the impulses $I_k^*\Big(x(t_k^-)\Big)$ are continuous operators on $PC^1\Big([-\tau^*,\mathbb{R}^n]\Big)(\zeta)$ for $k=1,2\cdots,m$. By using $(A1)$ and lemma

4.2, then for each $t \in [0, t_1]$, we have

$$\begin{aligned}
\|\Omega x_n - \Omega x\| &\leq \|\sigma(t, x_{n_t}) - \sigma(t, x_t)\| + \Big\|{}_0^{AB}I^v \mathcal{B}\Big(t, x_{n_t}, Px_n(t), Qx_n(t)\Big) \\
&\quad - \mathcal{B}\Big(t, x_t, Px(t), Qx(t)\Big)\Big\| \\
&\leq \mu(t)\|x_n - x\| + \frac{1-v}{B^*(v)}\Big\|\mathcal{B}\Big(t, x_{n_t}, Px_n(t), Qx_n(t)\Big) \\
&\quad - \mathcal{B}\Big(t, x_t, Px(t), Qx(t)\Big)\Big\| + \frac{v}{B^*(v)}\Big\|I_0^v \mathcal{B}\Big(t, x_{n_t}, Px_n(t), Qx_n(t)\Big) \\
&\quad - I_0^v \mathcal{B}\Big(t, x_t, Px(t), Qx(t)\Big)\Big\| \\
&\leq \mu(t)\|x_n - x\| + \frac{1-v}{B^*(v)}\Big(\mathbb{L}\|x_n - x\| + \mathbb{N}t\|x_n - x\| + \mathbb{C}T\|x_n - x\|\Big) \\
&\quad + \frac{v}{B^*(v)}\Big(\mathbb{L}\|x_n - x\| + \mathbb{N}t\|x_n - x\| + \mathbb{C}T\|x_n - x\|\Big) I_0^v(t) \\
&\leq \mu(t)\|x_n - x\| + \Big[\frac{1-v}{B^*(v)} + \frac{t^v}{B^*(v)\Gamma v}\Big] p\|x_n - x\|
\end{aligned}$$

$\|\Omega x_n - \Omega x\| \to 0$ as $x_n \to x$.
Similarly for $t \in (t_1, t_2]$, by using $(A4)$ and $(A5)$, we have

$$\begin{aligned}
\|\Omega x_n - \Omega x\| &\leq \|\sigma(t, x_{n_t}) - \sigma(t, x_t)\| + \Big\|{}_0^{AB}I^v \mathcal{B}\Big(t, x_{n_t}, Px_n(t), Qx_n(t)\Big) \\
&\quad - \mathcal{B}\Big(t, x_t, Px(t), Qx(t)\Big)\Big\| + \|I_1 x_n(t_1^-) - I_1 x(t_1^-)\| \\
&\leq \mu(t)\|x_n - x\| + \frac{1-v}{B^*(v)}\Big\|\mathcal{B}\Big(t, x_{n_t}, Px_n(t), Qx_n(t)\Big) \\
&\quad - \mathcal{B}\Big(t, x_t, Px(t), Qx(t)\Big)\Big\| + \frac{v}{B*(v)}\Big\|I_0^v \mathcal{B}\Big(t, x_{n_t}, Px_n(t), Qx_n(t)\Big) \\
&\quad - I_0^v \mathcal{B}\Big(t, x_t, Px(t), Qx(t)\Big)\Big\| + \vartheta^*\|x_n - x\| \\
&\leq \Big[\mu(t) + \vartheta^*(t)\Big]\|x_n - x\| + \frac{1-v}{B^*(v)}\Big(\mathbb{L}\|x_n - x\| + \mathbb{N}t\|x_n - x\| \\
&\quad + \mathbb{C}T\|x_n - x\|\Big) + \frac{v}{B^*(v)}\Big(\mathbb{L}\|x_n - x\| + \mathbb{N}t\|x_n - x\| \\
&\quad + \mathbb{C}T\|x_n - x\|\Big) I_0^v(t) \\
&\leq \Big[\mu(t) + \vartheta^*(t)\Big]\|x_n - x\| + \Big[\frac{1-v}{B^*(v)} + \frac{t^v}{B^*(v)\Gamma v}\Big] p\|x_n - x\|
\end{aligned}$$

Hence $\|\Omega x_n - \Omega x\| \to 0$ as $x_n \to x$. Ultimately for $t \in (t_m, T]$, we have

$$\begin{aligned}
\|\Omega x_n - \Omega x\| &\leq \|\sigma(t, x_{n_t}) - \sigma(t, x_t)\| + \Big\|{}_0^{AB}I^v \mathcal{B}\Big(t, x_{n_t}, Px_n(t), Qx_n(t)\Big) \\
&\quad -{}_0^{AB} I^v \mathcal{B}\Big(t, x_t, Px(t), Qx(t)\Big)\Big\| + \sum_{k=1}^{m} \Big\|I_1^*\Big(x_n(t_1^-)\Big) - I_1^*\Big(x(t_1^-)\Big)\Big\|
\end{aligned}$$

$$\leq \Big[\mu(t) + m\vartheta^*(t)\Big]\|x_n - x\| + \Big[\frac{1-v}{B^*(v)} + \frac{t^v}{B^*(v)\Gamma v}\Big]p\|x_n - x\|$$

$\|\Omega x_n - \Omega x\| \to 0$ as $n \to \infty$. Hence, Ω is continuous.
Part 2: Ω is bounded in $PC^1\Big([-\tau^*, T]\Big)$
It is enough to show that for any $\delta^* > 0$, then $x \in \mathbb{B}_{\delta^*} = \{x \in PC^1\Big([-\tau^*, T], \mathbb{R}^n]\Big);$ $\|x\| \leq \delta^*\}$. By using the inequality (4.1) and lemma 4.2, for each $t \in [0, t_1]$,

$$\begin{aligned}
\|\Omega x(t)\| \leq & \|\zeta(0) + \mathcal{F}^*(0)\| + \|\sigma(t, x_t)\| + \frac{1-v}{B^*(v)}\Big\|\mathcal{B}\Big(t, x_t, Px(t), Qx(t)\Big)\Big\| \\
& + \frac{v}{B^*(v)}\Big\|\mathcal{B}\Big(t, x_t, Px(t), Qx(t)\Big)\Big\| I_0^v(t) \\
\leq & \|\zeta(0) + \mathcal{F}^*(0)\| + \|\sigma(t, x_t)\| + \Big[\frac{1-v}{B^*(v)} \\
& + \frac{t^v}{B^*(v)\Gamma(v)}\Big]\Big(\mathbb{L}\|x_t\| + \mathbb{N}t\|x_t\| + \mathbb{C}T\|x_t\|\Big) \\
\leq & \|\zeta(0) + \mathcal{F}^*(0)\| + \|\sigma(t, x_t)\| + \Big[\frac{1-v}{B^*(v)} + \frac{t^v}{B^*(v)\Gamma(v)}\Big]p\|x_t\| \\
\leq & \varrho^*.
\end{aligned}$$

$|||^{ly}$, by using $(A4)$, for each $t \in (t_m, T]$.

$$\begin{aligned}
\|\Omega t_(t)\| \leq & \|\zeta(0) + \mathcal{F}^*(0)\| + \|\sigma(t, x_t)\| + \frac{1-v}{B^*(v)}\Big\|\mathcal{B}\Big(t, x_t, Px(t), Qx(t)\Big)\Big\| \\
& + \frac{v}{B^*(v)}\Big\|\mathcal{B}\Big(t, x_t, Px(t), Qx(t)\Big)\Big\| I_0^v(t) + \sum_{k=1}^{m}\Big\|I_k^*\Big(x(t_k^-)\Big)\Big\| \\
\leq & \|\zeta(0) + \mathcal{F}^*(0)\| + \|\sigma(t, x_t)\| \\
& + \Big[\frac{1-v}{B^*(v)} + \frac{t^v}{B^*(v)\Gamma(v)}\Big]\Big(\mathbb{L}\|x_t\| + \mathbb{N}t\|x_t\| + \mathbb{C}T\|x_t\|\Big) + m\mathcal{N}^* \\
\leq & \|\zeta(0) + \mathcal{F}^*(0)\| + \|\sigma(t, x_t)\| + \Big[\frac{1-v}{B^*(v)} + \frac{t^v}{B^*(v)\Gamma(v)}\Big]p\|x_t\| + m\mathcal{N}^* \\
\leq & \varrho^*.
\end{aligned}$$

This proves that Ω is bounded.
Part 3: Ω is equicontinuous in $PC^1\Big([-\tau^*, T], \mathbb{R}^n\Big)$.
Let $t_1, t_2 \in (0, t_1]$

$$\begin{aligned}
\|\Omega x(t_2) - \Omega x(t_1)\| \leq & \frac{1-v}{B^*(v)}\Big\|\mathcal{B}\Big(t_2, x_{t_2}, Px(t_2), Qx(t_2)\Big) \\
& - \mathcal{B}\Big(t_1, x_{t_1}, Px(t_1), Qx(t_1)\Big)\Big\|
\end{aligned}$$

$$
\begin{aligned}
&+\frac{v}{B^*(v)}\Big[\Big\|\mathcal{B}\Big(t_2,x_{t_2},Px(t_2),Qx(t_2)\Big)I_0^v(t_2)\\
&-\mathcal{B}\Big(t_1,x_{t_1},Px(t_1),Qx(t_1)\Big)I_0^v(t_1)\Big\|\Big]\\
&+\|\sigma(t_2,x_{t_2})-\sigma(t_1,x_{t_1})\|\\
\leq&\frac{1-v}{B^*(v)}\Big(\mathbb{L}\|x_{t_2}-x_{t_1}\|+\mathbb{N}t\|x_{t_2}-x_{t_1}\|+\mathbb{C}T\|x_{t_2}-x_{t_1}\|\Big)\\
&+\frac{v}{B^*(v)\Gamma(v)}\Big(\mathbb{L}\|x_{t_2}-x_{t_1}\|+\mathbb{N}t\|x_{t_2}-x_{t_1}\|\\
&+\mathbb{C}T\|x_{t_2}-x_{t_1}\|\Big)\Big[\frac{(t_2^v-t_1^v)}{v}\Big]+\mu(t)\|x_{t_2}-x_{t_1}\|\\
\leq&\mu(t)\|x_{t_2}-x_{t_1}\|+\Big[\frac{1-v}{B^*(v)}+\frac{(t_2^v-t_1^v)}{B^*(v)\Gamma v}\Big]\|q(t_2-t_1)\|\to 0 \text{ as } t_2\to t_1.
\end{aligned}
$$

Similarly, for $t\in(t_k,t_{k+1}]$, $\|\Omega x(t_{k+1})-\Omega x(t_k)\|\to 0$ as $t_{k+1}\to t_k$ where $k=1,2,.....,m$.
With the help of $(A4)$ and $(A5)$, we have proved Ω is equicontinuous.
As result of part (1)-(3) Ω is completely continuous.
Part 4: A priori bounds.
Now it is left to show that that the set

$$\Delta=\{t\in PC^1\Big([-\tau^*,T],\mathbb{R}^n\Big);x=\lambda\Omega(x),\text{for some } 0<\lambda<1\} \text{ is bounded.}$$

Let $x\in\Delta$, for some $0<\lambda<1$, for each $t\in[0,T]$,
we have $x(t)=\lambda\Omega x(t)$,

$$
\begin{aligned}
x(t)=&\lambda\Big[\zeta(0)+\mathcal{F}^*(0)+\sigma(t,x_t)+\frac{1-v}{B^*(v)}\mathcal{B}\Big(t,x_t,Px(t),Qx(t)\Big)\\
&+\frac{v}{B^*(v)}I_0^v\mathcal{B}\Big(t,x_t,Px(t),Qx(t)\Big)+\sum_{k=1}^m I_k^*\Big(x(t_k^-)\Big)\Big].
\end{aligned}
$$

This implies by $(A4)$ and for each $t\in[0,T]$, we have

$$
\begin{aligned}
|x(t)|=&|\lambda\Omega x(t)|\\
&\leq\lambda|\zeta(0)+\mathcal{F}^*(0)|+\lambda|\sigma(t,x_t)|+\frac{\lambda(1-v)}{B^*(v)}\Big|\mathcal{B}\Big(t,x_t,Px(t),Qx(t)\Big)\Big|\\
&+\frac{\lambda v}{B^*(v)\Gamma(v)}\Big|\mathcal{B}\Big(t,x_t,Px(t),Qx(t)\Big)\Big|I_0^v(t)+\lambda\sum_{k=1}^m\Big|I_k^*\Big(x(t_k^-)\Big)\Big|\\
&\leq\lambda|\zeta(0)+\mathcal{F}^*(0)|+\lambda|\sigma(t,x_t)|+\Big[\frac{\lambda(1-v)}{B^*(v)}+\frac{\lambda t^v}{B^*(v)\Gamma(v)}\Big]\\
&(\times)\Big(\mathbb{L}|x|+\mathbb{N}t|x|+\mathbb{C}T|x|\Big)+\lambda\sum_{k=1}^m\Big|I_k^*\Big(x(t_k^-)\Big)\Big|.
\end{aligned}
$$

Thus by using the inequality (4.1), for every $t \in [0,T]$ and for $\lambda < 1$,

$$\|x\| \leq \lambda\|\zeta(0)+\mathcal{F}^*(0)\| + \lambda\|\sigma(t,x_t)\| + \Big[\frac{\lambda(1-v)}{B^*(v)} + \frac{\lambda t^v}{B^*(v)\Gamma(v)}\Big]p\|x\| + \lambda m\mathcal{N}^*$$
$$\leq \varrho^*.$$

As it is true for every $x \in \Delta$, Δ is bounded. By Schaefer's fixed point theorem, we claim that Ω has a fixed point which is a solution of the ABC- fractional order impulsive neutral mixed integrodifferential equation (1.3). □

Theorem 4.3. *If the assumptions* $(A1) - (A5)$ *hold, then the ABC- fractional order impulsive neutral mixed integrodifferential equation (1.1) has a unique solution, provided that*

$$m\vartheta^* + \mu(t) + \Big[\frac{1-v}{B^*(v)} + \frac{t^v}{B^*(v)\Gamma(v)}\Big]p < 1 \tag{4.2}$$

where $\sum_{k=1}^{m}\Big\|I_k^*\Big(x_1(t_k)\Big) - I_k^*\Big(x_2(t_k)\Big)\Big\| = m\vartheta^*(t)\|x_1 - x_2\|$.

Proof. :We assume the contradiction to prove uniqueness, let x_1 and x_2 be two solutions of (1.1).
Case 1:For $t \in [-\tau^*, 0]$, $\quad \|\Omega x_1 - \Omega x_2\| \to 0$ implies $x_1 = x_2$.
Case 2: For $t \in [0, t_1]$,

$$\|\Omega x_1 - \Omega x_2\| \leq \mu(t)\|x_1 - x_2\| + \Big[\frac{1-v}{B^*(v)} + \frac{t^v}{B^*(v)\Gamma v}\Big]$$
$$(\times)\Big(\mathbb{L}\|x_1 - x_2\| + \mathbb{N}t\|x_1 - x_2\| + \mathbb{C}T\|x_1 - x_2\|\Big)$$
$$\leq \mu(t)\|x_1 - x_2\| + \Big[\frac{1-v}{B^*(v)} + \frac{t^v}{B^*(v)\Gamma v}\Big]p\|x_1 - x_2\|.$$

Case 3: For $t \in [t_k, t_{k+1}]$,

$$\|\Omega x_1 - \Omega x_2\| \leq \Big(\mu(t) + m\vartheta^*(t)\Big)\|x_1 - x_2\| + \Big[\frac{1-v}{B^*(v)} + \frac{t^v}{B^*(v)\Gamma v}\Big]\Big(\mathbb{L}\|x_1 - x_2\|$$
$$+ \mathbb{N}t\|x_1 - x_2\| + \mathbb{C}T\|x_1 - x_2\|\Big)$$
$$\leq \Big(\mu(t) + m\vartheta^*(t)\Big)\|x_1 - x_2\| + \Big[\frac{1-v}{B^*(v)} + \frac{t^v}{B^*(v)\Gamma v}\Big]p\|x_1 - x_2\|.$$

By the assumptions $(A1)$ to $(A5)$ and the inequality (4.2), Ω is a contraction. Then by Banach fixed - point theorem, the impulsive neutral FIDE (1.1) has a unique solution. □

5. Illustration

Consider the following problem

$$
\begin{aligned}
{}_0^{ABC}D^v\Big[x(t) - \frac{e^{-t}x(t)}{(9+e^t)(1+x(t))}\Big] &= \frac{1}{(t+12)}\frac{x(t)}{1+x(t)} + \int_0^t t\sqrt{2s+1}x(s)ds \\
&+ \int_0^t s^3 cos(s^4 - t)x(s)ds
\end{aligned}
$$

$$
x(t) = 1 + \phi_0, \quad t \in [-\tau^*, 0], \quad \phi_0 = \sum_{\lambda_i=1}^{n} \lambda_i x_i
$$

$$
\Delta x(t_i) = \frac{1}{2}x\Big(\frac{1^-}{2}\Big), \quad \text{where} \quad t_i = \frac{1}{2}, \quad B^*(v) = 1.
$$

By assuming

$$
\begin{aligned}
&\sigma(t, x_t) = \frac{e^{-t}x(t)}{(9+e^t)(1+x(t))}; Px(t) = \int_0^t t\sqrt{2s+1}x(s)ds; \\
&Qx(t) = \int_0^t s^3 cos(s^4 - t)x(s)ds; \\
&\mathcal{B}\Big(t, x_t, Px(t), Qx(t)\Big) = \frac{1}{(t+12)}\frac{x(t)}{1+x(t)} + \int_0^t t\sqrt{2s+1}x(s)ds \\
&\qquad + \int_0^t s^3 cos(s^4 - t)x(s)ds. \\
&\text{where} \quad \sigma(0, x_0) = \frac{e^{-0}x(0)}{(9+e^0)(1+x(0))} = \frac{0}{(9+1)(1+0)} = \frac{0}{10} = 0; \\
&\mathcal{B}\Big(0, x_0, Px(0), Qx(0)\Big) = \frac{1}{(0+12)}\frac{x(0)}{1+x(0)} + \int 0\sqrt{2s+1}x(s)ds \\
&\qquad + \int s^3 cos(s^4 - 0)x(s)ds. \\
&\mathcal{B}\Big(0, x_0, Px(0), Qx(0)\Big) = \frac{0}{12} + 0 + \frac{1}{4}\sin 0 = 0
\end{aligned}
$$

Then $\Big\|Px(t) - Px_1(t)\Big\| \leq \frac{1}{3}\|t - t_1\|$ and $\Big\|Qx(t) - Qx_1(t)\Big\| \leq \frac{1}{4}\|t - t_1\|$.

$$
\begin{aligned}
&\Big\|\mathcal{B}\Big(t, x_t, Px(t), Qx(t)\Big) - \mathcal{B}\Big(t_1, x_1t, Px_1(t), Qx_1(t)\Big)\Big\| \\
&\leq \frac{1}{12}\|x(t) - x_1(t)\| + \frac{1}{3}\|x(t) - x_1(t)\| \\
&\quad + \frac{1}{4}\|x(t) - x_1(t)\| \\
&\leq \frac{2}{3}\|x - x_1\|.
\end{aligned}
$$

Hence the conditions $A1 - A4$ hold true with $\mathcal{L}_1 = \frac{2}{3}, \quad \mathbb{N} = \frac{1}{3}, \quad \mathbb{C} = \frac{1}{4}, \quad \mathcal{N}^* = \frac{1}{2},$

$\|\sigma(t, x_t)\| \leq \frac{1}{10}$ and $\varrho^* < 1$. Therefore, the above problem has following solution

$$x_n(t) = 1 + \phi_0 + \sigma_{n-1}(t) + (1 - v)f_{n-1}(t) + \frac{1 - v}{\Gamma v} \int_0^t (t - s)^{v-1} f_{n-1}(s)ds.$$

6. Conclusion

The theory of fractional operators with non-singularity is contemporary, and it is essential to study qualitative attributes of mixed integrodifferential equations involving ABC- derivatives. In this paper, we have proved EUS of ABC - fractional dynamical system using Schaefer's and Banach's fixed - point theorem. As a future work, it would be interesting to find new properties and numerical methods of this ABC - fractional operator and some real applications for main results of this paper.

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