

## AN APPLICATION OF DOMINATION IN VAGUE FUZZY INCIDENCE GRAPHS

K.KALAIARASI \*, P.GEETHANJALI

**Abstract.** Fuzzy Graphs (FGs), also known as Fuzzy Incidence Graphs (FIGs), are a well-organized and useful tool for capturing and resolving a range of real-world scenarios involving ambiguous data and information. In this paper, the Composition of Two Vague Fuzzy Incidence Graphs (CT-VFIGs) and use incidence pairs to extend the idea of FG dominance to CT-VFIGs defined. Examples are used to clarify the concepts of Edge Incidentally Dominating Set (EIDS), Strong Edge Incidentally Dominating Set (SEIDS), and Weak Edge Incidentally Dominating Set (WEIDS). CT-VFIGs have an Edge Incidentally Domination Number (EIDN), a Strong Edge Incidentally Domination Number (SEIDN), and a Weak Edge Incidentally Domination Number (WEIDN). In the research field, CT-VFIGs are used to find the best combinations of journal publications that express the most progress and the least amount of non-progress. The results of our investigation are compared to those of other studies. Our research will help us fully appreciate and comprehend the additional properties of CT-VFIGs. Another benefit of our research is that it will aid in determining the maximum percentage of progress and the minimum percentage of non-progress in various journal publications.

**KEYWORDS:** Vague Fuzzy Incidence Graph, Composition of two VFIGs, Strong Edge Incidentally Dominating Set, Weak Edge Incidentally Dominating Set.

### 1. Introduction

Zadeh [40] [42] [43] introduced fuzzy set theory and related fuzzy logic as a technique of dealing with and addressing a wide range of situations in which variables, parameters, and relationships are only approximated, necessitating the employment of approximate reasoning systems. This is true of practically all nontrivial occurrences, processes, and systems that exist in reality, and standard binary logic mathematics cannot sufficiently characterize them. We summarise Gorzalczy's work on interval-valued fuzzy sets (IVFSs) [8] and Roy *et al.* [29] works on fuzzy relations because interval-valued fuzzy graphs (IVFGs) are commonly employed. Vague sets (VSs) were first proposed by W.L Gau and D.J Buehrer [7]. FG operations were investigated by R. Parvathi *et al.* [22]. In vague graphs (VGs), N. Ramakrishna [6] developed the concepts. In IFGs, A. N. Gani [9] developed the concepts of degree, order, and size. S. Samanta and M. Pal [30] have also expressed many FGs. H. Rashmanlou and M. Pal [26] advised irregular IVFGs. Akram. M [2] proposed vague hyper graphs. Degree of vertices in VGs were proposed by Borzooei [3]. Dinesh [5] looked at the topic of FIGs. Borzooei *et al.* [4] suggested and implemented regularity of VGs. Kalaiarasi & Mahalakshmi have also articulated and Kalaiarasi & Gopinath discussed fuzzy strong graphs. Akram *et al.* [1] proposed the concept Cayley VGs. S. Mathew and J.N.

Mordeson [17] proposed concepts in FIGs. Mordeson *et al.* [19] talked about VFIGs. Properties of VGs extended by Rao *et al.* [27].

Ore and Berge were the first to introduce dominance. IrfanNazeer *et al.* [11], developed the new graph's product. Haynes and Hedetniemi [10] looked into dominance in graphs further. Somasundaram and Somasundaram [33] have gained supremacy in FGs by utilising effective edges. In FGs, Xavior *et al.* [38] suggested dominance. PradipDebnath [23] has also shown dominance in IVFGs. For FGs, Revathi and Harinarayanan [28] developed an equitable domination number. Sunitha & Manjusha [34] have also declared that they have a stronghold. Nagoorgani & Chandrasekaran [21] have also demonstrated dominance in a FG. Sarala & Kavitha [35] have also expressed (1,2)-domination for FGs. Dharmalingam & Nithya [6] have also provided dominance values for FGs. Manjusha *et al.* [18] have studied paired domination. In FIGs, IrfanNazeer *et al.* [12] have achieved dominance. AN Shain and MMQ Shubatah [36] advocated the inverted dominating set of IVFGs. Kalaiarasi & Sabina have also expressed fuzzy inventory EOQ optimization mathematical model [15]. Kalaiarasi & Gopinath suggested fuzzy inventory order EOQ model with machine learning [16]. A new path graph definition was proposed by Tushar *et al.* [32]. A. Nagoor Gani *et al.* [10] addressed domination in FGs. AM Ismayil and HS Begum [4] have both accurately depicted split dominance. In ambiguous graphs, Yongsheng Rao *et al.* [39] established dominance. Shanmugavadivu and Gopinath suggested non homogeneous ternary five degrees equation [31]. Shanmugavadivu and Gopinath have also expressed on the homogeneous five degree equation [32]. Priyadharshini *et al.* have also expressed a fuzzy MCDM approach for measuring the business impact of employee selection [24]. and Mapreduce Methodology for Elliptical Curve Discrete Logarithmic Problems [41].

Section 2 gives some preliminary findings that are required in order to understand the rest of the paper. A definition of CT-VIFGs is given in section 3. In section 4, we look at the relationship between CT-VFIG order and size. Domination in CT-VFIGs is discussed in Section 5. Strong and weak domination in CT-VFIGs is discussed in section 6. The application of CT-VFIGs is discussed in section 7. A comparative analysis is offered in section 8.

## 2. Preliminaries

### Definition 2.1 [12]

Assume  $G_I = (V_I, E_I)$  is a graph. Then,  $G_I = (V_I, E_I, I_I)$  is named as an incidence graph, where  $I_I \subseteq V_I \times E_I$ .

### Definition 2.2 [12]

Assume  $G_{FS} = (V_{FS}, E_{FS})$  is a graph,  $\mu_{FS}$  is a fuzzy subset of  $V_{FS}$ , and  $\gamma_{FS}$  is a fuzzy subset of  $V_{FS} \times V_{FS}$ . Let  $\psi_{FS}$  be a fuzzy subset of  $V_{FS} \times E_{FS}$ . If  $\psi_{FS}(w_{11}, w_{11}w_{22}) \leq \min\{\mu_{FS}(w_{11}), \gamma_{FS}(w_{11}w_{22})\}$  for every  $w_{11} \in V_{FS}, w_{11}w_{22} \in E_{FS}$ , then  $\psi_{FS}$  is a fuzzy incidence of  $G_{FS}$ .

### Definition 2.3 [12]

Assume  $G_I$  is a graph and  $(\mu_I, \gamma_I)$  is a fuzzy sub graph of  $G_I$ . If  $\psi_I$  is a fuzzy incidence of  $G_I$ , then  $G_I = (\mu_I, \gamma_I, \psi_I)$  is named as FIG of  $G_I$ .

**Definition 2.4 [4]**

A VS  $A$  is a pair  $(t_{VA}, f_{VA})$  on set  $V$  where  $t_{VA}$  and  $f_{VA}$  are taken as real valued functions which can be defined on  $V \rightarrow [0,1]$ , so that  $t_{VA}(w_{11}) + f_{VA}(w_{11}) \leq 1$ , for all  $w_{11}$  belongs  $V$ . The interval  $[t_{VA}(w_{11}), 1 - f_{VA}(w_{11})]$  is known as the vague value of  $w_{11}$  is  $A$ .

**Definition 2.5[6]**

A pair  $G_V = (A, B)$  is said to be a VG on a crisp graph  $G = (V, E)$ , where  $A = (t_{VA}, f_{VA})$  is a VS on  $V$  and  $B = (t_{VB}, f_{VB})$  is a VS on  $E \subseteq V \times V$  such that  $t_{VB}(w_{11}w_{22}) \leq \min(t_{VA}(w_{11}), t_{VA}(w_{22}))$  and  $f_{VB}(w_{11}w_{22}) \geq \max(f_{VA}(w_{11}), f_{VA}(w_{22}))$ , for each edge  $w_{11}w_{22} \in E$

**Definition 2.6**

An VFIG is of the form  $G_{VI} = (V_{VI}, E_{VI}, I_{VI}, A_{VI}, B_{VI}, C_{VI})$  where  $A_{VI} = (t_{A_{VI}}, f_{A_{VI}})$ ,  $B_{VI} = (t_{B_{VI}}, f_{B_{VI}})$ ,  $C_{VI} = (t_{C_{VI}}, f_{C_{VI}})$  and  $V_{VI} = \{w_0, w_1, \dots, w_n\}$  such that  $t_{A_{VI}}: V_{VI} \rightarrow [0,1]$  and  $f_{A_{VI}}: V_{VI} \rightarrow [0,1]$  represent the degree (DG) of membership (MS) and non membership (NMS) of the vertex  $w_{ii} \in V_{VI}$  respectively, and  $0 \leq t_{A_{VI}} + f_{A_{VI}} \leq 1$  for each  $w_{ii} \in V_{VI} (i = 1, 2, \dots, n)$ ,  $t_{B_{VI}}: V_{VI} \times V_{VI} \rightarrow [0,1]$  and  $f_{B_{VI}}: V_{VI} \times V_{VI} \rightarrow [0,1]$  show the DG of MS and NMS of the edge  $(w_{11}, w_{22})$  respectively, such that  $t_{B_{VI}}(w_{11}, w_{22}) \leq \min\{t_{A_{VI}}(w_{11}), t_{A_{VI}}(w_{22})\}$  and  $f_{B_{VI}}(w_{11}, w_{22}) \geq \max\{f_{A_{VI}}(w_{11}), f_{A_{VI}}(w_{22})\}$ ,  $0 \leq t_{B_{VI}}(w_{11}, w_{22}) + f_{B_{VI}}(w_{11}, w_{22}) \leq 1$  for every  $(w_{11}, w_{22})$ ,  $t_{C_{VI}}: V_{VI} \times E_{VI} \rightarrow [0,1]$  and  $f_{C_{VI}}: V_{VI} \times E_{VI} \rightarrow [0,1]$ ,  $t_{C_{VI}}(w_{11}, w_{11}w_{22})$  and  $f_{C_{VI}}(w_{11}, w_{11}w_{22})$  show the DG of MS and NMS of the incidence pair respectively, such that  $t_{C_{VI}}(w_{11}, w_{11}w_{22}) \leq \min\{t_{A_{VI}}(w_{11}), t_{B_{VI}}(w_{11}, w_{22})\}$  and  $f_{C_{VI}}(w_{11}, w_{11}w_{22}) \geq \max\{f_{A_{VI}}(w_{11}), f_{B_{VI}}(w_{11}, w_{22})\}$ ,  $0 \leq t_{C_{VI}}(w_{11}, w_{11}w_{22}) + f_{C_{VI}}(w_{11}, w_{11}w_{22}) \leq 1$  for every  $(w_{11}, w_{11}w_{22})$ .

### 3. Composition of two VFIGs

**Definition 3.1**

The Composition of two VFIGs (CT-VFIGs)  $G_{VI}^1 = (V_{VI}^1, E_{VI}^1, I_{VI}^1, A_{VIIP}^1, B_{VIL}^1, C_{VII}^1)$  and  $G_{VI}^2 = (V_{VI}^2, E_{VI}^2, I_{VI}^2, A_{VIIP}^2, B_{VIL}^2, C_{VII}^2)$  is defined as an VFIG

$$G_{CVI} = G_{VI}^1 \Theta G_{VI}^2 = (V_{VI}, E_{VI}, I_{VI}, A_{VIIP}^1 \Theta A_{VIIP}^2, B_{VIL}^1 \Theta B_{VIL}^2, C_{VII}^1 \Theta C_{VII}^2) \text{ where } V_{CVI} = V_{VI}^1 \Theta V_{VI}^2$$

$$\text{and } E_{CVI} = \{((m_{11}, n_{11}), (m_{22}, n_{22})) / m_{11} = m_{22}, (n_{11}, n_{22}) \in E_{VI}^2 \text{ or } n_{11} = n_{22}, (m_{11}, m_{22}) \in E_{VI}^1\}$$

$$I_{CVI} =$$

$$\{(m_{11}, n_{11}), (m_{11}, n_{11})(m_{11}, n_{22}) / m_{11} = m_{22}, (n_{11}, n_{11}n_{22}) \in I_{VI}^2, (n_{22}, n_{11}n_{22}) \in I_{VI}^2 \text{ or } n_{11} = n_{22}, (m_{11}, m_{11}m_{22}) \in I_{VI}^1, (m_{22}, m_{11}m_{22}) \in I_{VI}^1\} \text{ with}$$

$$(A_{VIIP}^1 \Theta A_{VIIP}^2)(m_{11}, n_{11}) = \min\{A_{VIIP}^1(m_{11}), A_{VIIP}^2(n_{11})\} \forall (m_{11}, n_{11}) \in V_{VI}^1 \Theta V_{VI}^2,$$

$$(A_{VIIP}^1 \boxtimes A_{VIIP}^2)(m_{11}, n_{11}) = \max\{A_{VIIP}^1(m_{11}), A_{VIIP}^2(n_{11})\} \forall (m_{11}, n_{11}) \in V_{VI}^1 \boxtimes V_{VI}^2$$

$$\begin{aligned}
 & (B_{1VIL}^1 \boxtimes B_{1VIL}^2)((m_{11}, n_{11})(m_{22}, n_{22})) \\
 &= \begin{cases} \min\{A_{1VIP}^1(m_{11}), B_{1VIL}^2(n_{11}, n_{22})\}, & \text{if } m_{11} = m_{22}, (n_{11}, n_{22}) \in E_{VI}^2 \\ \min\{B_{1VIL}^1(m_{11}, m_{22}), A_{1VIP}^2(n_{11})\}, & \text{if } n_{11} = n_{22}, (m_{11}, m_{22}) \in E_{VI}^1 \\ \min\{B_{1VIL}^1(m_{11}, m_{22}), A_{1VIP}^2(n_{11}), A_{1VIP}^2(n_{22})\}, & \text{if } n_{11} \neq n_{22}, (m_{11}, m_{22}) \in E_{VI}^1 \end{cases} \\
 & (B_{2VIL}^1 \boxtimes B_{2VIL}^2)((m_{11}, n_{11})(m_{22}, n_{22})) \\
 &= \begin{cases} \max\{A_{2VIP}^1(m_{11}), B_{2VIL}^2(n_{11}, n_{22})\}, & \text{if } m_{11} = m_{22}, (n_{11}, n_{22}) \in E_{VI}^2 \\ \max\{B_{2VIL}^1(m_{11}, m_{22}), A_{2VIP}^2(n_{11})\}, & \text{if } n_{11} = n_{22}, (m_{11}, m_{22}) \in E_{VI}^1 \\ \max\{B_{1VIL}^1(m_{11}, m_{22}), A_{1VIP}^2(n_{11}), A_{1VIP}^2(n_{22})\}, & \text{if } n_{11} \neq n_{22}, (m_{11}, m_{22}) \in E_{VI}^1 \end{cases} \\
 & \quad (C_{1VII}^1 \boxtimes C_{1VII}^2)[(m_{11}, n_{11}), (m_{11}, n_{11})(m_{11}, n_{22})] \\
 & \quad \quad = \min\{A_{1VIP}^1(m_{11}), C_{1VII}^2(n_{11}, n_{11}n_{22})\} \text{ if } m_{11} \\
 & \quad \quad = m_{22}, (n_{11}, n_{11}n_{22}) \in I_{VI}^2 \\
 & \quad (C_{1VII}^1 \boxtimes C_{1VII}^2)[(m_{11}, n_{22}), (m_{11}, n_{11})(m_{11}, n_{22})] \\
 & \quad \quad = \min\{A_{1VIP}^1(m_{11}), C_{1VII}^2(n_{22}, n_{11}n_{22})\} \text{ if } m_{11} \\
 & \quad \quad = m_{22}, (n_{22}, n_{11}n_{22}) \in I_{VI}^2 \\
 & \quad (C_{1VII}^1 \boxtimes C_{1VII}^2)[(m_{11}, n_{11}), (m_{11}, n_{11})(m_{22}, n_{11})] \\
 & \quad \quad = \min\{C_{1VII}^1(m_{11}, m_{11}m_{22}), A_{1VIP}^2(n_{11})\} \text{ if } n_{11} \\
 & \quad \quad = n_{22}, (m_{11}, m_{11}m_{22}) \in I_{VI}^1 \\
 & \quad (C_{1VII}^1 \boxtimes C_{1VII}^2)[(m_{22}, n_{11}), (m_{11}, n_{11})(m_{22}, n_{11})] \\
 & \quad \quad = \min\{C_{1VII}^1(m_{22}, m_{11}m_{22}), A_{1VIP}^2(n_{11})\} \text{ if } n_{11} \\
 & \quad \quad = n_{22}, (m_{22}, m_{11}m_{22}) \in I_{VI}^1 \\
 & \quad (C_{1VII}^1 \boxtimes C_{1VII}^2)[(m_{11}, n_{11}), (m_{11}, n_{11})(m_{22}, n_{22})] \\
 & \quad \quad = \min\{C_{1VII}^1(m_{11}, m_{11}m_{22}), A_{1VIP}^2(n_{11}), A_{1VIP}^2(n_{22})\}, \text{ if } m_{11} \\
 & \quad \quad \neq m_{22}, n_{11} \neq n_{22}, (m_{11}, m_{11}m_{22}) \in I_{VI}^1 \\
 & \quad (C_{1VII}^1 \boxtimes C_{1VII}^2)[(m_{22}, n_{22}), (m_{11}, n_{11})(m_{22}, n_{22})] \\
 & \quad \quad = \min\{C_{1VII}^1(m_{22}, m_{11}m_{22}), A_{1VIP}^2(n_{11}), A_{1VIP}^2(n_{22})\}, \text{ if } m_{11} \\
 & \quad \quad \neq m_{22}, n_{11} \neq n_{22}, (m_{22}, m_{11}m_{22}) \in I_{VI}^1 \\
 & \quad (C_{1VII}^1 \boxtimes C_{1VII}^2)[(m_{11}, n_{22}), (m_{11}, n_{22})(m_{22}, n_{11})] \\
 & \quad \quad = \min\{C_{1VII}^1(m_{11}, m_{11}m_{22}), A_{1VIP}^2(n_{11}), A_{1VIP}^2(n_{22})\}, \text{ if } m_{11} \\
 & \quad \quad \neq m_{22}, n_{11} \neq n_{22}, (m_{11}, m_{11}m_{22}) \in I_{VI}^1 \\
 & \quad (C_{1VII}^1 \boxtimes C_{1VII}^2)[(m_{22}, n_{11}), (m_{11}, n_{22})(m_{22}, n_{11})] \\
 & \quad \quad = \min\{C_{1VII}^1(m_{22}, m_{11}m_{22}), A_{1VIP}^2(n_{11}), A_{1VIP}^2(n_{22})\}, \text{ if } m_{11} \\
 & \quad \quad \neq m_{22}, n_{11} \neq n_{22}, (m_{22}, m_{11}m_{22}) \in I_{VI}^1 \\
 & \quad (C_{2VII}^1 \boxtimes C_{2VII}^2)[(m_{11}, n_{11}), (m_{11}, n_{11})(m_{11}, n_{22})] \\
 & \quad \quad = \max\{A_{2VIP}^1(m_{11}), C_{2VII}^2(n_{11}, n_{11}n_{22})\} \text{ if } m_{11} \\
 & \quad \quad = m_{22}, (n_{11}, n_{11}n_{22}) \in I_{VI}^2 \\
 & \quad (C_{2VII}^1 \boxtimes C_{2VII}^2)[(m_{11}, n_{22}), (m_{11}, n_{11})(m_{11}, n_{22})] \\
 & \quad \quad = \max\{A_{2VIP}^1(m_{11}), C_{2VII}^2(n_{22}, n_{11}n_{22})\} \text{ if } m_{11} \\
 & \quad \quad = m_{22}, (n_{22}, n_{11}n_{22}) \in I_{VI}^2 \\
 & \quad (C_{2VII}^1 \boxtimes C_{2VII}^2)[(m_{11}, n_{11}), (m_{11}, n_{11})(m_{22}, n_{11})] \\
 & \quad \quad = \max\{C_{2VII}^1(m_{11}, m_{11}m_{22}), A_{2VIP}^2(n_{11})\} \text{ if } n_{11} \\
 & \quad \quad = n_{22}, (m_{11}, m_{11}m_{22}) \in I_{VI}^1 \\
 & \quad (C_{2VII}^1 \boxtimes C_{2VII}^2)[(m_{22}, n_{11}), (m_{11}, n_{11})(m_{22}, n_{11})] \\
 & \quad \quad = \max\{C_{2VII}^1(m_{22}, m_{11}m_{22}), A_{2VIP}^2(n_{11})\} \text{ if } n_{11} \\
 & \quad \quad = n_{22}, (m_{22}, m_{11}m_{22}) \in I_{VI}^1
 \end{aligned}$$

$$\begin{aligned}
 & (C_{2VII}^1 \boxtimes C_{2VII}^2)[(m_{11}, n_{11}), (m_{11}, n_{11})(m_{22}, n_{22})] \\
 & \quad = \max\{C_{2VII}^1(m_{11}, m_{11}m_{22}), A_{1VIP}^2(n_{11}), A_{1VIP}^2(n_{22})\}, \text{ if } m_{11} \\
 & \quad \neq m_{22}, n_{11} \neq n_{22}, (m_{11}, m_{11}m_{22}) \in I_{VI}^1 \\
 & (C_{2VII}^1 \boxtimes C_{2VII}^2)[(m_{22}, n_{22}), (m_{11}, n_{11})(m_{22}, n_{22})] \\
 & \quad = \max\{C_{2VII}^1(m_{22}, m_{11}m_{22}), A_{1VIP}^2(n_{11}), A_{1VIP}^2(n_{22})\}, \text{ if } m_{11} \\
 & \quad \neq m_{22}, n_{11} \neq n_{22}, (m_{22}, m_{11}m_{22}) \in I_{VI}^1 \\
 & (C_{2VII}^1 \boxtimes C_{2VII}^2)[(m_{11}, n_{22}), (m_{11}, n_{22})(m_{22}, n_{11})] \\
 & \quad = \max\{C_{2VII}^1(m_{11}, m_{11}m_{22}), A_{1VIP}^2(n_{11}), A_{1VIP}^2(n_{22})\}, \text{ if } m_{11} \\
 & \quad \neq m_{22}, n_{11} \neq n_{22}, (m_{11}, m_{11}m_{22}) \in I_{VI}^1 \\
 & (C_{2VII}^1 \boxtimes C_{2VII}^2)[(m_{22}, n_{11}), (m_{11}, n_{22})(m_{22}, n_{11})] \\
 & \quad = \max\{C_{2VII}^1(m_{22}, m_{11}m_{22}), A_{1VIP}^2(n_{11}), A_{1VIP}^2(n_{22})\}, \text{ if } m_{11} \\
 & \quad \neq m_{22}, n_{11} \neq n_{22}, (m_{22}, m_{11}m_{22}) \in I_{VI}^1
 \end{aligned}$$

Example 3.2

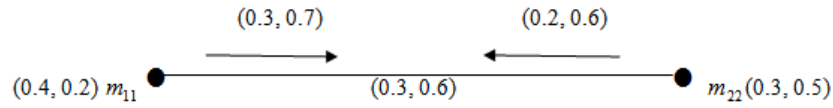


Figure1. VFIG  $G_{VI}^1$

Figure 1 indicates a VFIG  $G_{VI}^1 = (V_{VI}^1, E_{VI}^1, I_{VI}^1, A_{VIP}^1, B_{VIL}^1, C_{VII}^1)$  with  
 $A_{VIP}^1(m_{11}) = (0.4, 0.2), A_{VIP}^1(m_{22}) = (0.3, 0.5), B_{VIL}^1(m_{11}m_{22}) = (0.3, 0.6),$   
 $C_{VII}^1(m_{11}, m_{11}m_{22}) = (0.3, 0.7), C_{VII}^1(m_{22}, m_{11}m_{22}) = (0.2, 0.6)$

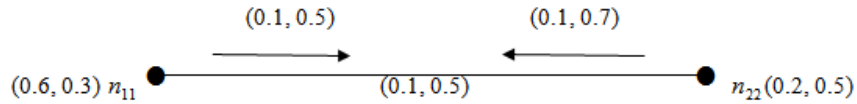


Figure2. VFIG  $G_{VI}^2$

Figure 2 indicates a VFIG  $G_{VI}^2 = (V_{VI}^2, E_{VI}^2, I_{VI}^2, A_{VIP}^2, B_{VIL}^2, C_{VII}^2)$  with  
 $A_{VIP}^2(n_{11}) = (0.6, 0.3), A_{VIP}^2(n_{22}) = (0.2, 0.5), B_{VIL}^2(n_{11}n_{22}) =$   
 $(0.1, 0.5), C_{VII}^2(n_{11}, n_{11}n_{22}) = (0.1, 0.5), C_{VII}^2(n_{22}, n_{11}n_{22}) = (0.1, 0.7).$

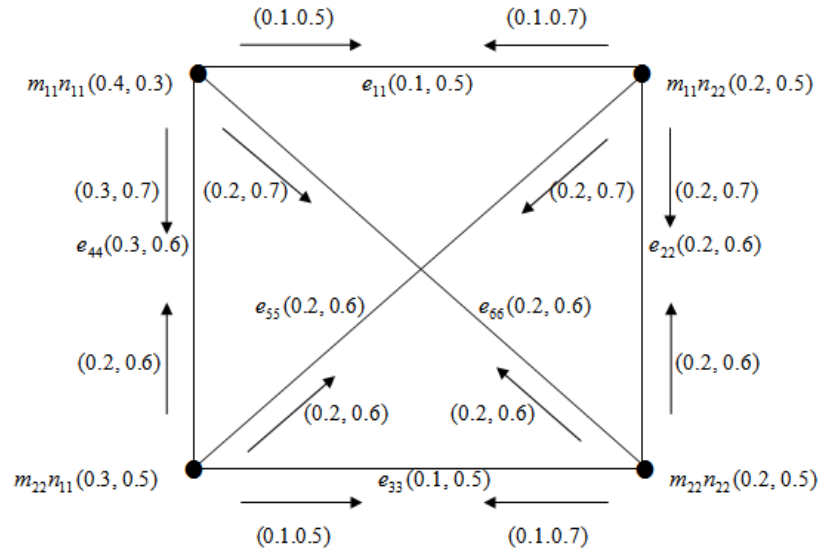


Figure3. Composition of figure 1 and figure 2

Figure 3 indicates a CT-VFIGs

$$\begin{aligned}
 G_{VI}^1 \boxtimes G_{VI}^2 &= (V_{VI}, E_{VI}, I_{VI}, A_{VIP}^1 \boxtimes A_{VIP}^2, B_{VIL}^1 \boxtimes B_{VIL}^2, C_{VII}^1 \boxtimes C_{VII}^2) \\
 (A_{VIP}^1 \boxtimes A_{VIP}^2)(m_{11}, n_{11}) &= (0.4, 0.3), (A_{VIP}^1 \theta A_{VIP}^2)(m_{11}, n_{22}) = (0.2, 0.5) \\
 (A_{VIP}^1 \Theta A_{VIP}^2)(m_{22}, n_{11}) &= (0.3, 0.5) \quad (A_{VIP}^1 \boxtimes A_{VIP}^2)(m_{22}, n_{22}) = (0.2, 0.5) \\
 (B_{VIL}^1 \boxtimes B_{VIL}^2)((m_{11}, n_{11})(m_{11}, n_{22})) &= \\
 0.1, 0.5, (B_{VIL}^1 \theta B_{VIL}^2)((m_{11}, n_{22})(m_{22}, n_{22})) &= 0.2, 0.6, \\
 (B_{VIL}^1 \Theta B_{VIL}^2)((m_{22}, n_{11})(m_{22}, n_{22})) &= 0.1, 0.5, \\
 (B_{VIL}^1 \theta B_{VIL}^2)((m_{11}, n_{11})(m_{22}, n_{11})) &= 0.3, 0.6, \\
 (B_{VIL}^1 \Theta B_{VIL}^2)((m_{11}, n_{11})(m_{22}, n_{22})) &= 0.2, 0.6, \\
 (B_{VIL}^1 \theta B_{VIL}^2)((m_{11}, n_{22})(m_{22}, n_{11})) &= 0.2, 0.6 \\
 (C_{VII}^1 \boxtimes C_{VII}^2)[(m_{11}, n_{11}), (m_{11}, n_{11})(m_{11}, n_{22})] &= (0.1, 0.5) \\
 (C_{VII}^1 \Theta C_{VII}^2)[(m_{11}, n_{22}), (m_{11}, n_{11})(m_{11}, n_{22})] &= \\
 (0.1, 0.7), (C_{VII}^1 \theta C_{VII}^2)[(m_{11}, n_{22}), (m_{11}, n_{22})(m_{22}, n_{22})] &= (0.2, 0.7), \\
 (C_{VII}^1 \boxtimes C_{VII}^2)[(m_{22}, n_{22}), (m_{11}, n_{22})(m_{22}, n_{22})] &= (0.2, 0.6), \\
 (C_{VII}^1 \Theta C_{VII}^2)[(m_{22}, n_{11}), (m_{22}, n_{11})(m_{22}, n_{22})] &= (0.1, 0.5), \\
 (C_{VII}^1 \theta C_{VII}^2)[(m_{22}, n_{22}), (m_{22}, n_{11})(m_{22}, n_{22})] &= (0.1, 0.7), \\
 (C_{VII}^1 \boxtimes C_{VII}^2)[(m_{11}, n_{11}), (m_{11}, n_{11})(m_{22}, n_{11})] &= (0.3, 0.7), \\
 (C_{VII}^1 \Theta C_{VII}^2)[(m_{22}, n_{11}), (m_{11}, n_{11})(m_{22}, n_{11})] &= (0.2, 0.6), \\
 (C_{VII}^1 \theta C_{VII}^2)[(m_{11}, n_{11}), (m_{11}, n_{11})(m_{22}, n_{22})] &= (0.2, 0.7), \\
 (C_{VII}^1 \boxtimes C_{VII}^2)[(m_{22}, n_{22}), (m_{11}, n_{11})(m_{22}, n_{22})] &= (0.2, 0.6), \\
 (C_{VII}^1 \Theta C_{VII}^2)[(m_{22}, n_{11}), (m_{22}, n_{11})(m_{11}, n_{22})] &= (0.2, 0.6), \\
 (C_{VII}^1 \theta C_{VII}^2)[(m_{11}, n_{22}), (m_{22}, n_{11})(m_{11}, n_{22})] &= (0.2, 0.7)
 \end{aligned}$$

**Definition 3.3**

Let  $G_{CVI}$  be a CT-VFIGs

- (i)  $G_{CVI}$  cardinality is determined by

$$|G_{CVI}| = \sum_{w_{11} \in V_{VI}} \frac{1 + t_{AVIP}(w_{11}) - f_{AVIP}(w_{11})}{2} + \sum_{w_{11}w_{22} \in E_{VI}} \frac{1 + t_{BVIL}(w_{11}w_{22}) - f_{BVIL}(w_{11}w_{22})}{2} + \sum_{w_{11}, w_{11}w_{22} \in I_{VI}} \frac{1 + t_{CVII}(w_{11}, w_{11}w_{22}) - f_{CVII}(w_{11}, w_{11}w_{22})}{2}$$

(ii)  $G_{CVI}$  vertex cardinality is determined by  $|V_{CVI}| = \sum_{w_{11} \in V_{CVI}} \frac{1 + t_{AVIP}(w_{11}) - f_{AVIP}(w_{11})}{2} \forall w_{11} \in V_{CVI}$

(iii)  $G_{CVI}$  edge cardinality is specified by  $|E_{CVI}| = \sum_{w_{11}w_{22} \in E_{CVI}} \frac{1 + t_{BVIL}(w_{11}w_{22}) - f_{BVIL}(w_{11}w_{22})}{2} \forall w_{11}w_{22} \in E_{CVI}$

(iv)  $G_{CVI}$  incidence pair cardinality is specified by

$$|I_{CVI}| = \sum_{\substack{w_{11}, w_{11}w_{22} \in I_{CVI} \\ \in I_{CVI}}} \frac{1 + t_{CVII}(w_{11}, w_{11}w_{22}) - f_{CVII}(w_{11}, w_{11}w_{22})}{2} \forall w_{11}, w_{11}w_{22}$$

#### 4. Relationship between order and size of CT-VFIGs

##### Definition 4.1

Assume  $G_{CVI}$  is a CT-VFIGs. Then  $O_{CVI}(G_{CVI}) = \sum_{w_{11} \neq w_{22}, w_{11}, w_{22} \in V_{CVI}} \left( \frac{1 + t_{CVII}(w_{11}, w_{11}w_{22}) - f_{CVII}(w_{11}, w_{11}w_{22})}{2} \right)$  is called order of  $G_{CVI}$  and  $S_{CVI}(G_{CVI}) = \sum_{w_{11}, w_{22} \in E_{CVI}} \left( \frac{1 + t_{BVIL}(w_{11}w_{22}) - f_{BVIL}(w_{11}w_{22})}{2} \right)$  is called size of  $G_{CVI}$ .

##### Definition 4.2

The edge degree of  $ae_{1VI}$  in a CT-VFIGs is defined as the sum of the weights of edges incident to  $e_{1VI}$ . It is defined by  $|d_{G_{CVI}}(e_{1VI})| = \{deg^t(e_{1VI}), deg^f(e_{1VI})\}$ . The minimum cardinality of edge degree of  $G_{CVI}$  is  $\delta_{CVI}(G_{CVI}) = \min\{d_{G_{CVI}}(e_{1VI})/e_{1VI} \in E_{CVI}\}$ . The maximum cardinality of edge degree of  $G_{CVI}$  is  $\Delta_{CVI}(G_{CVI}) = \max\{d_{G_{CVI}}(e_{1VI})/e_{1VI} \in E_{CVI}\}$

##### Proposition 4.3

In a CT-VFIGs  $O_{CVI}(G_{CVI}) \geq S_{CVI}(G_{CVI})$

**Proof.** Let  $G_{CVI}$  be a CT-VFIGs with one node. Then  $O_{CVI}(G_{CVI}) = S_{CVI}(G_{CVI}) = 0$ .

That is  $O_{CVI}(G_{CVI}) = S_{CVI}(G_{CVI})$  (1)

It is a trivial case. Assume  $G_{CVI}$  with more than one nodes.  $O_{CVI}(G_{CVI})$  is the sum of all incidence pairs cardinality of  $G_{CVI}$ . Since incidence pairs are two times of edges. Therefore, the total sum of all the incidence pairs cardinality will always greater than the total sum of all the edge cardinality.

$$O_{CVI}(G_{CVI}) > S_{CVI}(G_{CVI}) \quad (2)$$

From equations (1) and (2), we get  $O_{CVI}(G_{CVI}) \geq S_{CVI}(G_{CVI})$ .

##### Proposition 4.4

For any CT-VFIGs the following inequality holds

$$\delta_{CVI}(G_{CVI}) \leq \square_{CVI}(G_{CVI}) \leq S_{CVI}(G_{CVI}) \leq O_{CVI}(G_{CVI}).$$

**Proof.** Assume  $G_{CVI}$  is a CT-VFIGs with non empty node set.

Since  $\delta_{CVI}(G_{CVI})$  represents lowest edge degree and  $\Delta_{CVI}(G_{CVI})$  denotes highest edge degree of  $G_{CVI}$ .

$$\delta_{CVI}(G_{CVI}) \leq \mathbb{Q}_{CVI}(G_{CVI}) \quad (3)$$

We know  $O_{CVI}(G_{CVI}) = \sum_{w_{11} \neq w_{22}, w_{11}, w_{22} \in V_{CVI}} \left( \frac{1+t_{CVI}(w_{11}, w_{11}w_{22})-f_{CVI}(w_{11}, w_{11}w_{22})}{2} \right)$

$$\text{and } S_{CVI}(G_{CVI}) = \sum_{w_{11}, w_{22} \in E_{CVI}} \left( \frac{1+t_{CVI}(w_{11}, w_{22})-f_{CVI}(w_{11}w_{22})}{2} \right)$$

By definition of size of  $G_{CVI}$ ,  $S_{CVI}(G_{CVI}) = \sum_{w_{11}, w_{22} \in E_{CVI}} \left( \frac{1+t_{CVI}(w_{11}, w_{22})-f_{CVI}(w_{11}w_{22})}{2} \right) \geq \max\{d_{G_{CVI}}(e_{1VI})/e_{1VI} \in E_{CVI}\}$

$$\text{That is } S_{CVI}(G_{CVI}) \geq \mathbb{Q}_{CVI}(G_{CVI}) \quad (4)$$

Also, in a CT-VFIGs,  $G_{CVI}$  by 4.3 proposition

$$O_{CVI}(G_{CVI}) \geq S_{CVI}(G_{CVI}) \quad (5)$$

From inequalities (3), (4) and (5), we obtained  $\delta_{CVI}(G_{CVI}) \leq \mathbb{Q}_{CVI}(G_{CVI}) \leq S_{CVI}(G_{CVI}) \leq O_{CVI}(G_{CVI})$

### 5. Domination in CT-VFIGs

#### Definition 5.1

A edge  $e_{VI}$  in an CT-VFIGs  $G_{CVI}$  is called incidentally dominate edge if  $t_{CVI}(w_{11}, w_{11}w_{22}) = \min\{t_{ACVI}(w_{11}), t_{BCVI}(w_{11}, w_{22})\}$  and  $f_{CVI}(w_{11}, w_{11}w_{22}) = \max\{f_{ACVI}(w_{11}), f_{BCVI}(w_{11}, w_{22})\}$

#### Definition 5.2

A edge  $e_{1VI}$  in an CT-VFIGs  $G_{CVI}$  dominates to edge  $e_{2VI}$  if they are incidentally dominate edges.

#### Definition 5.3

A subset  $R_{CVI}$  of  $E_{CVI}$  is said to be edge incidentally dominating set (EIDS) if for each edge  $e_{1VI}$  not in  $R_{CVI}$ ,  $e_{1VI}$  is dominate at least one edge in  $R_{CVI}$ .

#### Definition 5.4

A edge incidentally dominating set  $R_{CVI}$  of the CT-VFIGs  $G_{CVI}$  is said to be a minimal EIDS of CT-VFIGs  $G_{CVI}$  if each edge in  $R_{CVI}$ , the set  $R_{CVI} - \{e_{1VI}\}$  is not a EIDS.

#### Definition 5.5

AEIDS with the lowest edge cardinality is called a minimum EIDS. The edge cardinality of a minimum EIDS is called edge incidentally dominating number of the CT-VFIGs  $G_{CVI}$ . It is denoted by  $\gamma_{VI}(G_{CVI})$

#### Example 5.6

In figure 3, the incidentally dominating edges are  $\{e_{11}\}, \{e_{22}\}, \{e_{33}\}, \{e_{44}\}, \{e_{55}\}, \{e_{66}\}$  and the EIDSs are  $S_{11} = \{e_{11}e_{22}\}, S_{22} = \{e_{11}e_{33}\}, S_{33} = \{e_{11}e_{44}\}, S_{44} = \{e_{11}e_{55}\}, S_{55} = \{e_{11}e_{66}\}, \dots$ . After calculating the edge cardinality of  $S_{11}, S_{22}, \dots, S_{33}, S_{44}, \dots$ , we obtain  $|S_{11}| = 0.6, |S_{22}| = 0.6, |S_{33}| = 0.65, |S_{44}| = 0.6, |S_{55}| = 0.6, \dots$ . The edge cardinality of a minimum EIDS is  $|S_{11}| = 0.6$  and  $\gamma_{VI}(G_{CVI}) = 0.6$ .

#### Theorem 5.7

Let  $G_{VI}^1 = (A_{VIP}^1, B_{VIL}^1, C_{VII}^1)$  and  $G_{VI}^2 = (A_{VIP}^2, B_{VIL}^2, C_{VII}^2)$  be two VFIGs. Then  $\gamma_{VI}(G_{CVI}) = \min\{A_{VIP}^1(m_{11}), A_{VIP}^2(n_{11})\}$  where  $m_{11} \in G_{VI}^1$  and  $n_{11} \in G_{VI}^2$



**Proof.** Assume  $G_{VI}^1 = (A_{VIP}^1, B_{VIL}^1, C_{VII}^1)$  and  $G_{VI}^2 = (A_{VIP}^2, B_{VIL}^2, C_{VII}^2)$  are two VFIGs. Since  $G_{VI}^1$  and  $G_{VI}^2$  are two VFIGs, then  $G_{VI}^1 \theta G_{VI}^2$  will be a VFIGs. So, each two edges in  $G_{VI}^1 \theta G_{VI}^2$  will dominates remaining edges. Then by definition of EIDN,  $\gamma_{VI}(G_{CVI}) = \min\{\text{cardinality of } (A_{VIP}^1(m_{11}), A_{VIP}^2(n_{11}))\}$ .

**Theorem 5.8**

Let  $G_{VI}^1 = (A_{VIP}^1, B_{VIL}^1, C_{VII}^1)$  and  $G_{VI}^2 = (A_{VIP}^2, B_{VIL}^2, C_{VII}^2)$  be two VFIGs with  $k \geq 2$  and  $l \geq 2$ , where  $k$  and  $l$  are representing the number of vertices in  $G_{VI}^1$  and  $G_{VI}^2$ , respectively. Then  $\frac{\gamma_{VI}(G_{CVI})}{2} = \min\{\text{cardinality of } (B_{VIL}^1(m_{11}m_{22}), B_{VIL}^2(n_{11}n_{22}))\}$ .

**Proof.** Consider  $G_{VI}^1 = (A_{VIP}^1, B_{VIL}^1, C_{VII}^1)$  and  $G_{VI}^2 = (A_{VIP}^2, B_{VIL}^2, C_{VII}^2)$  are two VFIGs. Since  $G_{VI}^1$  and  $G_{VI}^2$  are VFIGs. Then  $G_{VI}^1 \theta G_{VI}^2$  will also a VFIG with  $\frac{\gamma_{VI}(G_{CVI})}{2} = \min\{\text{cardinality of } (B_{VIL}^1(m_{11}m_{22}), B_{VIL}^2(n_{11}n_{22}))\}$  because each two edges in  $G_{VI}^1 \theta G_{VI}^2$  dominates to all remaining edges.

## 6. Strong and Weak Domination in CT-VFIGs

**Definition 6.1**

Let  $G_{CVI}$  be a CT-VFIGs. For any two edges  $e_{1VI}, e_{2VI} \in E_{CVI}$ ,  $e_{1VI}$  strongly dominates  $e_{2VI}$  in CT-VFIGs  $G_{CVI}$  if

- (i) they are incidentally dominate edges
- (ii)  $deg^t(e_{1VI}) \geq deg^t(e_{2VI}), deg^f(e_{1VI}) \leq deg^f(e_{2VI})$

Similarly  $e_{1VI}$  weakly dominates  $e_{2VI}$  if

- (i) they are incidentally dominate edges
- (ii)  $deg^t(e_{2VI}) \geq deg^t(e_{1VI}), deg^f(e_{2VI}) \leq deg^f(e_{1VI})$

**Definition 6.2**

An edge incidentally dominating set  $R_{CVI} \subseteq E_{CVI}$  is called a strong (weak) edge incidentally dominating set (SEIDS, WEIDS) of  $G_{CVI}$  if, for each  $e_{1VI} \in E_{CVI} - R_{CVI}$ , there exist at least one edge  $e_{2VI} \in R_{CVI}$ , so that  $e_{1VI}$  strongly (weakly) dominates  $e_{2VI}$ . The strong (weak) edge incidentally domination number of  $G_{CVI}$  denoted by  $\gamma_{SVI}(G_{CVI})$   $\gamma_{WVI}(G_{CVI})$ , is called as the minimum cardinality of a strong (weak) edge incidentally dominating set of  $G_{CVI}$ .

**Example 6.3**

In figure 3, the incidentally dominating edges are  $\{e_{11}\}, \{e_{22}\}, \{e_{33}\}, \{e_{44}\}, \{e_{55}\}, \{e_{66}\}$  and the SEIDSs are  $S_{11} = \{e_{11}e_{22}\}, S_{22} = \{e_{11}e_{44}\}, S_{33} = \{e_{22}e_{33}\}, S_{44} = \{e_{33}e_{44}\}$ . After calculating the edge cardinality of  $S_{11}, S_{22}, S_{33}, S_{44}$  we obtain  $|S_{11}| = 0.6, |S_{22}| = 0.65, |S_{33}| = 0.6, |S_{44}| = 0.6$ . The edge cardinality of a minimum SEIDS is  $|S_{11}| = 0.6$  and  $\gamma_{SVI}(G_{CVI}) = 0.6$ . The WEIDSs are  $S_{55} = \{e_{11}e_{55}\}, S_{66} = \{e_{11}e_{66}\}, S_{77} = \{e_{33}e_{66}\}$ . After calculating the edge cardinality of  $S_{55}, S_{66}, S_{77}$  we obtain  $|S_{55}| = 0.6, |S_{66}| = 0.6, |S_{77}| = 0.6$ . The edge cardinality of a minimum WEIDS is  $|S_{55}| = 0.6$  and  $\gamma_{WVI}(G_{CVI}) = 0.6$ .

**Theorem 6.4**

Let  $G_{CVI}$  be a CT-VFIGs without single node and  $R_{CVI}$  be a minimum SEIDS of  $G_{CVI}$ , then  $E_{CVI} - R_{CVI}$  is an SEIDS of CT-VFIGs.

**Proof.** Let  $G_{CVI}$  be a CT-VFIGS with minimum SEIDS, then for each edge  $e_{2VI} \in R_{CVI}$ , there is at least one edge  $e_{1VI} \in E_{CVI} - N_{CVI}$  so that  $deg^t(e_{1VI}) \geq deg^t(e_{2VI}), deg^f(e_{1VI}) \leq deg^f(e_{2VI})$  and  $t_{C_{CVI}}(w_{11}, w_{11}w_{22}) = \min\{t_{A_{CVI}}(w_{11}), t_{B_{CVI}}(w_{11}, w_{22})\}, f_{C_{CVI}}(w_{11}, w_{11}w_{22}) = \max\{f_{A_{CVI}}(w_{11}), f_{B_{CVI}}(w_{11}, w_{22})\}$ . Hence  $E_{CVI} - R_{CVI}$  strongly dominates each edge of  $R_{CVI}$ . So,  $E_{CVI} - R_{CVI}$  is an SEIDS of CT-VFIGS.

**Theorem 6.5**

Let  $G_{CVI}$  be a CT-VFIGS without single node and  $R_{CVI}$  be a minimum WEIDS of  $G_{CVI}$ , then  $E_{CVI} - R_{CVI}$  is an WEIDS of CT-VFIGS.

**Theorem 6.6**

For any CT-VFIGS with  $t_{C_{CVI}}(w_{11}, w_{11}w_{22}) = \min\{t_{A_{CVI}}(w_{11}), t_{B_{CVI}}(w_{11}, w_{22})\}$  and  $f_{C_{CVI}}(w_{11}, w_{11}w_{22}) = \max\{f_{A_{CVI}}(w_{11}), f_{B_{CVI}}(w_{11}, w_{22})\}$  for all  $w_{11} \in V_{CVI}, w_{11}w_{22} \in E_{CVI}$ , then  $\gamma_{SVI} = \gamma_{WVI}$ .

**Proof.** Let  $G_{CVI}$  be a CT-VFIGS with  $t_{C_{CVI}}(w_{11}, w_{11}w_{22}) = \min\{t_{A_{CVI}}(w_{11}), t_{B_{CVI}}(w_{11}, w_{22})\}$  and  $f_{C_{CVI}}(w_{11}, w_{11}w_{22}) = \max\{f_{A_{CVI}}(w_{11}), f_{B_{CVI}}(w_{11}, w_{22})\}$ . Assume for every node have same or different value. Since  $G_{CVI}$  is CT-VFIGS with  $t_{B_{CVI}}(w_{11}, w_{22}) = \min\{t_{A_{CVI}}(w_{11}), t_{A_{CVI}}(w_{22})\}$  and  $f_{B_{CVI}}(w_{11}, w_{22}) = \max\{f_{A_{CVI}}(w_{11}), f_{A_{CVI}}(w_{22})\}$  for all  $w_{11}, w_{22} \in V_{CVI}$  and  $t_{C_{CVI}}(w_{11}, w_{11}w_{22}) = \min\{t_{A_{CVI}}(w_{11}), t_{B_{CVI}}(w_{11}, w_{22})\}$  and  $f_{C_{CVI}}(w_{11}, w_{11}w_{22}) = \max\{f_{A_{CVI}}(w_{11}), f_{B_{CVI}}(w_{11}, w_{22})\}$  for all  $w_{11} \in V_{CVI}, w_{11}w_{22} \in E_{CVI}$ . Thus every  $e_{1VI}e_{2VI} \in E_{CVI}$  is SEIDS as well as WEIDS. Therefore  $\gamma_{SVI} = \gamma_{WVI}$ .

**Theorem 6.7**

For a CT-VFIGS, the below inequalities are true.

- (i)  $\gamma_{VI} \leq \gamma_{SVI} \leq O_{CVI}(G_{CVI}) - \max i mum d_{G_{CVI}} \text{ of } G_{CVI}$ .
- (ii)  $\gamma_{VI} \leq \gamma_{WVI} \leq O_{CVI}(G_{CVI}) - \min i mum d_{G_{CVI}} \text{ of } G_{CVI}$ .

**Proof.** (i) From definition 6.1 and 6.2 we have  $\gamma_{VI} \leq \gamma_{SVI}$  (6)

We know  $O_{CVI}(G_{CVI}) =$  the sum of the incidence pair of CT-VFIGS.

Also  $O_{CVI}(G_{CVI}) -$  not including the maximum  $d_{G_{CVI}}$  of CT-VFIGS

$$= O_{CVI}(G_{CVI}) - \mathbb{Q}_{CVI}(G_{CVI})$$

(7)

From equation (6) and (7)

$$\gamma_{VI} \leq \gamma_{SVI} \leq O_{CVI}(G_{CVI}) - \max i mum d_{G_{CVI}} \text{ of } G_{CVI}$$

- (ii) From definition 6.1 and 6.2 domination number  $\gamma_{VI}$  of CT-VFIGS is less than or equal to the  $\gamma_{WVI}$  of CT-VFIGS, because the edges of WEIDSM $_{CVI}$ , it weakly dominates any one of the edges of  $E_{CVI} - M_{CVI}$ .

Therefore

$$\gamma_{CVI}(G_{CVI}) \geq \gamma_{VI}(G_{CVI})$$

(8)

Also  $O_{CVI}(G_{CVI}) -$  not including the minimum  $d_{G_{CVI}}$  of CT-VFIGS

$$=O_{CVI}(G_{CVI}) - \delta_{CVI}(G_{CVI})$$

(9)

From equation (8) and (9), we get

$$\gamma_{VI} \leq \gamma_{WVI} \leq O_{CVI}(G_{CVI}) - \text{minimum } d_{G_{CVI}} \text{ of } G_{CVI}$$

### 7. Real-Life Application of CT-VFIGs

An application of CT-VFIGs is included here. Consider two networks (CT-VFIGs)  $G_{VI}^1$  and  $G_{VI}^2$ , which have two and two vertices, respectively, and show distinct journal publications from different journals of a research filed. The vertices MS value indicates the percentage of accepted research papers in journal publishing, while the NMS value represents the rejected research papers. The MS value of the edges indicates that the journal publications are mutually collaborative, whereas the NMS value indicates that the journal publications are not mutually collaborative. The MS value of the incidence pairs represents the percentage of progress, whereas the NMS value represents the percentage of journal publications that have not progressed. As in figure 3 composition of  $G_{VI}^1$  and  $G_{VI}^2$  show the percentage of progress of journal publication  $m_{11}$  with journal publications  $n_{11}$  and  $n_{22}$  has the maximum MS value and the percentage of non progress of journal publication  $m_{11}$  with journal publications  $n_{11}$  and  $n_{22}$  has the lesser NMS value. As a result, the best suited combinations of journal publications demonstrating the largest percentage of progress and the lowest percentage of non-progress in the research field exist.

### 8. Comparative Analysis

In figure 3, calculate the edge cardinality of each edge, we get all the edges have same value. In our study the edge degree cardinality of the CT-VIFGs  $|d_{G_{CVI}}(e_{1VI})| = 0.2$  and  $|d_{G_{CVI}}(e_{2VI})| = 0.3$  are not all the same. It can be observed that the edge degree of the edges  $|d_{G_{CVI}}(e_{1VI})| = \{0.9, 2.4\}$  shows the percentage of progress of journal publication  $m_{11}$  with journal publications  $n_{11}$  and  $n_{22}$  has the maximum MS value and the percentage of non progress of journal publication  $m_{11}$  with journal publications  $n_{11}$  and  $n_{22}$  has the lesser NMS value. As a result, the current method is ineffective in determining which journal publications have the highest percentage of progress and the lowest percentage of non-progress. The current method is useful for single networks, but it is insufficient to explain the overall impact of different networks' products. However, we may use composition to discuss the overall impact of combining multiple networks in our strategy. Our technique works with several networks as well as a single network. This allows us to discuss the overall influence of various networks products. As a result, our proposed strategy outperforms the existing one.

### 9. Conclusion

CT-VFIGs are extremely useful tools for researching a variety of computational intelligence and computer science topics. CT-VFIGs are used in a variety of fields, including natural networks and operations research. Three new CT-VIFG concepts in this article EIDS, SEIDS, and WEIDS. In the CT-VFIGs, some advantageous and

instrumental theorems of domination are also explained. A study of the makeup of VFIGs in the field of research is also included. Our research into CT-VFIG coloring, Hamiltonian CT-VFIGs, and CT-VFIG chromaticity in the future. The results of future research on these concepts will be revealed in upcoming papers.

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