

MATHEMATICAL MODEL FOR INSULIN THERAPY USING FUZZY RELIABILITY

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ABSTRACT. The total working principle of this project concentrates towards the study in determining the diabetic level and its effect in parallel intake of insulin. This model leads to determine the diabetic level and the effectiveness as well as securing insulin, Glargine measure up to the Neutral Protamine Hagedorn (NPH), in pregnancy ladies having type 1 diabetes mellitus (T1DM) by using fuzzy reliability function and its alpha cuts. By using this model, level of diabetes would be controlled with the support of insulin therapy treatment more than non insulin therapy patients. This mathematical modelling assists in determining the level of insulin during the treatment.

1. Introduction

As per the Classical Reliability Theory, the life time density functions of every parameter are specific. But on collaborating Fuzziness and randomness together in this model the real time situation problems are handled with positive result. Hence, Zadeh established the Fuzzy Set Theory in 1965 and the mathematics of fuzzy sets were analyzed. Various researchers developed only fuzzy reliability theory, whereas this mathematical model analysis during the treatment in pregnancy ladies to find out the effectiveness of insulin therapy will be more flexible to produce required output or result on time. Nevertheless, prior works analyzed the α -cuts of Fuzzy Reliability using its Gamma distribution.

The Glargine, a basal insulin analog once per day, has the effect of long time of action and absence is obviously high. It is synonymous with low frequency of hypoglycemic episodes and the effectual control of glycemic. Before beginning and throughout pregnancy, it is significant to maintain the good metabolic function in order to reduce the risk of fetal malformations. Owing to the complications by diabetes mellitus in the management of pregnancy, Glargine would be the advisable alternative. But, Due to Glargines clinical utility has not been established, it is not advisable to avail it during pregnancy.

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This section deals with a Fuzzy Reliability Analysis to examine the treatment of insulin therapy in pregnancy ladies as per the fuzzy gamma distribution.

Fuzzification provides varying value instead exact value that will fit into the defined set. The defined set is appropriate or possible to find a particular way. The unique values of the phenomenon would be redefined in the value set of continuum by the predefined membership functions.

Thus unique characteristics of membership in the set are defined in the process of fuzzification. In the phenomenon, every value which is identified specifically to the characteristics of the set would be termed as 1 and not specifically as 0. Values which are between two extremes level lies in the boundary, i.e transitional zone of the set. Values which are not up to the mark of the ideal characteristics of the set have given a decreasing value from 1 to 0 as a continuous scale. The unique phenomenon value would have a low possibility to acts as the member set that is assigned to decreased value.

Be the member of the set as the assigned value decreases. Value 0.5 in the Fuzzification is termed as the cross over point. If it is higher than 0.5 means that then the unique phenomenon value might be the member of the set. If the value goes lesser than 0.5, then it would be consider as the member of the set. But, the values are not probably as the part of the set.

2. FUZZY GAMMA DISTRIBUTION

If λ and r are unknown then we estimate them from a sample and obtain a fuzzy estimator $\bar{\lambda}$ for λ and \bar{r} for r . Now we consider the probability density function of fuzzy gamma distribution for the fuzzy number $\bar{\alpha}$ and $\bar{\beta}$.

$$\bar{f}(t, \lambda, r) = \frac{\lambda^r t^{r-1} e^{-\lambda t}}{\Gamma(r)}, t = 0,,$$

If α and β are unknown we must estimate them from a random sample and we obtain a fuzzy estimator $\bar{\lambda}$ for λ and \bar{r} for r . Now consider the probability density function of fuzzy gamma distribution for the fuzzy numbers $\bar{\lambda}$ and \bar{r} ,

$$\bar{f}(t, \lambda, r) = \frac{\lambda^r t^{(r-1)} e^{-\lambda t}}{\Gamma(r)}, t \geq 0, \lambda \in \bar{\lambda}[\alpha], r \in \bar{r}[\alpha]$$

The fuzzy probability of obtaining a value in the interval $[c, d]$, $c > 0$ is $p(c \leq X \leq d)$ and it's α - cut is defined as

$$\begin{aligned}\bar{P}[c, d][\alpha] &= \int_c^d \bar{f}(t, \lambda, r) dt / \lambda \in \bar{\lambda}[\alpha], r \in \bar{r}[\alpha] \\ \bar{p}[c, d][\alpha] &= [p^L[\alpha], p^U[\alpha]] \\ \text{where } p^L[\alpha] &= \text{Min} \left\{ \int_c^d \bar{f}(t, \lambda, r) dt \right\} \\ p^U[\alpha] &= \text{Max} \left\{ \int_c^d \bar{f}(t, \lambda, r) dt \right\}\end{aligned}$$

3. FUZZY RELIABILITY FUNCTION

Assume that X and U are two crisp sets. Let failure rate function be fuzzy and represented by a fuzzy set $\bar{H}(t)$, $\bar{H}(t) = \{h, \mu_{\bar{H}(t)} h \in X\}$. The α cut fuzzy set of $\bar{H}(t)$ is $\bar{H}_\alpha(t) = \{h \in X / \mu_{\bar{H}(t)} \geq \alpha\}$. Note that $\bar{H}_\alpha(t)$ is a crisp set.

Suppose that $\bar{H}(t)$ is a fuzzy number. Then for each choice of α cut, we have interval $\bar{H}_\alpha(t) = \{h_1(t), h_2(t)\}$. By the convexity of the fuzzy number, the bounds of the interval are function of α and can be obtained as $\bar{h}_{1\alpha} = \min \mu_{\bar{H}(t)}(\alpha)$ and $\bar{h}_{2\alpha} = \max \mu_{\bar{H}(t)}(\alpha)$ respectively.

Let $\mu : X \rightarrow V$ be a bounded continuously differentiable function from X to V . We wish to calculate the fuzzy set (fuzzy reliability functions) induced on V by applying μ to the set $\bar{U}(T)$. If we write $v = \mu(h)$, where $\mu \in \bar{U}(T)$. and $\bar{R}(t) = \{u, \mu_{\bar{R}(t)}(u) / u = \phi(h), u \in U\}$ then the membership function of $\bar{R}(t)$ is defined by the extension principle $\mu_{\bar{R}(t)}(u) = \sup_{h \in X} \{\mu_{\bar{H}(t)}(h) / u = \phi(h)\}$.

$$\begin{aligned}r_1(t) &= \min. \mu(h) \quad ; h_1(t) = h = h_2(t) \\ r_2(t) &= \max. \mu(h) \quad ; h_1(t) = h = h_2(t)\end{aligned}$$

The crisp reliability function of an object is $R(t) = P(T \geq t) = 1 - F(t)$. Now we define the fuzzy reliability by means of the fuzzy distribution function

$$\bar{R}(t) = \bar{P}(\bar{T} > t) = 1 - \bar{F}(t) \quad \forall t \in [0, \infty)$$

where \bar{T} is a fuzzy random variable which describes the vagueness of the time "t" and the uncertainty of the probability distribution whose distribution functions is $\bar{F}(x) = \bar{P}(X < x)$ and X is the random variable with gamma parameters.

The reliability function for gamma distribution is defined by

$$\begin{aligned}R(t) &= \frac{1}{\Gamma(r)} \int_t^\infty \lambda^r u^{r-1} e^{-\lambda u} du / \lambda \in \bar{\lambda}[\alpha], r \in \bar{r}[\alpha] \\ &= \frac{1}{\Gamma(r)} \Gamma(r, \lambda t) / \lambda \in \bar{\lambda}[\alpha], r \in \bar{r}[\alpha]\end{aligned}$$

The α - cut of fuzzy reliability function for gamma distribution is

$$\bar{R}(t)[\alpha] = [R_1(\alpha), R_2(\alpha)]$$

Where $R_1[\alpha] = \text{Max}_{\frac{1}{\Gamma(r)}} \Gamma(r, \lambda t) / \lambda \in \bar{\lambda}[\alpha], r \in \bar{r}[\alpha]$

$R_2[\alpha] = \text{Min}_{\frac{1}{\Gamma(r)}} \Gamma(r, \lambda t) / \lambda \in \bar{\lambda}[\alpha], r \in \bar{r}[\alpha]$

4. APPLICATION

The proposal comprised pregnant ladies affected by T1DM, and was followed up in action to the Diabetic and Pregnancy Outpatient Clinic at the University of Palermo, Italy, within 8 +/- 3.4 weeks subsequent to a positive pregnancy test. Thus they are treated with under conventional basal-bolus insulin therapy (aspart or lispro analogs at the 3 main meals plus glargine or NPH at bedtime). Healthy pregnant ladies have been subjected as controls for neonatal and fetal parameters. All their results are recorded then and there. Every patient metabolic status had been determined by mean glycemic values (2-hour postprandial blood glucose) and glycosylated hemoglobin (HbA1c) values (at 3-month intervals). Fetal measurements (< 50th and > 90th centiles of the head circumference, abdomen circumference, and femoral length) were estimated using ultrasound during the second and third trimesters. Weight and femoral length were assessed at birth, and neonates had been classified according to the fetal growth curve for the Italian population (< 10th centile = small for gestational age; and > 90th centile = large for gestational age (LGA)).

Table - 4.1 INSULIN LEVEL

Days	0	30	60	90	120	150	180	210	240	270
Level of Diabetes ($\mu\text{g/ml}$)	126	129	139	140	137	150	151	143	134	139

During insulin therapy, the parameters of gamma distribution were found and there are $\lambda = 0.26$ and $r = 4.874$. and the value of t will be assumed be $t = 60$

Thus the corresponding Triangular fuzzy numbers will be

$$\bar{\lambda} = [0.260, 0.268, 0.273]$$

$$\bar{r} = [4.80, 4.874, 4.910]$$

and the corresponding α cuts will be

$$\bar{\lambda}[\alpha] = [0.260 + 0.008\alpha, 0.273 - 0.005\alpha] \bar{\lambda}[\alpha] = [0.260 + 0.008\alpha, 0.273 - 0.005\alpha]$$

$$\bar{r}[\alpha] = [4.80 + 0.074\alpha, 4.910 - .036\alpha]$$

Table : 4.2 Alpha - cuts of the fuzzy reliability

Values of α	$\bar{\lambda}_l$	$\bar{\lambda}_i$	\bar{r}_l	\bar{r}_u	$R_1[\alpha]$	$R_2[\alpha]$
0	0.2600	0.2730	0.4800	4.9100	0.0368	0.0460
0.1	0.2608	0.2725	0.4804	4.9064	0.0371	0.0456
0.2	0.2616	0.2720	0.4808	4.9028	0.0373	0.0451
0.22	0.2624	0.2715	0.4812	4.8992	0.0379	0.0441
0.4	0.2632	0.2710	0.4816	4.8956	0.0382	0.0437
0.5	0.2640	0.2705	0.4820	4.8920	0.0384	0.0432
0.6	0.2648	0.2700	0.4824	4.8884	0.0387	0.0428
0.7	0.2656	0.2695	0.4828	4.8848	0.0390	0.0423
0.8	0.2664	0.2690	0.4832	4.8812	0.0392	0.0419
0.9	0.2672	0.2685	0.4836	4.8776	0.0398	0.0414
1	0.2680	0.2680	0.4840	4.8740	0.0368	0.0460

5. CONCLUSION

Out of 73 pregnant ladies 30 were with T1M whereas other 43 were healthy even though monitored under treatment. Both people set didn't find differences in required insulin level (IU/kg of body weight) and glycemic profile. On taking fasting and time after 2 hours of breakfast glycemic values in the glargine group on the first ($P = 0.008$ and $P < 0.001$, respectively) and the second ($P = 0.015$ and $P = 0.016$) trimesters the lower HbA1c levels in the first trimester ($P = 0.037$) may be represented.

The frequency of femoral length < 50 th centile at both second, third trimesters was 4/15 (26.7%) in the glargine-treated group ($P = 0.033$ and $P = 0.013$, respectively, vs control), 3/15 (20.0%) and 1/15 (6.7%), respectively, in the NPH-treated group (both, $P = NS$ vs control), and 2/43 (4.7%) and 1/43 (2.3%), respectively, in the control group. The prevalence of LGA was 7/15 (46.7%) in the glargine group ($P < 0.001$ vs control), 4/15 (27.6%) in the NPH group ($P = 0.033$ vs control), and 2/43 (4.7%) in the control group. Further larger prospective studies are necessary to assess the safety profile of glargine in T1DM during pregnancy.

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