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THE RR, RRR AND RM_2 INDICES OF GENERALIZED TRANSFORMATION GRAPHS G^{XY}

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Abstract. Recently, Gutman et al., defined three vertex-degree-based topological invarients and obtained mathematical properties with their applications to physico-chemical properties of alkanes. In this paper, the expressions for the Reciprocal Randić index, Reduced Zagreb index and Reduced reciprocal Randić index of the generalized transformation graphs G^{xy} and their complement graphs are obtained.

1. Introduction

In this paper we consider the simple graphs with non direction, typically known as undirected graphs. Consider any graph G, having vertex/node set with cardinality denoted as $|V(G)| = n$ and the edge set with cardinality denoted as $|E(G)| = m$. Moving towards recalling the basic definition *degree* of any vertex say a as the number of edges incident to it and is denoted by $deg(a)$ or $d_G(a)$ or $\delta(a)$.

The first and second *Zagreb indices* of a graph G are defined as [7]

$$M_1(G) = \sum_{uv\in E(G)} [d_G(u) + d_G(v)] \quad \text{and} \quad M_2(G) = \sum_{uv\in E(G)} d_G(u)d_G(v).$$

The Zagreb indices were used in the structure property model. More results can be found in [2, 5, 6, 10].

The first and second *Zagreb coindices* of a graph G are defined as [3]

$$\overline{M_1}(G) = \sum_{uv\notin E(G)} [d_G(u) + d_G(v)] \quad \text{and} \quad \overline{M_2}(G) = \sum_{uv\notin E(G)} d_G(u)d_G(v).$$

Mathematical properties of these coindices can be found in [6].

From the existing literature survey in the area of applications of graph theory it is evidentially true that the Randić index works out to be one of the best used topological index or say a molecular descriptors used to understand of structure-property and structure-activity relationships of different chemical compounds in

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the field of graph theory[4, 12, 13, 15].For mathematical properties of this graph invariant one can refer [9, 14].
The *Randić index* [19] $R(G)$ is defined as the sum of the weights $(d_G(u)d_G(v))^{-1/2}$ over all edges uv of G. That is,

$$R(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{d_G(u)d_G(v)}}.$$

The topological indices we will be studying here were produced by Gutman et al,[8] in his very recent work, defined as,
The *Reciprocal Randić index* $RR(G)$ is defined as the sum of the weights $(d_G(u)d_G(v))^{1/2}$ over all edges uv of G. That is,

$$RR(G) = \sum_{uv \in E(G)} \sqrt{d_G(u)d_G(v)}.$$

The *Reduced Zagreb index* $RM_2(G)$ is defined as

$$RM_2(G) = \sum_{uv \in E(G)} (d_G(u) - 1)\,(d_G(v) - 1).$$

The *Reduced Reciprocal Randić index* $RRR(G)$ is defined as,

$$RR(G) = \sum_{uv \in E(G)} \sqrt{(d_G(u) - 1)\,(d_G(v) - 1)}.$$

2. Generalized transformation graphs G^{xy}

A new concept of graph transformation came into consideration after Basavanagoud et al,[1] introduced the construction of new graphs defined as Generalized transformation graphs G^{xy}, keeping the conceptual definition of *semitotal-point graph* $T_2(G)$ as the base, where the new graph generated will have the vertex set as $V(G) \cup E(G)$ where, the vertex-vertex adjacency and vertex-edge incidence in G^{xy} depends on two conditions:

(a) $\alpha, \beta \in V(G)$, α, β are adjacent in G if $x = +$ and α, β are not adjacent in G if $x = -$.
(b) $\alpha \in V(G)$ and $\beta \in E(G)$, α, β are incident in G if $y = +$ and α, β are not incident in G if $y = -$.

Further, applying the combination of these two binary signs + and − the four generalized transformation graph, G^{++}, G^{+-}, G^{-+} and G^{--} were obtained. An example of generalized graph transformations and their complements are depicted in the Figure 1.

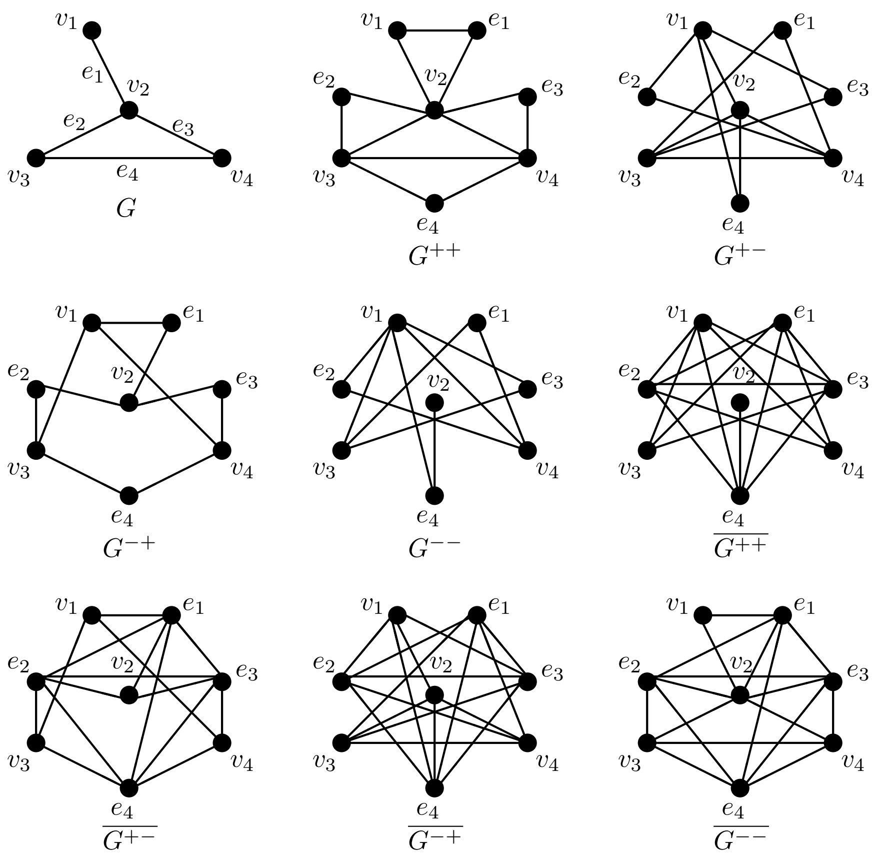

Figure 1: Graph, its generalized transformations and their complements

Observe that, G^{++} is just the semitotal-point graph $T_2(G)$ of G. The vertex v of G^{xy} corresponding to a vertex v of G is referred to as a *point vertex.* The vertex e of G^{xy} corresponding to an edge e of G is referred to as a *line vertex.* Later on, substantial works on graph invarients of generalized transformation graphs was carried out, refer [1, 11, 16, 17, 18].

In this paper, the computation for the Reciprocal Randić index, Reduced second Zagreb index and Reduced reciprocal Randić index of the generalized transformation graphs G^{xy} and for their complement graphs are obtained. Note that, The complement of G will be denoted by $\overline{G}$. If G has n vertices and m edges then the number of vertices of G^{xy} is $n+m$ and $d_{\overline{G}}(u) = n-1-d_G(u)$.

3. The Reciprocal Randić index of genralized transformation graphs G^{xy}

Theorem 3.1. *Let G be a graph with $|V(G)| = n$ and $|E(G)| = m$. Then*

$$RR(G^{++}) = 2\left[RR(G) + \sum_{u\in V(G)} d_G(u)\sqrt{d_G(u)}\right].$$

Proof. According to the study of edge set partition of $E(G^{++})$ we consider that the graph has two edge subsets E_1 and E_2, such that

	Edge set	cardinality
E_1	$\{uv \mid uv \in E(G)\}$	m
E_2	$\{ue \mid$ the vertex u is incident to the edge e in $G\}$	$2m$

Then for $u \in V(G)$we have $d_{G^{++}}(u) = 2d_G(u)$ and for $e \in E(G)$ we have $d_{G^{++}}(e) = 2$.(refer [1], Prop. 4.1)

$$\begin{aligned} RR(G^{++}) &= \sum_{uv\in E(G)} \sqrt{d_{G^{++}}(u)d_{G^{++}}(v)} \\ &= \sum_{uv\in E_1} \sqrt{d_{G^{++}}(u)d_{G^{++}}(v)} + \sum_{ue\in E_2} \sqrt{d_{G^{++}}(u)d_{G^{++}}(e)} \\ &= \sum_{uv\in E(G)} \sqrt{(2d_G(u))\,(2d_G(v))} + \sum_{ue\in E_2} \sqrt{(2d_G(u))\,(2)} \\ &= 2\left[RR(G) + \sum_{ue\in E_2} \sqrt{d_G(u)}\right]. \end{aligned}$$

Since the quantity $\sqrt{d_G(u)}$ repeats $d_G(u)$ times in the set E_2, the above expression reduces to

$$RR(G^{++}) = 2\left[RR(G) + \sum_{u\in V(G)} d_G(u)\sqrt{d_G(u)}\right]$$

□

Theorem 3.2. *Let G be a graph with $|V(G)| = n$ and $|E(G)| = m$. Then*

$$RR(G^{+-}) = m^2 + m(n-2)\sqrt{m(n-2)}.$$

Proof. According to the study of edge set partition of $E(G^{+-})$ we consider that the graph has two edge subsets E_1 and E_2, such that

	Edge set	cardinality
E_1	$\{uv \mid uv \in E(G)\}$	m
E_2	$\{ue \mid$ the vertex u is not incident to the edge e in $G\}$	$2m(n-2)$

Then for $u \in V(G)$we have $d_{G^{+-}}(u) = m$ and for $e \in E(G)$ we have $d_{G^{+-}}(e) = n-2$.(refer [1], Prop. 4.1)

$$\begin{aligned}
RR(G^{+-}) &= \sum_{uv \in E(G)} \sqrt{d_{G^{+-}}(u)d_{G^{+-}}(v)} \\
&= \sum_{uv \in E_1} \sqrt{d_{G^{+-}}(u)d_{G^{+-}}(v)} + \sum_{ue \in E_2} \sqrt{d_{G^{+-}}(u)d_{G^{+-}}(e)} \\
&= \sum_{uv \in E(G)} \sqrt{(m)(m)} + \sum_{ue \in E_2} \sqrt{(m)(n-2)} \\
&= m^2 + m(n-2)\sqrt{m(n-2)}
\end{aligned}$$

□

Remark 3.3. From the above theorem we observe that all graphs G having same order and size thenc the Reciprocal Randić index of genralized transformation graphs $RR(G^{+-})$ is same.

Theorem 3.4. *Let G be a graph with $|V(G)| = n$ and $|E(G)| = m$. Then*

$$RR(G^{-+}) = \left[\frac{n(n-1)-2m}{2}\right](n-1) + 2m\sqrt{2(n-1)}$$

Proof. According to the study of edge set partition of $E(G^{-+})$ we consider that the graph has two edge subsets E_1 and E_2, such that

	Edge set	cardinality
E_1	$\{uv \mid uv \notin E(G)\}$	$\binom{n}{2} - m$
E_2	$\{ue \mid$ the vertex u is incident to the edge e in $G\}$	$2m$

For $u \in V(G)$ we have $d_{G^{-+}}(u) = n-1$ and for $e \in E(G)$ we have $d_{G^{-+}}(e) = 2$.(refer [1], Prop. 4.1)

$$\begin{aligned}
RR(G^{-+}) &= \sum_{uv \in E(G)} \sqrt{d_{G^{-+}}(u)d_{G^{-+}}(v)} \\
&= \sum_{uv \in E_1} \sqrt{d_{G^{-+}}(u)d_{G^{-+}}(v)} + \sum_{ue \in E_2} \sqrt{d_{G^{-+}}(u)d_{G^{-+}}(e)} \\
&= \sum_{uv \notin E(G)} \sqrt{(n-1)(n-1)} + \sum_{ue \in E_2} \sqrt{(n-1)(2)} \\
&= \left[\binom{n}{2} - m\right](n-1) + 2m\sqrt{2(n-1)} \\
&= \left(\frac{n(n-1)-2m}{2}\right)(n-1) + 2m\sqrt{2(n-1)}.
\end{aligned}$$

□

Remark 3.5. From the above theorem, we observe that all graphs G having same order and size then the Reciprocal Randić index of genralized transformation graphs $RR(G^{-+})$ is same.

Theorem 3.6. *Let G be a graph with $|V(G)| = n$ and $|E(G)| = m$. Then*

$$\begin{aligned} RR(G^{--}) &= \sum_{uv \notin E(G)} \sqrt{(n+m-1-2d_G(u))\,(n+m-1-2d_G(v))} \\ &\quad + \sum_{u \in V(G)} (m-d_G(u))\sqrt{(n+m-1-2d_G(u))\,(n-2)}. \end{aligned}$$

Proof. According to the study of edge set partition of $E(G^{--})$ we consider that the graph has two edge subsets E_1 and E_2, such that

	Edge set	cardinality
E_1	$\{uv \mid uv \notin E(G)\}$	$\binom{n}{2} - m$
E_2	$\{ue \mid$ the vertex u is not incident to the edge e in $G\}$	$m(n-2)$

For $u \in V(G)$we have $d_{G^{--}}(u) = n+m-1-2d_G(u)$ and for $e \in E(G)$ we have $d_{G^{--}}(e) = n-2$.(refer [1], Prop. 4.1)

$$\begin{aligned} RR(G^{--}) &= \sum_{uv \in E(G)} \sqrt{d_{G^{--}}(u)d_{G^{--}}(v)} \\ &= \sum_{uv \in E_1} \sqrt{d_{G^{--}}(u)d_{G^{--}}(v)} + \sum_{ue \in E_2} \sqrt{d_{G^{--}}(u)d_{G^{--}}(e)} \\ &= \sum_{uv \notin E(G)} \sqrt{(n+m-1-2d_G(u))\,(n+m-1-2d_G(v))} \\ &\quad + \sum_{ue \in E_2} \sqrt{(n+m-1-2d_G(u))\,(n-2)} \\ &= \sum_{uv \notin E(G)} \sqrt{(n+m-1-2d_G(u))\,(n+m-1-2d_G(v))} \\ &\quad + \sum_{u \in V(G)} (m-d_G(u))\sqrt{(n+m-1-2d_G(u))\,(n-2)}. \end{aligned}$$

□

4. The Reciprocal Randić index of complement of genaralized transformation $\overline{G^{xy}}$

Theorem 4.1. *Let G be a graph with $|V(G)| = n$ and $|E(G)| = m$. Then*

$$\begin{aligned} RR\left(\overline{G^{++}}\right) &= \sum_{uv \notin E(G)} \sqrt{(n+m-1-2d_G(u))\,(n+m-1-2d_G(v))} \\ &\quad + \sum_{u \in V(G)} (m-d_G(u))\sqrt{(n+m-1-2d_G(u))\,(n+m-3)} + \frac{m(m-1)\,(n+m-3)}{2}. \end{aligned}$$

Proof. Similar to the previous section of generalized transformation graph we can obtain the edge set partition of $\overline{G^{++}}$ into subsets E_1, E_2 and E_3, such that

	Edge set	cardinality
E_1	$\{uv \mid uv \notin E(G)\}$	$\binom{n}{2} - m$
E_2	$\{ue \mid$ the vertex u is not incident to the edge e in $G\}$	$m(n-2)$
E_3	$\{ef \mid e, f \in E(G)\}$	$\binom{m}{2}$

Now, For $u \in V(G)$we have $d_{\overline{G^{++}}}(u) = n + m - 1 - 2d_G(u)$ and for $e \in E(G)$ we have $d_{\overline{G^{++}}}(e) = n + m - 3$(refer [17], Prop. 1.2).

$$\begin{aligned}
RR\left(\overline{G^{++}}\right) &= \sum_{uv \in E(\overline{G^{++}})} \sqrt{d_{\overline{G^{++}}}(u) d_{\overline{G^{++}}}(v)} \\
&= \sum_{uv \in E_1} \sqrt{d_{\overline{G^{++}}}(u) d_{\overline{G^{++}}}(v)} + \sum_{ue \in E_2} \sqrt{d_{\overline{G^{++}}}(u) d_{\overline{G^{++}}}(e)} + \sum_{ef \in E_3} \sqrt{d_{\overline{G^{++}}}(e) d_{\overline{G^{++}}}(f)} \\
&= \sum_{uv \notin E(G)} \sqrt{\left(n + m - 1 - 2d_G(u)\right)\left(n + m - 1 - 2d_G(v)\right)} \\
&\quad + \sum_{ue \in E_2} \sqrt{\left(n + m - 1 - 2d_G(u)\right)\left(n + m - 3\right)} + \sum_{ef \in E_3} \sqrt{\left(n + m - 3\right)\left(n + m - 3\right)} \\
&= \sum_{uv \notin E(G)} \sqrt{\left(n + m - 1 - 2d_G(u)\right)\left(n + m - 1 - 2d_G(v)\right)} \\
&\quad + \sum_{ue \in E_2} \sqrt{\left(n + m - 1 - 2d_G(u)\right)\left(n + m - 3\right)} + \sum_{ef \in E_3} \left(n + m - 3\right) \\
&= \sum_{uv \notin E(G)} \sqrt{\left(n + m - 1 - 2d_G(u)\right)\left(n + m - 1 - 2d_G(v)\right)} \\
&\quad + \sum_{u \in V(G)} \left(m - d_G(u)\right) \sqrt{\left(n + m - 1 - 2d_G(u)\right)\left(n + m - 3\right)} + \binom{m}{2} \left(n + m - 3\right) \\
&= \sum_{uv \notin E(G)} \sqrt{\left(n + m - 1 - 2d_G(u)\right)\left(n + m - 1 - 2d_G(v)\right)} \\
&\quad + \sum_{u \in V(G)} \left(m - d_G(u)\right) \sqrt{\left(n + m - 1 - 2d_G(u)\right)\left(n + m - 3\right)} + \frac{m(m-1)\left(n + m - 3\right)}{2}.
\end{aligned}$$

□

Theorem 4.2. *Let G be a graph with $|V(G)| = n$ and $|E(G)| = m$. Then*

$$RR\left(\overline{G^{+-}}\right) = \frac{\left[n(n-1) - 2m\right](n-1)}{2} + 2m\sqrt{(n-1)(m+1)} + \frac{m\left(m^2 - 1\right)}{2}$$

Proof. Here the edge set $E(\overline{G^{+-}})$ can be partitioned into E_1, E_2 and E_3 subsets, where

	Edge set	cardinality
E_1	$\{uv \mid uv \notin E(G)\}$	$\binom{n}{2} - m$
E_2	$\{ue \mid$ the vertex u is incident to the edge e in $G\}$	$2m$
E_3	$\{ef \mid e, f \in E(G)\}$	$\binom{m}{2}$

For $u \in V(G)$we have $d_{\overline{G^{+-}}}(u) = n-1$ and for $e \in E(G)$ we have $d_{\overline{G^{+-}}}(e) = m+1$(refer [17], Prop. 1.2).

$$\begin{aligned}
RR\left(\overline{G^{+-}}\right) &= \sum_{uv \in E(\overline{G^{+-}})} \sqrt{d_{\overline{G^{+-}}}(u) d_{\overline{G^{+-}}}(v)} \\
&= \sum_{uv \in E_1} \sqrt{d_{\overline{G^{+-}}}(u) d_{\overline{G^{+-}}}(v)} + \sum_{ue \in E_2} \sqrt{d_{\overline{G^{+-}}}(u) d_{\overline{G^{+-}}}(e)} + \sum_{ef \in E_3} \sqrt{d_{\overline{G^{+-}}}(e) d_{\overline{G^{+-}}}(f)} \\
&= \sum_{uv \notin E(G)} \sqrt{(n-1)(n-1)} + \sum_{ue \in E_2} \sqrt{(n-1)(m+1)} + \sum_{ef \in E_3} \sqrt{(m+1)(m+1)} \\
&= \sum_{uv \notin E(G)} (n-1) + \sum_{ue \in E_2} \sqrt{(n-1)(m+1)} + \sum_{ef \in E_3} (m+1) \\
&= \left[\binom{n}{2} - m\right](n-1) + 2m\sqrt{(n-1)(m+1)} + \binom{m}{2}(m+1) \\
&= \frac{[n(n-1) - 2m](n-1)}{2} + 2m\sqrt{(n-1)(m+1)} + m\left(m^2 - 1\right).
\end{aligned}$$

□

Remark 4.3. From the above theorem we observe that all graphs G having same order and size then the Reciprocal Randić index of complement of genralized transformation graphs $RR\left(\overline{G^{+-}}\right)$ is same.

Theorem 4.4. *Let G be a graph with $|V(G)| = n$ and $|E(G)| = m$. Then*

$$RR\left(\overline{G^{-+}}\right) = m^2 + m(n-2)\sqrt{m(n+m-3)} + \frac{m(m-1)(n+m-3)}{2}.$$

Proof. The edge set $E(\overline{G^{-+}})$ can be partitioned into E_1, E_2 and E_3 subsets, where

	Edge set	cardinality
E_1	$\{uv \mid uv \in E(G)\}$	m
E_2	$\{ue \mid$ the vertex u is not incident to the edge e in $G\}$	$m(n-2)$
E_3	$\{ef \mid e, f \in E(G)\}$	$\binom{m}{2}$

For $u \in V(G)$we have $d_{\overline{G^{-+}}}(u) = m$ and for $e \in E(G)$ we have $d_{\overline{G^{-+}}}(e) = n+m-3$.(refer [17], Prop. 1.2).

$$
\begin{aligned}
RR\left(\overline{G^{-+}}\right) &= \sum_{uv\in E(\overline{G^{-+}})} \sqrt{d_{\overline{G^{-+}}}(u)d_{\overline{G^{-+}}}(v)} \\
&= \sum_{uv\in E_1} \sqrt{d_{\overline{G^{-+}}}(u)d_{\overline{G^{-+}}}(v)} + \sum_{ue\in E_2} \sqrt{d_{\overline{G^{-+}}}(u)d_{\overline{G^{-+}}}(e)} + \sum_{ef\in E_3} \sqrt{d_{\overline{G^{-+}}}(e)d_{\overline{G^{-+}}}(f)} \\
&= \sum_{uv\in E(G)} \sqrt{(m)(m)} + \sum_{ue\in E_2} \sqrt{(m)(n+m-3)} + \sum_{ef\in E_3} \sqrt{(n+m-3)(n+m-3)} \\
&= \sum_{uv\in E(G)} m + \sum_{ue\in E_2} \sqrt{m(n+m-3)} + \sum_{ef\in E_3} (n+m-3) \\
&= m^2 + m(n-2)\sqrt{m(n+m-3)} + \binom{m}{2}(n+m-3) \\
&= m^2 + m(n-2)\sqrt{m(n+m-3)} + \frac{m(m-1)\,(n+m-3)}{2}.
\end{aligned}
$$

□

Remark 4.5. Above theorem shows that all graphs G having same order and size then the Reciprocal Randić index of complement of genralized transformation graphs $RR\left(\overline{G^{-+}}\right)$ is same.

Theorem 4.6. *Let G be a graph with $|V(G)| = n$ and $|E(G)| = m$. Then*

$$RR\left(\overline{G^{--}}\right) = 2RR(G) + \sum_{u\in V(G)} d_G(u)\sqrt{2d_G(u)\,(m+1)} + \frac{m(m^2-1)}{2}.$$

Proof. The edge set $E(\overline{G^{--}})$ can be partitioned into E_1, E_2 and E_3 subsets, where

	Edge set	cardinality
E_1	$\{uv \mid uv \in E(G)\}$	m
E_2	$\{ue \mid$ the vertex u is incident to the edge e in $G\}$	$2m$
E_3	$\{ef \mid e, f \in E(G)\}$	$\binom{m}{2}$

For $u \in V(G)$we have$d_{\overline{G^{--}}}(u) = 2d_G(u)$ and for $e \in E(G)$ we have $d_{\overline{G^{--}}}(e) = m+1$.(refer [17], Prop. 1.2).

$$\begin{aligned}
RR\left(\overline{G^{--}}\right) &= \sum_{uv\in E(\overline{G^{--}})} \sqrt{d_{\overline{G^{--}}}(u)d_{\overline{G^{--}}}(v)} \\
&= \sum_{uv\in E_1} \sqrt{d_{\overline{G^{--}}}(u)d_{\overline{G^{--}}}(v)} + \sum_{ue\in E_2} \sqrt{d_{\overline{G^{--}}}(u)d_{\overline{G^{--}}}(e)} + \sum_{ef\in E_3} \sqrt{d_{\overline{G^{--}}}(e)d_{\overline{G^{--}}}(f)} \\
&= \sum_{uv\in E(G)} \sqrt{(2d_G(u))\,(2d_G(v))} + \sum_{ue\in E_2} \sqrt{(2d_G(u))\,(m+1)} + \sum_{ef\in E_3} \sqrt{(m+1)\,(m+1)} \\
&= 2\sum_{uv\in E(G)} \sqrt{(d_G(u))\,(d_G(v))} + \sum_{ue\in E_2} \sqrt{2d_G(u)\,(m+1)} + \sum_{ef\in E_3} (m+1) \\
&= 2RR(G) + \sum_{u\in V(G)} d_G(u)\sqrt{2d_G(u)\,(m+1)} + \binom{m}{2}(m+1) \\
&= 2RR(G) + \sum_{u\in V(G)} d_G(u)\sqrt{2d_G(u)\,(m+1)} + \frac{m(m^2-1)}{2}.
\end{aligned}$$

□

Corollary 4.7. *If G_1 and G_2 are two different graphs having same number of vertices and edges, then*

$$\begin{aligned}
RR(G_1^{+-}) &= RR(G_2^{+-}) \\
RR(G_1^{-+}) &= RR(G_2^{-+}) \\
RR\left(\overline{G_1^{+-}}\right) &= RR\left(\overline{G_2^{+-}}\right) \\
RR\left(\overline{G_1^{-+}}\right) &= RR\left(\overline{G_2^{-+}}\right).
\end{aligned}$$

5. Reduced Zagreb index of genralized transformation graphs G^{xy} with their complements $\overline{G^{xy}}$

Incorporating the computational methods, those followed in the above section *3* and *4*, we derive the below *theorem.*

Theorem 5.1. *Let G be a graph with $|V(G)| = n$ and $|E(G)| = m$. Then*

$$\begin{aligned}
RM_2(G^{++}) &= 4M_2(G) - m \\
RM_2(G^{+-}) &= m(m-1)^2 + m(m-1)(n-2)(n-3) \\
RM_2(G^{-+}) &= \frac{1}{2}\left[n(n-1) - 2m\right](n-2)^2 + 2m\,(n-2) \\
RM_2(G^{--}) &= \frac{1}{2}\left[n(n-1) - 2m\right]\left(n^2 + m^2 + 2nm - 4(n+m-1)\right) + 2\overline{M}_1(G)(-n-m+2) \\
&\quad + 4\overline{M}_2(G) + (n-3)\left[n(nm + m^2 - 2m) + 2m(-2m-n-m+2) + 2M_1(G)\right] \\
RM_2\left(\overline{G^{++}}\right) &= \frac{1}{2}\left[n(n-1) - 2m\right]\left(n^2 + m^2 + 2nm - 4(n+m-1)\right) + 2\overline{M}_1(G)(-n-m+2) \\
&\quad + 4\overline{M}_2(G) + (n+m-4)\left[m(n-2)(n+m-2) - 4m^2 + 2M_1(G)\right] \\
&\quad + \frac{1}{2}\left[m(m-1)(n+m-4)^2\right]. \\
RM_2\left(\overline{G^{+-}}\right) &= \frac{1}{2}\left[n(n-1) - 2m\right](n-2)^2 + 2m^2(n-2) + \frac{1}{2}m^3(m-1). \\
RM_2\left(\overline{G^{-+}}\right) &= m(m-1)^2 + m(n-2)(m-1)(n+m-4) + \frac{1}{2}m(m-1)(n+m-4)^2 \\
RM_2\left(\overline{G^{--}}\right) &= 4M_2(G) - 2M_1(G)(1-m) + m(1-2m) + \frac{1}{2}m^3(m-1) \\
&.
\end{aligned}$$

Corollary 5.2. *If G_1 and G_2 are two different graphs having same order and size, then*

$$\begin{aligned}
RM_2(G_1^{+-}) &= RM_2(G_2^{+-}) \\
RM_2(G_1^{-+}) &= RM_2(G_2^{-+}) \\
RM_2\left(\overline{G_1^{+-}}\right) &= RM_2\left(\overline{G_2^{+-}}\right) \\
RM_2\left(\overline{G_1^{-+}}\right) &= RM_2\left(\overline{G_2^{-+}}\right).
\end{aligned}$$

6. Reduced Receprocal Randić index of genralized transformation graphs G^{xy} with their complements $\overline{G^{xy}}$

Incorporating the computational methods, those followed in the above section *3* and *4*, we can see that,

$$RM_2(G) = M_2(G) - M_1(G) + m$$

Theorem 6.1. *Let G be a graph with $|V(G)| = n$ and $|E(G)| = m$. Then*

$$\begin{aligned}
RRR(G^{++}) &= \sum_{uv\in E(G)} \sqrt{(2d_G(u)-1)\,(2d_G(v)-1)} + \sum_{u\in V(G)} d_G(u)\sqrt{d_G(u)-1} \\
RRR(G^{+-}) &= m(m-1) + m(n-2)\sqrt{(m-1)(n-3)} \\
RRR(G^{-+}) &= \frac{1}{2}\left[n(n-1)-2m\right](n-2) + 2m\sqrt{n-2} \\
RRR(G^{--}) &= \sum_{uv\notin E(G)} \sqrt{(n+m-2-2d_G(u))(n+m-2-2d_G(v))} \\
&\quad + \sum_{u\in V(G)} (m-d_G(u))\sqrt{(n-3)(n+m-2-2d_G(u))} \\
RRR\left(\overline{G^{++}}\right) &= . \\
RRR\left(\overline{G^{+-}}\right) &= \frac{1}{2}\left[n(n-1)-2m\right](n-2) + 2m\sqrt{m(n-2)} + \frac{1}{2}m^2(m-1). \\
RRR\left(\overline{G^{-+}}\right) &= m(m-1) + m(n-2)\sqrt{(m-1)(n+m-4)} + \frac{1}{2}m(m-1)(n+m-4) \\
RRR\left(\overline{G^{--}}\right) &= \sum_{uv\in E(G)} \sqrt{(2d_G(u)-1)\,(2d_G(v)-1)} + \sum_{u\in V(G)} d_G(u)\sqrt{m\,(d_G(u)-1)} + \frac{1}{2}m^2(m-1)
\end{aligned}$$

.

Corollary 6.2. *If G_1 and G_2 are two different structures having same number of vertices and edges, then*

$$\begin{aligned}
RRR(G_1^{+-}) &= RRR(G_2^{+-}) \\
RRR(G_1^{-+}) &= RRR(G_2^{-+}) \\
RRR\left(\overline{G_1^{+-}}\right) &= RRR\left(\overline{G_2^{+-}}\right) \\
RRR\left(\overline{G_1^{-+}}\right) &= RRR\left(\overline{G_2^{-+}}\right).
\end{aligned}$$

Conclusion

After studying the above results obtained, one can easily justify that for any set of graphs having same order and size have equal $RR(G^{-+})$, equal $RR(G^{+-})$, equal $RR\left(\overline{G^{-+}}\right)$and equal $RR\left(\overline{G^{+-}}\right)$ value.

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