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CONTROL CHART FOR NUMBER OF DEFECTIVES USING TRIANGULAR FUZZY NUMBERS

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Abstract. Quality control is to give a feasible structure to joining the undertakings of those in the affiliation obligated for the new development, backing, and improvement of thing quality and to oblige creation, movement, and organization at the most reasonable levels giving full satisfaction of the customer. In this investigation article, the soft control diagram reliant upon the amount of defectives (np) is assembled and moreover the sensible model is given.

1. Introduction

Control traces have two sorts: variable and quality. Techniques of verifiable cooperation control are comprehensively used by the collecting industry to perceive and take out relinquishes during creation. Control diagram method is prominent as a basic development in progress cycle noticing (Montgomery, 2008). The control diagram has a huge limit in distinctive the occasion of assignable causes, so the significant change can be made before non-changing things are manufactured in a gigantic aggregate (Rungsarit Intaramo, 2012). The control chart technique may be considered as both the graphical enunciation and movement of real theory testing. It is proposed that if a control chart is used to screen measure, some test limits should be settled, for instance, the model size, the inspecting stretch between reformist models, and quite far or fundamental regions of the layout. SPC is a viable system for improvement of an affiliation's quality and productivity. The essential objective of SPC resembles that of the control graph procedure, that is, to rapidly assess the occasion of assignable causes or cycle shifts. Roland and Wang (2000) introduced cushioned SPC theory reliant upon the use of feathery reasoning to the SPC-zone rules. El-Shal and Morris (2000) changed SPC-zone rules to reduce sham alert and perceive the veritable error. Surely, the issue with control traces is achieved by questionable data for instance human, assessment devices or biological conditions. This investigation paper is summarized as the theoretical plan of cushioned standard with control graph for number of defectives (np) is given under with a portrayal.

Key words and phrases. Fuzzy, Fuzzy control blueprint and Process limit.

2. Methods and materials

In the current Shewhart (1931) method, the arrangement of the control outline for number of defectives is dictated by the going with condition:

$$\begin{aligned} UCL_{np} &= n\overline{p} + 3\sqrt{n\overline{p}(1-\overline{p})} \\ CL_{np} &= n\overline{p} \\ LCL_{np} &= n\overline{p} - 3\sqrt{n\overline{p}(1-\overline{p})} \end{aligned}$$

Where UCL is the upper control limit, CL is the centre line and LCL is the lower control limit of np control chart. Fuzzy numbers (Pb_x, Pb_y, Pb_z) are represented as $(\tilde{\overline{p}}_{b_{x_j}}, \tilde{\overline{p}}_{b_{y_j}}, \tilde{\overline{p}}_{b_{z_j}})$ for each fuzzy observation on the control chart for number of defectives. The centre line (CL) for the $n\tilde{p}$ -control chart is as follows:

$$CL = (n\tilde{\overline{p}}_{b_{x_j}}, n\tilde{\overline{p}}_{b_{y_j}}, n\tilde{\overline{p}}_{b_{z_j}}), \text{ where } j = 1, 2, 3, \ldots, n.$$

By contemplating the meanings of $n\tilde{p}$ - control limits and soft numbers chiefly reliant upon three-sided enlistment works, the cushioned center line, cushy upper and feathery lower cutoff points of the fleecy norm $n\tilde{p}$ - control diagram are given as follows:

$$\begin{aligned} (UCL_{n\tilde{p}_{b_x}}, UCL_{n\tilde{p}_{b_y}}, UCL_{n\tilde{p}_{b_z}}) &= \begin{pmatrix} n\tilde{\overline{p}}_{b_x} + 3\sqrt{n\tilde{\overline{p}}_{b_x}(1-\tilde{\overline{p}}_{b_x})}, \\ n\tilde{\overline{p}}_{b_y} + 3\sqrt{n\tilde{\overline{p}}_{b_y}(1-\tilde{\overline{p}}_{b_y})}, \\ n\tilde{\overline{p}}_{b_z} + 3\sqrt{n\tilde{\overline{p}}_{b_z}(1-\tilde{\overline{p}}_{b_z})} \end{pmatrix} \\ (CL_{n\tilde{p}_{b_x}}, CL_{n\tilde{p}_{b_y}}, CL_{n\tilde{p}_{b_z}}) &= (n\tilde{\overline{p}}_{b_{x_j}}, n\tilde{\overline{p}}_{b_{y_j}}, n\tilde{\overline{p}}_{b_{z_j}}) \\ (LCL_{n\tilde{p}_{b_x}}, LCL_{n\tilde{p}_{b_y}}, LCL_{n\tilde{p}_{b_z}}) &= \begin{pmatrix} n\tilde{\overline{p}}_{b_x} - 3\sqrt{n\tilde{\overline{p}}_{b_x}(1-\tilde{\overline{p}}_{b_x})}, \\ n\tilde{\overline{p}}_{b_y} - 3\sqrt{n\tilde{\overline{p}}_{b_y}(1-\tilde{\overline{p}}_{b_y})}, \\ n\tilde{\overline{p}}_{b_z} - 3\sqrt{n\tilde{\overline{p}}_{b_z}(1-\tilde{\overline{p}}_{b_z})} \end{pmatrix} \end{aligned}$$

The proposed and approved standard deviations $(\tilde{\sigma}_{i.n\overline{p}:F-C_p, i=x,y,z})$ for $n\tilde{p}$-fuzzy control chart are estimated (Radhakrishnan and Balamurugan, 2011) and cautiously evaluated by adopting process capability

$$C_p = \frac{USL_{i.np:F-C_p} - LSL_{i.np:F-C_p}}{6\sigma}, i = x, y, z.$$

So, the results of the proposed fuzzy control limits for number of defectives $(n\tilde{p})$ with the assist of process capability are as follows:

$$
\begin{aligned}
(UCL_{n\tilde{p}_{b_x}:C_p}, UCL_{n\tilde{p}_{b_y}:C_p}, UCL_{n\tilde{p}_{b_z}:C_p}) &= \begin{pmatrix} n\tilde{\bar{p}}_{b_x} + 3\tilde{\sigma}_{x.n\tilde{p}:F-C_p}, \\ n\tilde{\bar{p}}_{b_y} + 3\tilde{\sigma}_{y.n\tilde{p}:F-C_p}, \\ n\tilde{\bar{p}}_{b_z} + 3\tilde{\sigma}_{z.n\tilde{p}:F-C_p} \end{pmatrix} \\
(CL_{n\tilde{p}_{b_x}:C_p}, CL_{n\tilde{p}_{b_y}:C_p}, CL_{n\tilde{p}_{b_z}:C_p}) &= (n\tilde{\bar{p}}_{b_{x_j}}, n\tilde{\bar{p}}_{b_{y_j}}, n\tilde{\bar{p}}_{b_{z_j}}) \\
(LCL_{n\tilde{p}_{b_x}:C_p}, LCL_{n\tilde{p}_{b_y}:C_p}, LCL_{n\tilde{p}_{b_z}:C_p}) &= \begin{pmatrix} n\tilde{\bar{p}}_{b_x} - 3\tilde{\sigma}_{x.n\tilde{p}:F-C_p}, \\ n\tilde{\bar{p}}_{b_y} - 3\tilde{\sigma}_{y.n\tilde{p}:F-C_p}, \\ n\tilde{\bar{p}}_{b_z} - 3\tilde{\sigma}_{z.n\tilde{p}:F-C_p} \end{pmatrix}
\end{aligned}
$$

3. Illustration

The going with information gives the outcomes of audit of a sheet-metal part for a plane super supercharger skin (Grant and Leavenworth, 2000). The part was looked into resulting to being framed by a drop hammer.

Table 1: Inspection of a sheet-metal part for an aircraft turbo-super charger skin

Production order number	Lot size	Number of rejects	Triangular Fuzzy numbers		
			P_{b_x}	P_{b_y}	P_{b_z}
1	200	23	19	23	27
2	200	15	10	15	19
3	200	17	13	17	22
4	200	15	12	15	18
5	200	41	37	41	43
6	200	0	0	0	4
7	200	25	21	25	30
8	200	31	26	31	34
9	200	29	23	29	35
10	200	0	0	0	4
11	200	8	3	8	11
12	200	16	13	16	22

The three-sided cushy numbers are gained by using PC program reliant upon the above discernments and given in a comparative Table-1.

Then the centre lines (CL) for the fuzzy $n\tilde{p}$ - control chart are as follows:

$$
\begin{aligned}
\text{The total number inspected } \sum n = 200 \times 12 &= 2400. \\
\text{The total number of defectives } \sum n\tilde{p}_{b_x} &= 177.
\end{aligned}
$$

Therefore,

$$\tilde{\bar{p}}_{b_x} = \frac{\sum np}{\sum n} = \frac{177}{2400} = 0.0738$$

and

$$
\begin{aligned}
n\tilde{\bar{p}}_{b_x} &= 200 \times 0.0738 \\
CL_{n\tilde{\bar{p}}_{b_x}} &= n\tilde{\bar{p}}_{b_x} = 14.7500
\end{aligned}
$$

The total number of defectives

$$\sum n\tilde{\bar{p}}_{b_y} = 220.$$

Therefore,

$$\tilde{\bar{p}}_{b_y} = \frac{\sum np}{\sum n} = \frac{220}{2400} = 0.0917$$

and

$$\begin{aligned} n\tilde{\bar{p}}_{b_y} &= 200 \times 0.0917 \\ CL_{n\tilde{\bar{p}}_{b_y}} &= n\tilde{\bar{p}}_{b_y} = 18.3333 \end{aligned}$$

The total number of defectives

$$\sum n\tilde{\bar{p}}_{b_z} = 269.$$

Therefore,

$$\tilde{\bar{p}}_{b_z} = \frac{\sum np}{\sum n} = \frac{269}{2400} = 0.1121$$

and

$$\begin{aligned} n\tilde{\bar{p}}_{b_z} &= 200 \times 0.1121 \\ CL_{n\tilde{\bar{p}}_{b_z}} &= n\tilde{\bar{p}}_{b_z} = 22.4167 \end{aligned}$$

The constructed fuzzy centre line, fuzzy upper and fuzzy lower limits of the fuzzy rule $n\tilde{p}$ - control chart are given as follows:

$$(UCL_{n\tilde{p}_{b_x}}, UCL_{n\tilde{p}_{b_y}}, UCL_{n\tilde{p}_{b_z}}) = \begin{pmatrix} n\tilde{\bar{p}}_{b_x} + 3\sqrt{n\tilde{\bar{p}}_{b_x}(1-\tilde{\bar{p}}_{b_x})}, \\ n\tilde{\bar{p}}_{b_y} + 3\sqrt{n\tilde{\bar{p}}_{b_y}(1-\tilde{\bar{p}}_{b_y})}, \\ n\tilde{\bar{p}}_{b_z} + 3\sqrt{n\tilde{\bar{p}}_{b_z}(1-\tilde{\bar{p}}_{b_z})} \end{pmatrix}$$

$$(UCL_{n\tilde{p}_{b_x}}, UCL_{n\tilde{p}_{b_y}}, UCL_{n\tilde{p}_{b_z}}) = \begin{pmatrix} 14.7500 + 3\sqrt{14.7500(1-0.0738)} = 25.9387, \\ 18.3333 + 3\sqrt{18.3333(1-0.0917)} = 30.5757, \\ 22.4167 + 3\sqrt{22.4167 + (1-0.1121)} = 35.8009 \end{pmatrix}.$$

$$(CL_{n\tilde{p}_{b_x}}, CL_{n\tilde{p}_{b_y}}, CL_{n\tilde{p}_{b_z}}) = (n\tilde{\bar{p}}_{b_{x_j}}, n\tilde{\bar{p}}_{b_{y_j}}, n\tilde{\bar{p}}_{b_{z_j}})$$

$$(CL_{n\tilde{p}_{b_x}}, CL_{n\tilde{p}_{b_y}}, CL_{n\tilde{p}_{b_z}}) = (14.7500, 18.333, 22.4167)$$

$$(LCL_{n\tilde{p}_{b_x}}, LCL_{n\tilde{p}_{b_y}}, LCL_{n\tilde{p}_{b_z}}) = \begin{pmatrix} n\tilde{\bar{p}}_{b_x} - 3\sqrt{n\tilde{\bar{p}}_{b_x}(1-\tilde{\bar{p}}_{b_x})}, \\ n\tilde{\bar{p}}_{b_y} - 3\sqrt{n\tilde{\bar{p}}_{b_y}(1-\tilde{\bar{p}}_{b_y})}, \\ n\tilde{\bar{p}}_{b_z} - 3\sqrt{n\tilde{\bar{p}}_{b_z}(1-\tilde{\bar{p}}_{b_z})} \end{pmatrix}$$

$$(LCL_{n\tilde{p}_{b_x}}, LCL_{n\tilde{p}_{b_y}}, LCL_{n\tilde{p}_{b_z}}) = \begin{pmatrix} 14.7500 - 3\sqrt{14.7500(1-0.0738)} = 3.6613, \\ 18.3333 - 3\sqrt{18.3333(1-0.0917)} = 6.0910, \\ 22.4167 - 3\sqrt{22.4167 + (1-0.1121)} = 9.0325 \end{pmatrix}.$$

The recommended standard deviations for $n\tilde{p}$ - fuzzy control chart are calculated by using process capability and presented in the Table-2.

Table 2: Calculation of proposed standard deviations from triangular fuzzy numbers for number of defectives

Production order number	$\tilde{\sigma}_{x.n\tilde{p}:F-C_p}$	$\tilde{\sigma}_{y.n\tilde{p}:F-C_p}$	$\tilde{\sigma}_{z.n\tilde{p}:F-C_p}$	$\tilde{\sigma}^{\alpha}_{x.n\tilde{p}:F-C_p}$	$\tilde{\sigma}^{\alpha}_{z.n\tilde{p}:F-C_p}$
1	4.1467	4.5117	4.8327	4.3894	5.0219
2	3.0822	3.7249	4.1467	3.5174	4.3894
3	3.4864	3.9440	4.4249	3.7925	4.6970
4	3.3586	3.7249	4.0472	3.6024	4.2379
5	5.4914	5.7092	5.8099	5.6355	5.8726
6	0.0000	0.0000	1.9799	0.0000	2.5263
7	4.3353	4.6771	5.0498	4.5624	5.2652
8	4.7560	5.1181	5.3122	4.9972	5.4303
9	4.5117	4.9795	5.3735	4.8251	5.5977
10	0.0000	0.0000	1.9799	0.0000	2.5263
11	1.7190	2.7713	3.2241	2.4606	3.4802
12	3.4864	3.8367	4.4249	3.7192	4.7483

The results of the proposed fuzzy control limits for number of defectives ($n\tilde{p}$) with the assist of process capability are as follows:

$$
\begin{aligned}
(UCL_{n\tilde{p}_{b_x}:C_p}, UCL_{n\tilde{p}_{b_y}:C_p}, UCL_{n\tilde{p}_{b_z}:C_p}) &= \begin{pmatrix} n\tilde{\bar{p}}_{b_x} + 3\tilde{\sigma}_{x.n\tilde{p}:F-C_p}, \\ n\tilde{\bar{p}}_{b_y} + 3\tilde{\sigma}_{y.n\tilde{p}:F-C_p}, \\ n\tilde{\bar{p}}_{b_z} + 3\tilde{\sigma}_{z.n\tilde{p}:F-C_p} \end{pmatrix} \\
(UCL_{n\tilde{p}_{b_x}:C_p}, UCL_{n\tilde{p}_{b_y}:C_p}, UCL_{n\tilde{p}_{b_z}:C_p}) &= \begin{pmatrix} 14.7500 + (3 \times 0.4576) = 16.1228, \\ 18.3333 + (3 \times 0.4758) = 19.7606, \\ 22.4167 + (3 \times 0.3192) = 23.3742 \end{pmatrix}. \\
(CL_{n\tilde{p}_{b_x}:C_p}, CL_{n\tilde{p}_{b_y}:C_p}, CL_{n\tilde{p}_{b_z}:C_p}) &= (14.7500, 18.3333, 22.4167) \\
(LCL_{n\tilde{p}_{b_x}:C_p}, LCL_{n\tilde{p}_{b_y}:C_p}, LCL_{n\tilde{p}_{b_z}:C_p}) &= \begin{pmatrix} n\tilde{\bar{p}}_{b_x} - 3\tilde{\sigma}_{x.n\tilde{p}:F-C_p}, \\ n\tilde{\bar{p}}_{b_y} - 3\tilde{\sigma}_{y.n\tilde{p}:F-C_p}, \\ n\tilde{\bar{p}}_{b_z} - 3\tilde{\sigma}_{z.n\tilde{p}:F-C_p} \end{pmatrix} \\
(LCL_{n\tilde{p}_{b_x}:C_p}, LCL_{n\tilde{p}_{b_y}:C_p}, LCL_{n\tilde{p}_{b_z}:C_p}) &= \begin{pmatrix} 14.7500 - (3 \times 0.4576) = 13.3772, \\ 18.3333 - (3 \times 0.4758) = 16.9060, \\ 22.4167 - (3 \times 0.3192) = 21.4592 \end{pmatrix}.
\end{aligned}
$$

4. Conclusion

The fleecy 'np' control chart reliant upon measure capacity is worked unprecedented for this paper and is applied to certified data. Undeniably the thing/organi zation isn't in worthy quality exactly as expected, suitably a change and improvement is required at the same time/structure. Also, because of non-conventionality, it is recommended to use proposed fleecy $n\tilde{p}$ - control chart as a choice as opposed to Shewhart control layout.

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