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A NEW METHOD FOR SOLVING MULTI-ITEM MULTI-OBJECTIVE SOLID FIXED CHARGED SHIPMENT MODEL WITH TYPE-2 FUZZY VARIABLES

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Abstract. A multi-item multi-objective fixed charged solid shipment model with criterion e.g. shipment penalty, amounts, demands and carriages as type-2 triangular fuzzy variables with condition on few components and carriages is proposed here. A nearest interval approximation model applying generalized credibility measure for the constraints is introduced for this particular model with the critical value based reductions of corresponding type-2 fuzzy parameters. An example is provided to explain the model with hypothetical data and is then worked out by applying generalized reduced gradient (GRG) technique.

1. Introduction

The solid shipment (transportation) model (SSM) is an exclusive form of linear programming model where we deal with condition of sources, stations and carriages. The classical shipment model is an exclusive form of solid shipment model if only one type of carriage is taken under consideration. During the shipment movement due to complex situation, a few important criterions in the SSM are always treated as unclear variables to fit the realistic positions. There are cases to form a shipment plan for the later months; the amount quantity at every origin, the requirement at every station and the carriage quantity are frequently necessary to be determined by experienced knowledge or probability statistics as a result of no definite data. It is much better to explore this issue by applying fuzzy or stochastic optimization models. It is difficult to predict the exact shipment cost for a sure time period. Shipment model is associated with additional costs along with shipping cost. These locked penalties might be due to road taxes, toll charges etc. In this case it is called fixed charge shipment model. Fuzzy set theory is the one of the popular approaches to deal with uncertainty. Type-2 fuzzy sets were introduced by [13] as a development of type-1 fuzzy sets [12]. Type-2 fuzzy sets have membership functions as type-1 fuzzy sets. The advantage of type-2 fuzzy sets is that they are helpful in a few cases where it is uncertain to find the definite membership functions for fuzzy sets. Multi-item SSM is a model of shipping multiple components from multiple sources to multiple destinations over a few carriages.

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While transporting a few components from source, a situation may arise when not all brands of components can be shipped over all brands of carriages because of quality of components (e.g. liquid, breakable etc.). Multi-item solid fixed charge shipment model (MISFCSM) with condition on carriages is a model of shipping goods to a few destinations over a particular carriage with additional fixed charge for that particular route. Multi-item multi-objective solid shipment models are models that are used to find optimal solutions of multiple objective functions of shipping multiple components from multiple sources to multiple destinations over a few carriage.
The main motivation of this paper is to study solid shipment model with type-2 fuzzy parameters. The solid shipment model with type-1 fuzzy parameters has been discussed by many researchers [2, 3, 11].
The paper is structured as follows: section 2 presents a few basic preliminaries related to the concept. We have discussed the nearest interval approximation of continuous type-2 fuzzy variables in section 3. We have formulated a multi-item multi-objective solid fixed charged shipment model with conditions on a few brands and carriages in the sense that a few specific brands are restricted to be shipped over a few particular carriages in section 4. The shipment criterion, e.g., unit shipment penalty, fixed costs, amounts, demands, carriage capacities are taken as type-2 triangular fuzzy variables. We have investigated the model by formulating a nearest approximation model applying the CV based reductions in section 5. The model is then solved numerically in section 6 applying fuzzy programming technique and LINGO 16 solver.

2. Preliminaries

Definition 2.1. A type-1 fuzzy variable [9] is defined as a function from the possibility space [1] to the set of real numbers, whereas a type-2 fuzzy variable [7] is defined as a function from the fuzzy possibility space [9] to the set of real numbers.

Definition 2.2. [13]. A type-2 fuzzy set $\tilde{B}$ defined on the universe of discourse Y is described by a membership function $\tilde{\mu}_{\tilde{B}} : Y \mapsto F([0,1])$ and is expressed by the following set notation : $\tilde{B} = \{\big(y, \tilde{\mu}_{\tilde{B}}(y)\big) : y \in Y\}$.

Example 2.3. [6] A type-2 triangular fuzzy variable $\tilde{\tau}'$ is expressed by $(r'_1, r'_2, r'_3;$ $\theta'_l, \theta'_r)$, where $r'_1, r'_2, r'_3 \in \mathbb{R}$ and θ'_l, θ'_r are two criterion defining the degree of ambiguity that $\tilde{\tau}'$ takes a value x and the secondary possibility distribution function $\tilde{\mu}_{\tilde{\tau}'}(x)$ of $\tilde{\tau}'$ is denoted as

$$\tilde{\mu}_{\tilde{\tau}'}(x') = \begin{cases} (\frac{x'-r'_1}{r'_2-r'_1} - \theta'_l \frac{x'-r'_1}{r'_2-r'_1}, \frac{x'-r'_1}{r'_2-r'_1}, \frac{x'-r'_1}{r'_2-r'_1} + \theta'_r \frac{x'-r'_1}{r'_2-r'_1}), if\, x' \in [r'_1, \frac{r'_1+r'_2}{2}]; \\ (\frac{x'-r'_1}{r'_2-r'_1} - \theta'_l \frac{r'_2-x'}{r'_2-r'_1}, \frac{x'-r'_1}{r'_2-r'_1}, \frac{x'-r'_1}{r'_2-r'_1} + \theta'_r \frac{r'_2-x'}{r'_2-r'_1}), if\, x' \in (\frac{r'_1+r'_2}{2}, r'_2]; \\ (\frac{r'_3-x'}{r'_3-r'_2} - \theta'_l \frac{x'-r'_2}{r'_3-r'_2}, \frac{r'_3-x'}{r'_3-r'_2}, \frac{r'_3-x'}{r'_3-r'_2} + \theta'_r \frac{x'-r'_2}{r'_3-r'_2}), if\, x' \in (r'_2, \frac{r'_2+r'_3}{2}]; \\ (\frac{r'_3-x'}{r'_3-r'_2} - \theta'_l \frac{r'_3-x'}{r'_3-r'_2}, \frac{r'_3-x'}{r'_3-r'_2}, \frac{r'_3-x'}{r'_3-r'_2} + \theta'_r \frac{r'_3-x'}{r'_3-r'_2}), if\, x' \in (\frac{r'_2+r'_3}{2}, r'_3]. \end{cases}$$

Example 2.4. The secondary possibility distribution $\tilde{\tau}' = (5, 6, 7; 0.5, 0.5)$ is given by

$$\tilde{\mu}_{\tilde{\tau}'}(x') = \begin{cases} ((0.5x' - 2.5), (x' - 5), (1.5x' - 7.5)), \; if\, x' \in [5, 5.5]; \\ ((1.5x' - 8), (x' - 5), (0.5x' - 2)), \; if\, x' \in (5.5, 6]; \\ ((10 - 1.5x'), (7 - x'), (4 - 0.5x')), \; if\, x' \in (6, 6.5]; \\ ((3.5 - 0.5x'), (7 - x'), (10.5 - 1.5x')), \; if\, x' \in (6.5, 7]. \end{cases}$$

2.1. Critical Values for RFVs. The different forms of critical values(CV) [10] of a regular fuzzy variable $\tilde{\tau'}$ is defined below.

(i) the optimistic CV of $\tilde{\tau}'$, denoted by $CV^*[\tilde{\tau}']$, is defined as

$$CV^*[\tilde{\tau}'] = \sup_{\alpha' \in [0,1]} [\alpha' \wedge Pos\{\tilde{\tau}' \geq \alpha'\}]$$

(ii) the pessimistic CV of $\tilde{\tau}'$, denoted by $CV_*[\tilde{\tau}']$, is defined as

$$CV_*[\tilde{\tau}'] = \sup_{\alpha' \in [0,1]} [\alpha' \wedge Nec\{\tilde{\tau}' \geq \alpha'\}]$$

(iii) the CV of $\tilde{\tau}'$, denoted by $CV[\tilde{\tau}']$, is defined as

$$CV[\tilde{\tau}'] = \sup_{\alpha' \in [0,1]} [\alpha' \wedge Cr\{\tilde{\tau}' \geq \alpha'\}].$$

Theorem 2.5. [10] Suppose that $\tilde{\tau}' = (s_1', s_2', s_3'; \eta_l', \eta_r')$ be a type-2 triangular fuzzy variable. Then we have:

(i) The reduction of $\tilde{\tau}'$ to τ_1' applying the optimistic CV reduction approach has the consecutive possibility distribution

$$\mu_{\tau_1'}(x') = \begin{cases} \frac{(1+\eta_r')(x'-s_1')}{s_2'-s_1'+\eta_r'(x'-s_1')}, \; if\, x' \in [s_1', \frac{s_1'+s_2'}{2}]; \\ \frac{(1-\eta_r')x'+\eta_r's_2'-s_1'}{s_2'-s_1'+\eta_r'(s_2'-s')}, \; if\, x' \in (\frac{s_1'+s_2'}{2}, s_2']; \\ \frac{(-1+\eta_r')x'-\eta_r's_2'+s_3'}{s_3'-s_2'+\eta_r'(x'-s_2')}, \; if\, x' \in (s_2', \frac{s_2'+s_3'}{2}]; \\ \frac{(1+\eta_r')(s_3'-x')}{s_3'-s_2'+\eta_r'(s_3'-x')}, \; if\, x' \in (\frac{s_2'+s_3'}{2}, s_3']. \end{cases}$$

(ii) The reduction of $\tilde{\tau}'$ to τ_2' applying the pessimistic CV reduction approach has the consecutive possibility distribution

$$\mu_{\tau_2'}(x') = \begin{cases} \frac{(x'-s_1')}{s_2'-s_1'+\eta_l'(x'-s_1')}, \; if\, x' \in [s_1', \frac{s_1'+s_2'}{2}]; \\ \frac{x'-s_1'}{s_2'-s_1'+\eta_l'(s_2'-x')}, \; if\, x' \in (\frac{s_1'+s_2'}{2}, s_2']; \\ \frac{(s_3'-x')}{s_3'-s_2'+\eta_l'(x'-s_2')}, \; if\, x' \in (s_2', \frac{s_2'+s_3'}{2}]; \\ \frac{(s_3'-x')}{s_3'-s_2'+\eta_l'(s_3'-x')}, \; if\, x' \in (\frac{s_2'+s_3'}{2}, s_3']. \end{cases}$$

(iii) The reduction of $\tilde{\tau}'$ to τ_3' applying the CV reduction approach has the consecutive possibility distribution

$$\mu_{\tau_3'}(x') = \begin{cases} \frac{(1+\eta_r')(x'-s_1')}{s_2'-s_1'+2\eta_r'(x'-s_1')}, \; if\, x' \in [s_1', \frac{s_1'+s_2'}{2}]; \\ \frac{(1-\eta_l')x'+\eta_l's_2'-s_1'}{s_2'-s_1'+2\eta_l'(s_2'-x')}, \; if\, x' \in (\frac{s_1'+s_2'}{2}, s_2']; \\ \frac{(-1+\eta_l')x'-\eta_l's_2'+s_3'}{s_3'-s_2'+2\eta_l'(x'-s_2')}, \; if\, x' \in (s_2', \frac{s_2'+s_3'}{2}]; \\ \frac{(1+\eta_r')(s_3'-x')}{s_3'-s_2'+2\eta_r'(s_3'-x')}, \; if\, x' \in (\frac{s_2'+s_3'}{2}, s_3']. \end{cases}$$

3. Nearest interval approximation of continuous type-2 fuzzy variables

Kundu et al.[4] proposed the interval approximation of type-2 fuzzy variables by applying the α cut of the optimistic, pessimistic and credibilistic approximation of type-2 triangular fuzzy variables given by theorem 3.1. Lastly, using interval approximation method to these α cuts estimated crisp intervals are obtained which are given below:

(i) **applying α cut of the optimistic CV based reduction(optimistic interval approximation)**:

The optimistic interval approximation of $\tilde{\tau}$ is $[C_L, C_R]$ where,

$$C_L = C_{L1} + C_{L2} \tag{3.1}$$

$$C_{L1} = \frac{(1+\eta_r')s_1'}{\eta_r'}\ln(\frac{1+\eta_r'}{1+0.5\eta_r'}) - \frac{s_2'-s_1'-\eta_r's_1'}{\eta_r'^2}\left[0.5\eta_r' - (1+\eta_r')\ln(\frac{1+\eta_r'}{1+0.5\eta_r'})\right],$$

$$C_{L2} = -\frac{s_1'-\eta_r's_2'}{\eta_r'}\ln(1-0.5\eta_r') + \frac{s_2'-s_1'+\eta_r's_2'}{\eta_r'^2}\left[0.5\eta_r' + (1-\eta_r')\ln(1-0.5\eta_r')\right].$$

$$C_R = C_{R1} + C_{R2}, \tag{3.2}$$

$$C_{R1} = \frac{(1+\eta_r')s_3'}{\eta_r'}\ln(\frac{1+\eta_r'}{1+0.5\eta_r'}) + \frac{s_3'-s_2'+\eta_r's_3'}{\eta_r'^2}\left[0.5\eta_r' - (1+\eta_r')\ln(\frac{1+\eta_r'}{1+0.5\eta_r'})\right],$$

$$C_{R2} = -\frac{s_3'-\eta_r's_2'}{\eta_r'}\ln(1-0.5\eta_r') - \frac{s_3'-s_2'-\eta_r's_2'}{\eta_r'^2}\left[0.5\eta_r' + (1-\eta_r')\ln(1-0.5\eta_r')\right].$$

(ii) **applying α cut of the pessimistic CV based reduction(pessimistic interval approximation)**:

The pessimistic interval approximation of $\tilde{\tau}$ is $[C_L, C_R]$ where,

$$C_L = C_{L1} + C_{L2}, \tag{3.3}$$

$$C_{L1} = -\frac{s_1'}{\eta_l'}\ln(1-0.5\eta_l') - \frac{s_2'-s_1'-\eta_l's_1'}{\eta_l'^2}\left[0.5\eta_l' + \ln(1-0.5\eta_l')\right],$$

$$C_{L2} = \frac{s_1'}{\eta_l'}\ln(\frac{1+\eta_l'}{1+0.5\eta_l'}) + \frac{s_2'-s_1'+\eta_l's_2'}{\eta_l'^2}\left[0.5\eta_l' - \ln(\frac{1+\eta_l'}{1+0.5\eta_l'})\right].$$

$$C_R = C_{R1} + C_{R2}, \tag{3.4}$$

$$C_{R1} = -\frac{s_3'}{\eta_l'}\ln(1-0.5\eta_l') + \frac{s_3'-s_2'+\eta_l's_3'}{\eta_l'^2}\left[0.5\eta_l' + \ln(1-0.5\eta_l')\right],$$

$$C_{R2} = \frac{s_3'}{\eta_l'}\ln(\frac{1+\eta_l'}{1+0.5\eta_l'}) + \frac{s_3'-s_2'-\eta_l's_2'}{\eta_l'^2}\left[0.5\eta_l' - \ln(\frac{1+\eta_l'}{1+0.5\eta_l'})\right].$$

(iii) **applying α cut of the CV reduction(credibilistic interval approximation)**:The credibilistic interval approximation of $\tilde{\tau}$ is $[C_L, C_R]$ where,

$$C_L = C_{L1} + C_{L2}, \tag{3.5}$$

$$C_{L1} = \frac{(1+\eta_r')s_1'}{2\eta_r'}\ln(1+\eta_r') - \frac{s_2'-s_1'-2\eta_r's_1'}{4\eta_r'^2}\left[\eta_r' - (1+\eta_r')\ln(1+\eta_r')\right],$$

$$C_{L2} = \frac{s_1'-\eta_l's_2'}{2\eta_l'}\ln(1+\eta_l') + \frac{s_2'-s_1'+2\eta_l's_2'}{4\eta_l'^2}\left[\eta_l' - (1-\eta_l')\ln(1+\eta_l')\right].$$

$$C_R = C_{R1} + C_{R2}, \tag{3.6}$$

$$C_{R1} = \frac{(1+\eta_r')s_3'}{2\eta_r'}\ln(1+\eta_r') + \frac{s_3'-s_2'+2\eta_r's_3'}{4\eta_r'^2}\left[\eta_r' - (1+\eta_r')\ln(1+\eta_r')\right],$$

$$C_{R2} = \frac{s_3'-\eta_l's_2'}{2\eta_l'}\ln(1+\eta_l') - \frac{s_3'-s_2'-2\eta_l's_2'}{4\eta_l'^2}\left[\eta_l' - (1-\eta_l')\ln(1+\eta_l')\right].$$

4. Model: Multi-item multi-objective solid fixed charged shipment model with condition on carriages

Suppose that $K(k = 1, 2, ..., K)$ different modes of carriages are necessary to transport l components from m sources $O_i(i = 1, 2, ..., m)$ to n stations $D_j(j = 1, 2, ..., n)$ and also$(t = 1, 2, ..., R)$ objectives are to be minimized. In addition to that there are a few conditions on a few particular components and carriages so that a few components can not be shipped over a few carriages. Suppose that I_k as the set of components which can be shipped over carriages $k(k = 1, 2, ..., K)$. We use the representation $p'(p' = 1, 2, ..., l)$ to stand for the components.

The solid fixed charge shipment model (SFCSM) is linked with two types of costs, unit shipment cost for shipping unit product from source i to station j and a fixed cost for the direction (i,j). Here we develop a multi-item multi-objective solid fixed charged shipment model(MIMOSFCSM) with m sources, n stations, k carriages, unit shipment costs and fixed costs criterion as T2 FVs as follows:

$$\begin{aligned} \text{Min } Z_t &= \sum_{p'=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K} d_{ijk}^{p'}(c_{ijk}^{tp'}x_{ijk}^{p'}) + e_{ijk}^{tp'}y_{ijk}^{p'},\ t=1,2,3,......R \\ \text{subject to} &\sum_{j=1}^{n}\sum_{k=1}^{K} d_{ijk}^{p'}x_{ijk}^{p'} \le a_i^{p'},\ i=1,2,...,m;p'=1,2,...,l, \\ &\sum_{i=1}^{m}\sum_{k=1}^{K} d_{ijk}^{p'}x_{ijk}^{p'} \ge b_j^{p'},\ j=1,2,...,n;p'=1,2,...,l, \\ \sum_{p'=1}^{l}\sum_{i=1}^{m}&\sum_{j=1}^{n} d_{ijk}^{p'}x_{ijk}^{p'} \le f_k, k=1,2,...,K, \\ &x_{ijk}^{p'} \ge 0,\ \forall\, i,j,k,p', \end{aligned} \tag{4.1}$$

where $d_{ijk}^{p'}$ are defined as

$d_{ijk}^{p'} = \begin{cases} 1, if p' \in I_K\ \forall\, i,j,k,p'; \\ 0, \text{otherwise}, \end{cases}$ and $y_{ijk}^{p'} = \begin{cases} 1, if x_{ijk}^{p'} > 0; \\ 0, \text{otherwise}. \end{cases}$

Here, $x_{ijk}^{p'}$is the decision variable representing the amount of p'-th article shipped from source i to station j, $e_{ijk}^{tp'}$ is the type-2 fuzzy fixed cost linked with direction (i,j) for the objective Z_t. The unit shipment cost $c_{ijk}^{tp'}$(from i-th origin to j-th station by k-th carriage for p'-th article) for the objective Z_t, total supply of p'-th article $a_i^{p'}$ at i-th source, total requirement of p'-th article $b_j^{p'}$ at j-th station and total quantity f_k of k-th carriage are all type-2 fuzzy variables.

5. Solution Procedure

5.1. applying nearest interval approximation. We consider $c_{ijk}^{tp'}$, $e_{ijk}^{tp'}$, $a_i^{p'}$, $b_j^{p'}$ and f_k are type-2 triangular fuzzy variables denoted by $c_{ijk}^{tp'}$=$(c_{ijk}^{tp'1}, c_{ijk}^{tp'2}, c_{ijk}^{tp'3}; \theta_{l,ijk}^{p'}, \theta_{r,ijk}^{p'})$, $e_{ijk}^{tp'}$=$(e_{ijk}^{tp'1}, e_{ijk}^{tp'2}, e_{ijk}^{tp'3}; \theta_{l,ijk}^{\prime p'}, \theta_{r,ijk}^{\prime p'})$,$a_i^{p'} = (a_i^{p'1}, a_i^{p'2}, a_i^{p'3}; \theta_{l,i}^{p'}, \theta_{r,i}^{p'})$, $b_j^{p'}$=$(b_j^{p'1}, b_j^{p'2}, b_j^{p'3}; \theta_{l,j}^{p'}, \theta_{r,j}^{p'})$, and f_k=$(f_k^1, f_k^2, f_k^3; \theta_{l,k}, \theta_{r,k})$. We find the credibilistic interval approximation of $c_{ijk}^{tp'}$, $e_{ijk}^{tp'}$, $a_i^{p'}$, $b_j^{p'}$, f_k from(3.5)-(3.6) and suppose these are $[c_{ijkL}^{tp'}, c_{ijkR}^{tp'}]$, $[e_{ijkL}^{tp'}, e_{ijkR}^{tp'}]$, $[a_{iL}^{p'}, a_{iR}^{p'}]$, $[b_{jL}^{p'}, b_{jR}^{p'}]$, and $[f_{kL}, f_{kR}]$. Then with these credibilistic interval approximations the problem (4.1) becomes

$$\text{Min } Z_t = \sum_{p'=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K} d_{ijk}^{p'}([c_{ijkL}^{tp'}, c_{ijkR}^{tp'}]x_{ijk}^{p'}) + [e_{ijkL}^{tp'}, e_{ijkR}^{tp'}]y_{ijk}^{tp'}, t = 1, 2, ..., R$$

$$\text{subject to } \sum_{j=1}^{n}\sum_{k=1}^{K} d_{ijk}^{p'} x_{ijk}^{p'} \leq [a_{iL}^{p'}, a_{iR}^{p'}],$$

$$\sum_{i=1}^{m}\sum_{k=1}^{K} d_{ijk}^{p'} x_{ijk}^{p'} \geq [b_{jL}^{p'}, b_{jR}^{p'}],$$

$$\sum_{p'=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ijk}^{p'} x_{ijk}^{p'} \leq [f_{kL}, f_{kR}], \tag{5.1}$$

$$x_{ijk}^{p'} \geq 0, \forall i, j, k, p', d_{ijk}^{p'} = \begin{cases} 1, if\, p' \in I_K \;\forall\, i, j, k, p'; \\ 0, \text{ otherwise,} \end{cases} \quad y_{ijk}^{p'} = \begin{cases} 1, if\, x_{ijk}^{p'} > 0; \\ 0, \text{ otherwise.} \end{cases}$$

5.1.1. Deterministic Form. The left hand side of the origin, station and carriage quantity constraints of the problem (5.1) are denoted by $S_i^{p'}, D_j^{p'}$ and E_k respectively. Here the right hand sides of these constraints are interval numbers and left hand sides are crisp, then the possibility degree [14] of satisfaction of these constraints are represented as

$$P_{S_i^{p'} \leq [a_{iL}^{p'}, a_{iR}^{p'}]} = \begin{cases} 1, & S_i^{p'} \leq a_{iL}^{p'}; \\ \frac{a_{iR}^{p'} - S_i^{p'}}{a_{iR}^{p'} - a_{iL}^{p'}}, & a_{iL}^{p'} < S_i^{p'} \leq a_{iR}^{p'}; \\ 0, & S_i^{p'} > a_{iR}^{p'}. \end{cases}$$

$$P_{D_j^{p'} \geq [b_{jL}^{p'}, b_{jR}^{p'}]} = \begin{cases} 0, & D_j^{p'} < b_{jL}^{p'}; \\ \frac{D_j^{p'} - b_{jL}^{p'}}{b_{jR}^{p'} - b_{jL}^{p'}}, & b_{jL}^{p'} \leq D_j^{p'} < b_{jR}^{p'}; \\ 1, & D_j^{p'} > b_{jL}^{p'}. \end{cases}$$

$$P_{E_k \leq [e_{kL}, e_{kR}]} = \begin{cases} 1, & E_k \leq e_{kL}; \\ \frac{e_{kR} - E_k}{e_{kR} - e_{kL}}, & e_{kR} < E_k \leq e_{kR}; \\ 0, & E_k > e_{kR}. \end{cases}$$

The constraints are allowed to be satisfied with a few predetermined possibility degree level $\alpha_i^{p'}, \beta_j^{p'}$ and $\gamma_k (0 < \alpha_i^{p'}, \beta_j^{p'}, \gamma_k \leq 1)$ respectively, i.e. $P_{S_i^{p'} \leq [a_{iL}^{p'}, a_{iR}^{p'}]} \geq \alpha_i^{p'}$, $P_{D_j^{p'} \geq [b_{jL}^{p'}, b_{jR}^{p'}]} \geq \beta_j^{p'}$ and $P_{E_k \leq [e_{kL}, e_{kR}]} \geq \gamma_k \forall\, i, j, k, p'$, then the corresponding inequalities of the constraints are found as follows:

$$S_i^{p'} \leq a_{iR}^{p'} - \alpha_i^{p'}[a_{iR}^{p'} - a_{iL}^{p'}], \tag{5.2}$$

$$D_j^{p'} \geq b_{jL}^{p'} + \beta_j^{p'}[b_{jR}^{p'} - b_{jL}^{p'}], \tag{5.3}$$

$$E_k \leq e_{kR} - \gamma_k[e_{kR} - e_{kL}]. \tag{5.4}$$

5.2. Fuzzy programming technique. Zimmermann [15] established that fuzzy linear programming technique regularly provides useful solutions and an optimal compromise solution for multiple objective problems. The following are the steps to solve the several objective models applying fuzzy programming technique:

Step 1: The several objective model is solved as a one objective model applying, every time, single objective $\bar{Z}_t$ to find the optimal solution $X^{t*} = x^{p'}_{ijk}$ of R distinct single objective model.
Step 2: The values of R objective functions at all these R optimal solutions X^{t*} are calculated and the upper and lower bound for every objective is fixed by $U_t = Max\{\bar{Z}_t(X^{1*}), \bar{Z}_t(X^{2*}), ..., \bar{Z}_t(X^{t*})\}$ and $L_t = \bar{Z}_t(X^{t*})$.
Step 3: The linear membership function $\mu_t(\bar{Z}_t)$ corresponding to t^{th} objective is calculated as

$$\mu_t(\bar{Z}_t) = \begin{cases} 1, if\, \bar{Z}_t \leq L_t; \\ \frac{U_t - \bar{Z}_t}{U_t - L_t}, if\, L_t < \bar{Z}_t < U_t; \\ 0, if\, \bar{Z}_t \geq U_t, \forall t. \end{cases}$$

Step 4: The fuzzy linear programming model is expressed applying max-min operator as
Max δ
subject to

$$\delta \leq \mu_t(\bar{Z}_t) = \frac{U_t - \bar{Z}_t}{U_t - L_t}, \forall t \tag{5.5}$$

and the constraints of (5.1)
$\delta \geq 0$ and $\delta = \min_t\{\mu_t(\bar{Z}_t)\}$.
Step 5: The diminished model is worked out and the optimum solutions are obtained.
We obtain minimum objective function value (say $\underline{Z}_t$) and maximum possible objective function value (say $\bar{Z}_t$) for $[c^{tp'}_{ijkL}, c^{tp'}_{ijkR}]$, $[e^{tp'}_{ijkL}, e^{tp'}_{ijkR}]$ by solving the succeeding two models:

$$\underline{Z}_t = \min_{c^{tp'}_{ijkL} \leq c^{tp'}_{ijk} \leq c^{tp'}_{ijkR}, e^{tp'}_{ijkL} \leq e^{tp'}_{ijk} \leq e^{tp'}_{ijkR}} \left[Min \sum_{p'=1}^{l} \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} d^{p'}_{ijk}(c^{tp'}_{ijk} x^{p'}_{ijk}) + e^{tp'}_{ijk} y^{p'}_{ijk}\right] \tag{5.6}$$

$$\bar{Z}_t = \max_{c^{tp'}_{ijkL} \leq c^{tp'}_{ijk} \leq c^{tp'}_{ijkR}, e^{tp'}_{ijkL} \leq e^{tp'}_{ijk} \leq e^{tp'}_{ijkR}} \left[Min \sum_{p'=1}^{l} \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} d^{p'}_{ijk}(c^{tp'}_{ijk} x^{p'}_{ijk}) + e^{tp'}_{ijk} y^{p'}_{ijk}\right] \tag{5.7}$$

subject to the above constraints (5.2) − (5.4) for both cases.

6. Numerical Model

The proposed model is illustrated numerically in this section with hypothetical data. The proposed approachability is solved numerically by taking one example of the model. Consider the model with objective functions ($t = 1, 2$), sources ($i = 1, 2, 3$), stations ($j = 1, 2, 3$), carriage ($k = 1, 2, 3, 4$) and components ($p' = 1, 2, 3$). Suppose that $I_1 = \{1, 2\}$, $I_2 = \{1, 2, 3\}$, $I_3 = \{3\}$, $I_4 = \{1, 2, 3\}$. The shipment penalty and fixed costs for this model are given in Tables 1 − 12. The supplies, demands and carriage capacities are the following data: $a^1_1 = [22.4651, 25.5349]$, $a^2_1 = [26.9808, 29.5287]$, $a^3_1 = [24.982, 28.0359]$, $a^1_2 = [26.9861,$

$30.5347], a_2^2 = [21.9722, 26.0278], a_2^3 = [33.4874, 36.0084], a_3^1 = [27.4935, 29.013]$, $a_3^2 = [22.9768, 25.5349], a_3^3 = [23.9768, 27.5581], b_1^1 = [9.9913, 13.0043]$, $b_1^2 = [14.9947, 16.5026], b_1^3 = [15.9894, 19.0053], b_2^1 = [11.4847, 14.0102]$, $b_2^2 = [12.0192, 13.9808], b_2^3 = [11.9739, 15.013], b_3^1 = [10.4836, 13.5164]$, $b_3^2 = [9.4823, 11.5059], b_3^3 = [12.4858, 16.0057], e_1 = [34.9898, 37.0102]$, $e_2 = [47.473, 50.018], e_3 = [28.987, 32.0261], e_4 = [41.4847, 44.5153]$.
The fixed credibility levels for the constraints are taken as $\alpha_i^p = 0.7, \beta_j^p = 0.7, \gamma_k = 0.7$. We found minimum and maximum value of the objective functions($t = 1, 2$) by solving (5.6) and (5.7) and resulting solutions are found as follows:
$\underline{Z}_1 = 329.5304$; $x_{131}^1 = 12.6066$, $x_{211}^1 = 12.1004$, $x_{321}^1 = 10.8889$, $x_{322}^1 = 2.3637$, $x_{122}^2 = 11.695, x_{114}^2 = 16.0502, x_{224}^2 = 1.6973, x_{234}^2 = 10.8988, x_{132}^3 = 12.9245$, $x_{312}^3 = 5.1268, x_{322}^3 = 14.1013, x_{332}^3 = 2.0252, x_{114}^3 = 12.9737$ and
$\bar{Z}_1 = 518.6123$; $x_{111}^1 = 10.7794, x_{131}^1 = 12.6066, x_{211}^1 = 1.321, x_{321}^1 = 10.8889$, $x_{322}^1 = 2.3637, x_{122}^2 = 9.6698, x_{114}^2 = 16.0502, x_{134}^2 = 2.0252, x_{224}^2 = 3.7225, x_{234}^2 = 8.8736, x_{132}^3 = 14.9497, x_{312}^3 = 7.152, x_{322}^3 = 14.1013$, $x_{114}^3 = 10.9485$.
$\underline{Z}_2 = 421.015$; $x_{131}^1 = 12.6066, x_{211}^1 = 12.1004, x_{322}^1 = 13.2526, x_{211}^2 = 1.6973$, $x_{122}^2 = 13.3923, x_{114}^2 = 14.3529, x_{234}^2 = 10.8988, x_{312}^3 = 0.9583, x_{322}^3 = 5.6836$, $x_{332}^3 = 14.9497, x_{123}^3 = 8.4177, x_{114}^3 = 17.1422$ and
$\bar{Z}_2 = 658.6885$; $x_{111}^1 = 10.7794, x_{131}^1 = 12.6066, x_{211}^1 = 1.321, x_{322}^1 = 13.2526$, $x_{122}^2 = 13.3923, x_{114}^2 = 14.3529, x_{214}^2 = 1.6973, x_{234}^2 = 10.8988, x_{332}^3 = 14.9497$, $x_{213}^3 = 16.7569, x_{114}^3 = 1.3436, x_{124}^3 = 14.1013$.
Here, $L_{11} = 329.5304, U_{11} = 383.6979$, $L_{12} = 518.6123, U_{12} = 559.8821$, $L_{21} = 421.015, U_{21} = 454.8664$, $L_{22} = 658.6885$, and $U_{22} = 697.5698$ are the lower (L_{t1}, L_{t2}) and upper bounds (U_{t1}, U_{t2}) corresponding to the first and second objective functions respectively. The compromise optimal solution of (5.5) applying LINGO 16 solver, based upon GRG technique are as follows: $x_{111}^1 = 10.7794$, $x_{131}^1 = 12.6066, x_{211}^1 = 1.321, x_{321}^1 = 2.277984, x_{322}^1 = 10.97462, x_{122}^2 = 13.3923$, $x_{114}^2 = 14.3529, x_{214}^2 = 1.6973, x_{234}^2 = 10.8988, x_{132}^3 = 10.4533, x_{322}^3 = 8.919884$, $x_{332}^3 = 4.4964, x_{213}^3 = 7.837016, x_{114}^3 = 10.26348, x_{124}^3 = 5.181416, \delta = 0.3489903$ and the minimum first and second shipment cost(first and second objective value) is $\underline{Z}_1^* = 360.8425$, $\bar{Z}_1^* = 545.4793$, $\underline{Z}_2^* = 442.9211$, $\bar{Z}_2^* = 684.0006$.
i.e. $[360.8425, 545.4793]$ and $[442.9211, 684.0006]$.

7. Conclusion

In this paper, we have projected and worked out a multi-item multi-objective solid fixed charge shipment model with type-2 triangular fuzzy variables. A nearest interval approximation approach is used to solve the model applying LINGO 16 solver.

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Appendix A. Data

TABLE 1. c_{ijk}^{11}

i/j	1	2	3	k
1	$[1.4975, 2.5025]$	$[2.5028, 3.4972]$	$[1.4974, 2.5026]$	
2	$[1.4926, 2.5074]$	$[3.4977, 4.5023]$	$[2.9795, 4.5102]$	1
3	$[2.0159, 3.4921]$	$[2.5, 3.5]$	$[3.5025, 4.4975]$	
1	$[2.4847, 4.5051]$	$[2.9947, 4.5026]$	$[4.9908, 7.5023]$	
2	$[6.4866, 8.5045]$	$[4.5, 6]$	$[5.5025, 6.4975]$	2
3	$[1.5, 2.5]$	$[1.5079, 2.4921]$	$[2.5048, 3.4952]$	
1	$[3.9939, 5.5031]$	$[4.4931, 5.5069]$	$[6.5059, 7.4941]$	
2	$[4.5074, 5.4926]$	$[6.5, 7.5]$	$[4.0205, 5.4898]$	4
3	$[6.5, 7.5]$	$[6, 7.5]$	$[8.4978, 9.5022]$	

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TABLE 2. c_{ijk}^{12}

i/j	1	2	3	k
1	$[5.5, 7]$	$[4.4975, 7.0099]$	$[5.9898, 8.0102]$	
2	$[4, 6]$	$[7.5, 8.5]$	$[7.5042, 8.9916]$	1
3	$[7.0041, 8.9959]$	$[7.9808, 10.0192]$	$[9.4884, 11.0232]$	
1	$[5.4821, 7.0359]$	$[1.5025, 2.9951]$	$[5.4974, 6.5026]$	
2	$[5.9852, 7.5074]$	$[3.9947, 6.0053]$	$[4.4936, 5.5064]$	2
3	$[9.498, 10.502]$	$[7.4955, 10.0179]$	$[7.502, 10.4898]$	
1	$[1.5092, 2.9816]$	$[4.5, 5.5]$	$[2.5051, 3.4949]$	
2	$[3.5028, 4.4972]$	$[3.0205, 4.9795]$	$[1.9898, 4.0102]$	4
3	$[8.0041, 9.498]$	$[3.9954, 6.0046]$	$[5.9959, 8.0041]$	

TABLE 3. c_{ijk}^{13}

i/j	1	2	3	k
1	$[4.4904, 5.5096]$	$[4.0102, 5.4949]$	$[2.5025, 3.4975]$	
2	$[6.4974, 7.5026]$	$[5.9916, 7.5042]$	$[6.4978, 8.0043]$	2
3	$[2.0148, 3.4926]$	$[2.5048, 3.4952]$	$[3.4978, 4.5022]$	
1	$[10.5048, 11.4952]$	$[5.5074, 6.9852]$	$[6.0046, 7.9954]$	
2	$[4.5145, 5.4855]$	$[10.5143, 11.9714]$	$[9.4974, 10.5026]$	3
3	$[10.4921, 11.5079]$	$[11.4941, 12.5059]$	$[12.4812, 13.5188]$	
1	$[2.5153, 4.4949]$	$[4.0049, 5.4975]$	$[4.9951, 6.5025]$	
2	$[10.9898, 12.5051]$	$[10.9796, 13.5051]$	$[12.5069, 14.4977]$	4
3	$[6.5672, 8.4776]$	$[8.0489, 9.4755]$	$[7.4898, 10.502]$	

TABLE 4. e_{ijk}^{11}

i/j	1	2	3	k
1	$[4.5102, 5.4898]$	$[3.9943, 5.5028]$	$[5.9947, 7.5026]$	
2	$[3.0148, 4.4926]$	$[2.0232, 3.9768]$	$[5.0102, 7.9796]$	1
3	$[3.9898, 5.5051]$	$[5.009, 6.4955]$	$[6.5065, 7.4935]$	
1	$[3.9795, 6.0205]$	$[4.0296, 6.4926]$	$[4.9659, 7.5085]$	
2	$[6.9947, 8.5026]$	$[7.9947, 9.5026]$	$[8.991, 10.5045]$	2
3	$[9.9852, 11.5074]$	$[11.4975, 12.5025]$	$[12.5116, 13.4884]$	
1	$[7.5074, 8.9852]$	$[8.0057, 9.9943]$	$[8.9947, 11.0053]$	
2	$[9.9752, 12.0248]$	$[10.9905, 13.0095]$	$[12.0046, 13.9954]$	4
3	$[4.0109, 5.9891]$	$[5.0118, 6.9882]$	$[6.0128, 7.9872]$	

TABLE 5. e^{12}_{ijk}

i/j	1	2	3	k
1	$[4.0232, 5.4884]$	$[5.0232, 6.4884]$	$[6.018, 7.491]$	
2	$[7.0109, 8.4945]$	$[8.5051, 9.4949]$	$[9.5051, 10.4949]$	1
3	$[9.9091, 11.5045]$	$[10.982, 12.509]$	$[11.9905, 13.5048]$	
1	$[6.0148, 7.4926]$	$[7.018, 8.491]$	$[8.013, 9.4935]$	
2	$[9.4955, 10.5045]$	$[10.4884, 11.5116]$	$[11.4952, 2.5048]$	2
3	$[12.5, 13.5]$	$[13, 14.5]$	$[13.0106, 15.4974]$	
1	$[4.5051, 5.4949]$	$[6.0053, 7.4974]$	$[6.9898, 8.5051]$	
2	$[2.9905, 4.5048]$	$[3.991, 5.5045]$	$[4.4874, 6.5042]$	4
3	$[5.5222, 7.4926]$	$[7, 8.5]$	$[7.4874, 9.5042]$	

TABLE 6. e^{13}_{ijk}

i/j	1	2	3	k
1	$[1.5026, 2.4974]$	$[2.5074, 3.4926]$	$[3.498, 4.502]$	
2	$[4.5074, 5.4926]$	$[5.4977, 6.5023]$	$[5.4935, 7.5022]$	2
3	$[6.4921, 8.5026]$	$[7.9951, 9.5025]$	$[8.9954, 10.5023]$	
1	$[4, 5.5]$	$[4.4921, 6.5026]$	$[5.4847, 7.5051]$	
2	$[8.9905, 10.5048]$	$[9.991, 11.5045]$	$[10.4921, 12.5026]$	3
3	$[14.0046, 15.4977]$	$[13.4885, 16.5023]$	$[15.9959, 17.502]$	
1	$[2.9894, 5.5026]$	$[4.5208, 6.4931]$	$[5.5349, 7.4884]$	
2	$[6.5, 8.5]$	$[8.0232, 9.4884]$	$[9.0192, 10.4904]$	4
3	$[9.9954, 11.5023]$	$[10.4931, 12.5023]$	$[11.9954, 13.5023]$	

TABLE 7. c^{21}_{ijk}

i/j	1	2	3	k
1	$[1.4975, 2.5025]$	$[2.5028, 3.4972]$	$[1.4974, 3.0053]$	
2	$[1.4926, 2.5074]$	$[3.4977, 4.5023]$	$[2.9795, 4.5102]$	1
3	$[2.5079, 3.4921]$	$[2, 3.5]$	$[3.5025, 4.4975]$	
1	$[3.4949, 4.5051]$	$[2.9947, 4.5026]$	$[4.4977, 5.5023]$	
2	$[4.4955, 8.5313]$	$[4.5, 7.5]$	$[6.5025, 8.9901]$	2
3	$[1.5, 3]$	$[1.5079, 2.9841]$	$[2.5048, 3.9905]$	
1	$[9.9939, 12.0061]$	$[9.9861, 11.5069]$	$[12.0118, 13.4941]$	
2	$[10.0148, 11.9852]$	$[12, 13.5]$	$[12.0205, 13.9795]$	4
3	$[12, 13.5]$	$[12, 13.5]$	$[12.9913, 15.5022]$	

TABLE 8. c^{22}_{ijk}

i/j	1	2	3	k
1	$[6, 8]$	$[4.9951, 8.0099]$	$[6.4847, 9.0102]$	
2	$[4.5, 7]$	$[8, 9.5]$	$[8.0084, 9.9916]$	1
3	$[7.5061, 9.9959]$	$[8.4713, 11.0192]$	$[9.9768, 12.0232]$	
1	$[5.9641, 8.0359]$	$[2.0049, 3.9951]$	$[5.9947, 7.5026]$	
2	$[6.4778, 8.5074]$	$[4.4921, 7.0053]$	$[4.9872, 6.5064]$	2
3	$[9.9959, 11.5020]$	$[7.991, 11.0179]$	$[8.0041, 11.4898]$	
1	$[2.0184, 3.9816]$	$[5, 6.5]$	$[3.0102, 4.4949]$	
2	$[4.0057, 5.4972]$	$[3.5307, 5.9795]$	$[2.4847, 5.0102]$	4
3	$[8.5061, 10.498]$	$[4.4931, 9.0139]$	$[6.4939, 9.0041]$	

TABLE 9. c^{23}_{ijk}

i/j	1	2	3	k
1	$[8.4713, 11.0192]$	$[8.0102, 9.494]$	$[6.0049, 7.4975]$	
2	$[8.9947, 11.5079]$	$[10.9916, 13.0084]$	$[6.9957, 9.0043]$	2
3	$[5.0148, 6.9852]$	$[6.0095, 7.9905]$	$[4.9957, 7.0043]$	
1	$[11.0095, 12.4952]$	$[6.0148, 7.9852]$	$[6.5069, 8.9954]$	
2	$[5.0289, 6.4855]$	$[11.0286, 12.9714]$	$[9.9947, 11.5026]$	3
3	$[10.9841, 12.5079]$	$[11.9882, 13.5059]$	$[12.9624, 14.5188]$	
1	$[3.0204, 5.4949]$	$[4.5074, 6.4975]$	$[5.4926, 7.5025]$	
2	$[11.4847, 13.5051]$	$[11.4745, 14.5051]$	$[13.0092, 15.4977]$	4
3	$[7.0896, 9.4776]$	$[8.5734, 10.4755]$	$[7.9877, 11.502]$	

TABLE 10. e^{21}_{ijk}

i/j	1	2	3	k
1	$[2.0205, 3.4898]$	$[2.4972, 3.5028]$	$[4.4974, 5.5026]$	
2	$[1.5074, 2.4926]$	$[1.5116, 2.4884]$	$[3.5051, 5.9796]$	1
3	$[2.4949, 3.5051]$	$[3.5045, 4.4955]$	$[4.5065, 5.4935]$	
1	$[2.4898, 4.0205]$	$[3.0148, 4.4926]$	$[3.9829, 5.5085]$	
2	$[5.4974, 6.5026]$	$[6.4974, 7.5026]$	$[7.4955, 8.5045]$	2
3	$[8.4926, 9.5074]$	$[9.4975, 10.5025]$	$[10.5116, 11.4884]$	
1	$[5.5074, 6.9852]$	$[6.5028, 7.9943]$	$[7.4974, 9.0053]$	
2	$[8.4876, 10.0248]$	$[9.4952, 11.0095]$	$[10.5023, 11.9954]$	4
3	$[2.5055, 3.9891]$	$[3.5059, 4.9882]$	$[4.5064, 5.9872]$	

TABLE 11. e_{ijk}^{22}

i/j	1	2	3	k
1	$[2.5116, 3.4884]$	$[3.5116, 4.4884]$	$[4.509, 5.491]$	
2	$[5.5055, 6.4945]$	$[6.5051, 7.4949]$	$[7.0102, 8.4949]$	1
3	$[8.4955, 9.5045]$	$[9.491, 10.509]$	$[10.4952, 11.5048]$	
1	$[4.5074, 5.4926]$	$[5.509, 6.491]$	$[6.5065, 7.4935]$	
2	$[7.4955, 8.5045]$	$[7.9768, 9.5116]$	$[9.4952, 10.5048]$	2
3	$[10.5, 11.5]$	$[11.5, 12.5]$	$[12.0053, 13.4974]$	
1	$[2.5051, 3.4949]$	$[4.5026, 5.4974]$	$[5.4949, 6.5051]$	
2	$[1.4952, 2.5048]$	$[2.4955, 3.5045]$	$[3.4958, 4.5042]$	4
3	$[4.5074, 5.4926]$	$[5, 6.5]$	$[6.4958, 7.5042]$	

TABLE 12. e_{ijk}^{23}

i/j	1	2	3	k
1	$[1.5026, 2.4974]$	$[1.5074, 2.4926]$	$[1.498, 2.502]$	
2	$[2.5074, 3.4926]$	$[3.4977, 4.5023]$	$[4.4978, 5.5022]$	2
3	$[5.4974, 6.5026]$	$[6.4975, 7.5025]$	$[7.4977, 8.5023]$	
1	$[2.5, 3.5]$	$[3.4974, 4.5026]$	$[4.4949, 5.5051]$	
2	$[7.4952, 8.5048]$	$[8.4955, 9.5045]$	$[9.4974, 10.5026]$	3
3	$[12.5023, 13.4977]$	$[12.4931, 14.5023]$	$[14.498, 15.502]$	
1	$[1.9947, 3.5026]$	$[3.5069, 4.4931]$	$[4.5116, 5.4884]$	
2	$[5.5, 6.5]$	$[6.5116, 7.4884]$	$[7.5096, 8.4904]$	4
3	$[8.4977, 9.5023]$	$[9.4977, 10.5023]$	$[10.4977, 11.5023]$	