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SOME COMMON FIXED POINT THEOREMS IN INTUITIONISTIC FUZZY METRIC SPACES USING THE PROPERTY (E. A)

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Abstract. In this paper, we establish some common fixed point theorems in intuitionistic fuzzy metric spaces for sequence of self mappings using implicit relation and the common property (E .A). Our results generalize and improve several results of metric spaces and fuzzy metric spaces to intuitionistic fuzzy metric spaces.

1. Introduction

The significant idea of fuzzy set was first introduced in 1965 by Iranian mathematician Zadeh [17]. Fuzzy set is defined by a membership function which assigns each object to a grade of membership between zero and one. Following the concept of fuzzy set Kramosil and Michalek [7] extended the concept of metric space as fuzzy metric space. George and Veeramani [5] modified the concept of fuzzy metric space given by Kramosil and Michalek [7] by imposing some stronger conditions using continuous t-norm and defined the hausdorff topology of fuzzy metric spaces. Vasuki [16] have proved fuzzy version of common fixed point theorem using a strong definition of Cauchy sequence in fuzzy metric space.

Atanassov [4] introduced the concept of intutionistic fuzzy sets as a generalization of fuzzy sets and later there has been much progress in the study of intutionistic fuzzy sets by many authors [12, 13, 14, 15]. Park [9] introduced the notion of intuitionistic fuzzy metric spaces using the idea of fuzzy metric space due to George and Veeramani [5]. Recently, in 2006, Alaca et al. [2] using the idea of intuitionistic fuzzy sets, defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norm and continuous t - conorms as a generalization of fuzzy metric space due to Kramosil and Michalek [7].

Jungck [6] introduced commuting mappings in metric space. Sessa [11] generalized commuting mappings in metric space as weakly commuting mappings. Pant [8] introduced R - weakly commuting mappings in metric space. Turkoglu et al. [14] introduced common fixed point theorems in intuitionistic fuzzy metric spaces. Aamri and El Moutawakil [1] defined the (E. A) property in metric space for self mappings whose class contains the class of non compatible as well as compatible mappings. Common property (E. A) is introduced by Ali et al. [3]. It is observed

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that (E. A) property and common property (E. A) require the closedness of the subspaces for the existence of fixed point.

In this paper, we establish some common fixed point theorems in intuitionistic fuzzy metric spaces for sequence of self mappings using implicit relation and the common property (E. A).

2. Preliminaries

Definition 2.1. (Schweizer and Sklar [10]): A binary operation $* : [0,1]\times[0,1] \to [0,1]$ is called continuous $t-$ norm if it satisfies the following conditions:

(1) $*$ is commutative and associative;
(2) $*$ is continuous;
(3) $a * 1 = a$, for all $a \in [0,1]$;
(4) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, for all $a, b, c, d \in [0,1]$.

Definition 2.2. (Schweizer and Sklar [10]): A binary operation $\diamond : [0,1]\times[0,1] \to [0,1]$ is called continuous $t-$ conorm if it satisfies the following conditions:

(1) $\diamond$ is commutative and associative;
(2) $\diamond$ is continuous;
(3) $a \diamond 0 = a$, for all $a \in [0,1]$;
(4) $a \diamond b \leq c \diamond d$ whenever $a \leq c$ and $b \leq d$, for all $a, b, c, d \in [0,1]$.

Remark 2.3. (Schweizer and Sklar [10]): The concepts of triangular norms (t-norms) and triangular conorms (t-conorms) are known as the axiomatic skeletons that we use for characterizing fuzzy intersections and unions respectively.

Definition 2.4. (Park [9]): An intutionistic fuzzy metric space (IFMS) is a 5 - tuple $(X, M, N, *, \diamond)$ such that X is a non empty set, $*$ is a continuous $t-$ norm, $\diamond$ is a continuous $t-$ conorm M and N are fuzzy set on $X \times X \times [0, \infty)$ satisfying the following conditions, for all $x, y, z \in X$ and $s, t > 0$:

(1) $M(x, y, t) + N(x, y, t) \leq 1$;
(2) $M(x, y, t) > 0$;
(3) $M(x, y, t) = 1$ if and only if $x = y$;
(4) $M(x, y, t) = M(y, x, t)$
(5) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$;
(6) $M(x, y, .) : [0, \infty) \to [0, 1]$ is continuous;
(7) $N(x, y, t) > 0$;
(8) $N(x, y, t) = 0$ if and only if $x = y$;
(9) $N(x, y, t) = N(y, x, t)$;
(10) $N(x, y, t) \diamond N(y, z, s) \geq N(x, z, t + s)$;
(11) $N(x, y, .) : [0, \infty) \to [0, 1]$ is continuous.

Here (M, N) is called an intuitionistic fuzzy metric on X. The functions $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and the degree of non-nearness between x and y with respect to t respectively.

Remark 2.5. (Park [9]) Every fuzzy metric space $(X, M, *)$ is an intuitionistic fuzzy metric space of the form $(X, M, N, *, \diamond)$such that t- norm $*$ and t- conorm $\diamond$ are associated as, $x \diamond y = 1 - ((1 - x) * (1 - y))$ for any $x, y \in X$.

Remark 2.6. (Park [9]) In intuitionistic fuzzy metric space X, $M(x,y,t)$ is non-decreasing and $N(x,y,t)$ is non-increasing for all $x,y \in X$.

Example 2.7. 2.7 (Park [9]) Let (X,d) be a metric space. $a * b = ab$ and $a \diamond b = min 1, a+b$ for all $a,b \in [0,1]$ and let,

$$M(x,y,t) = \frac{ht^n}{ht^n + md(x,y)}$$

and

$$N(x,y,t) = \frac{d(x,y)}{kt^n + md(x,y)}$$

for all $h,k,m,n \in R$, then $(X,M,N,*,\diamond)$ is an intuitionistic fuzzy metric space.

Definition 2.8. (Turkoglu et al. [14]): Two self mappings f and g of an intuitionistic fuzzy metric space $(X,M,N,*,\diamond)$ are said to be weakly commuting if,
$M(fgx,gfx,t) \geq M(fx,gx,t)$ and $N(fgx,gfx,t) \leq N(fx,gx,t)$,
for all $x \in X, t > 0$.

Definition 2.9. (Turkoglu et al. [14]): Two self mappings f and g of an intuitionistic fuzzy metric space $(X,M,N,*,\diamond)$ are said to be R-weakly commuting if,
$M(fgx,gfx,t) \geq M(fx,gx,\frac{t}{R})$ and $N(fgx,gfx,t) \leq N(fx,gx,\frac{t}{R})$,
for all $x \in X, t > 0$ and $R > 0$.

Definition 2.10. (Aamri and Moutawakil aam): Two self mappings A and B of an intuitionistic fuzzy metric space $(X,M,N,*,\diamond)$ are said to satisfy the $(E.A)$ property if there exist a sequence $\{x_n\}$ in X such that for all $t > 0$,

$$lim_{n\to\infty} Ax_n = lim_{n\to\infty} Bx_n = x,$$

for some $x \in X$.

Definition 2.11. (Ali et al. [3]): Two pairs (A,S) and (B,T) of self mappings of a fuzzy metric space $(X,M,*)$ are said to satisfy the common $(E.A)$ property if there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that for all $t > 0$,

$$lim_{n\to\infty} Ax_n = lim_{n\to\infty} Sx_n = lim_{n\to\infty} By_n = lim_{n\to\infty} Ty_n = x,$$

for some $x \in X$.

3. Main Results

Definition 3.1. Implicit Relation: Let Φ be the class of all real valued and continuous functions, $\phi : (R^+)^4 \to R$, satisfying the following conditions:
(I) $\phi(x,y,x,y) \geq 0$ or $\phi(x,y,y,x) \geq 0$ or $\phi(x,x,y,y) \geq 0$ implies $x \geq y$, for all $x,y \geq 0$;
(II) $\phi(x,y,x,y) \leq 1$ or $\phi(x,y,y,x) \leq 1$ or $\phi(x,x,y,y) \leq 1$ implies $x \leq y$, for all $x,y \geq 0$.

Theorem 3.2. *Let A, B and $< F_i >$ where $i \in N\{0\}$, be self-mappings of an intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$, satisfying the following conditions,*
(I) $F_i \subseteq B$ and $F_0 \subseteq A$;
(II) $\phi(M(F_ix, F_0y, kt), M(Ax, By, t), M(F_ix, Ax, t), M(F_0y, By, kt)) \geq 0$;
(III) $\phi(N(F_ix, F_0y, kt), N(Ax, By, t), N(F_ix, Ax, t), N(F_0y, By, kt)) \leq 1$;
(IV) The pairs (F_i, A) and (F_0, B) share the common property $(E.A)$;
(V) The pairs (F_i, A) and (F_0, B) are R- weakly commuting,
for all $x, y \in X$, $t > 0$, $k \in (0, 1)$.
If range of one of A and B is closed subspace of X, then $< F_i >$ where $i \in N\{0\}$, A and B have a unique common fixed point.

Proof. Since the pairs (F_i, A) and (F_0, B) share the common property $(E.A)$, there exist two sequences $\{x_n\}$ and $\{y_n\}$ such that,

$$lim_{n\to\infty}F_0x_n = lim_{n\to\infty}Bx_n = lim_{n\to\infty}F_iy_n = lim_{n\to\infty}Ay_n = z,$$

for some $z \in X$. Suppose that $A(X)$ is a closed subspace of X, then there exists some $u \in X$ such that $z = Au$.
Now we show that, $F_iu = z$. Using condition (II) with $x = u$ and $y = x_n$.

$$\phi(M(F_iu, F_0x_n, kt), M(Au, Bx_n, t), M(F_iu, Au, t), M(F_0x_n, Bx_n, kt)) \geq 0.$$

Taking limit as $n \to \infty$, we get

$$\phi(M(F_iu, z, kt), M(z, z, t), M(F_iu, Au, t), M(z, z, kt)) \geq 0$$

or

$$\phi(M(F_iu, z, kt), 1, M(F_iu, Au, t), 1) \geq 0.$$

Since ϕ is non-decreasing therefore,

$$\phi(M(F_iu, z, t), 1, M(F_iu, z, t), 1) \geq 0.$$

From the definition 3.1,

$$M(F_iu, z, t) \geq 1.$$

Using condition (III) with $x = u$ and $y = x_n$, we have

$$\phi(N(F_iu, F_0x_n, kt), N(Au, Bx_n, t), N(F_iu, Au, t), N(F_0x_n, Bx_n, kt)) \leq 1.$$

Taking limit as $n \to \infty$, we get

$$\phi(N(F_iu, z, kt), N(z, z, t), N(F_iu, Au, t), N(z, z, kt)) \leq 1$$

or

$$\phi(N(F_iu, z, kt), 0, N(F_iu, Au, t), 0) \leq 1$$

or

$$\phi(N(F_iu, z, t), 0, N(F_iu, z, t), 0) \leq 1.$$

From the definition 3.1,

$$N(F_iu, z, t) \leq 0.$$

Therefore, $F_i u = z = Au$.
Since $F_i \subseteq B$, there exists some $v \in X$, such that $F_i u = z = Bv$.
Now we show that $F_0 v = z$. Using condition (II) with $y = v$ and $x = y_n$, we have

$$\phi(M(F_i y_n, F_0 v, kt), M(Ay_n, Bv, t), M(F_i y_n, Ay_n, t), M(F_0 v, Bv, kt)) \geq 0.$$

Taking limit as $n \to \infty$, we get

$$\phi(M(z, F_0 v, kt), M(z, z, t), M(z, z, t), M(F_0 v, z, kt)) \geq 0$$

or

$$\phi(M(z, F_0 v, kt), 1, 1, M(F_0 v, z, kt)) \geq 0.$$

From the definition 3.1, we get

$$M(F_0 v, z, kt) \geq 1.$$

Using condition (II) with $y = v$ and $x = y_n$, we have

$$\phi(N(F_i y_n, F_0 v, kt), N(Ay_n, Bv, t), N(F_i y_n, Ay_n, t), N(F_0 v, Bv, kt)) \leq 1.$$

Taking limit as $n \to \infty$, we get

$$\phi(N(z, F_0 v, kt), N(z, z, t), N(z, z, t), N(F_0 v, z, kt)) \leq 1$$

or

$$\phi(N(z, F_0 v, kt), 0, 0, N(F_0 v, z, kt)) \leq 1.$$

From the definition 3.1

$$N(F_0 v, z, kt) \leq 0.$$

Therefore, $F_0 v = z = Bv$. Hence $F_i u = Au = F_0 v = Bv = z$.
Since F_i and A are R-weakly commutting, there exists $R > 0$ such that,

$$M(F_i Au, AF_i u, t) \geq M(F_i u, Au, \frac{t}{R}) = 1$$

and

$$N(F_i Au, AF_i u, t) \leq N(F_i u, Au, \frac{t}{R}) = 0,$$

hence $F_i Au = AF_i u = F_i F_i u = AAu$.
Similarly F_0 and B are R- weakly commutting, there exists $R > 0$ such that,

$$M(F_0 Bv, BF_0 v, t) \geq M(F_0 v, Bv, \frac{t}{R}) = 1$$

and

$$N(F_0 Bv, BF_0 v, t) \leq N(F_0 v, Bv, \frac{t}{R}) = 0,$$

hence $F_0 Bv = BF_0 v = F_0 F_0 v = BBv$.
Using condition (II) with $y = v$ and $x = F_i u$,

$$\phi(M(F_i F_i u, F_0 v, kt), M(AF_i u, Bv, t), M(F_i F_i u, AF_i u, t), M(F_0 v, Bv, kt)) \geq 0$$

or

$$\phi(M(F_i F_i u, F_i u, kt), M(F_i F_i u, F_i u, t), 1, 1) \geq 0.$$

Since ϕ is non-decreasing, therefore

$$\phi(M(F_i F_i u, F_i u, t), M(F_i F_i u, F_i u, t), 1, 1) \geq 0.$$

From the definition 3.1,

$$M(F_i F_i u, F_i u, t) \geq 1.$$

Using condition (III) with $y = v$ and $x = F_i u$, we have

$$\phi(N(F_iF_iu, F_0v, kt), N(AF_iu, Bv, t), N(F_iF_iu, AF_iu, t), N(F_0v, Bv, kt)) \leq 1$$

or

$$\phi(N(F_iF_iu, F_iu, kt), N(F_iF_iu, F_iu, t), 0, 0) \leq 1.$$

Since ϕ is non-decreasing, therefore

$$\phi(N(F_iF_iu, F_iu, t), N(F_iF_iu, F_iu, t), 0, 0) \leq 1.$$

From the definition 3.1,

$$N(F_iF_iu, F_iu, t) \leq 0.$$

It is possible only when, $F_iF_iu = F_iu$. Hence, $F_iz = z$. Therefore, $F_iz = z = Az$. Thus z is a fixed point of F_i and A.
Now using condition (II) with $y = F_0v$ and $x = u$, we get

$$\phi(M(F_iu, F_0F_0v, kt), M(Au, BF_0v, t), M(F_iu, Au, t), M(F_0F_0v, BF_0v, kt)) \geq 0$$

or

$$\phi(M(F_0v, F_0F_0v, kt), M(F_0v, F_0F_0v, t), 1, 1) \geq 0.$$

Since ϕ is non-decreasing, therefore

$$\phi(M(F_0v, F_0F_0v, t), M(F_0v, F_0F_0v, t), 1, 1) \geq 0.$$

From the definition 3.1,

$$M(F_0v, F_0F_0v, t) \geq 1.$$

Now using condition (III) with $y = F_0v$ and $x = u$, we have

$$\phi(N(F_iu, F_0F_0v, kt), N(Au, BF_0v, t), N(F_iu, Au, t), N(F_0F_0v, BF_0v, kt)) \leq 1$$

or

$$\phi(N(F_0v, F_0F_0v, kt), N(F_0v, F_0F_0v, t), 0, 0) \leq 1.$$

Since ϕ is non-decreasing therefore,

$$\phi(N(F_0v, F_0F_0v, t), N(F_0v, F_0F_0v, t), 0, 0) \leq 1.$$

From the definition 3.1,

$$N(F_0v, F_0F_0v, t) \leq 0.$$

It is possible only when, $F_0v = F_0F_0v = z$. Hence, $F_0z = z$.Therefore, $F_0z = Bz = z$. Thus z is a fixed point of F_0 and B. Which gives, $F_iz = Az = F_0z = Bz = z$. Hence z is a common fixed point of $< Fi >, A$ and B.
Now to show that z is unique common fixed point of $< Fi >$, where $i \in N \cup \{0\}$, A and B. Suppose z_1 is another common fixed point of $< Fi >$, where $i \in N \cup \{0\}$, A and B.

Now using condition (II) with $y = z_1$ and $x = z$, we get

$$\phi(M(F_iz, F_0z_1, kt), M(Az, Bz_1, t), M(F_iz, Az, t), M(F_0z_1, Bz_1, kt)) \geq$$

or

$$\phi(M(z, z_1, kt), M(z, z_1, t), 1, 1) \geq 0.$$

Since ϕ is non-decreasing therefore,

$$\phi(M(z, z_1, t), M(z, z_1, t), 1, 1) \geq 0.$$

From the definition 3.1,

$$M(z, z_1, t) \geq 1.$$

Now using condition (III) with $y = z_1$ and $x = z$,

$$\phi(N(F_iz, F_0z_1, kt), N(Az, Bz_1, t), N(F_iz, Az, t), N(F_0z_1, Bz_1, kt)) \leq 1$$

or

$$\phi(N(z, z_1, kt), N(z, z_1, t), 0, 0) \leq 1.$$

Since ϕ is non-decreasing therefore,

$$\phi(N(z, z_1, t), N(z, z_1, t), 0, 0) \leq 1.$$

From the definition 3.1,

$$M(z, z_1, t) \leq 0.$$

It is possible only when $z = z_1$. Hence z is unique common fixed point of $< F_i >$, where $i \in N \cup \{0\}$, $A$and B.

Theorem 3.3. *Let A, B, F and G be self-maps of intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$, satisfying the following conditions:*
(I) $F \subseteq B$ and $G \subseteq A$;
(II) $\phi(M(Fx, Gy, kt), M(Ax, By, t), M(Fx, Ax, t), M(Gy, By, kt)) \geq 0$;
(III) $\phi(N(Fx, Gy, kt), N(Ax, By, t), N(Fx, Ax, t), N(Gy, By, kt)) \leq 1$;
(IV) The pairs (F, A) and (G, B) share the common property $(E.A)$;
(V) The pairs (F, A) and (G, B) are R-weakly commuting,
for all $x, y \in X$, $t > 0$, $k \in (0, 1)$. If range of one of A and B is closed subspace of X, then F, G, A and B have unique common fixed point.

Proof. Since the pairs (F, A) and (G, B) share the common property $(E.A)$, there exist two sequences $\{x_n\}$ and $\{y_n\}$ such that,

$$lim_{n\to\infty}Gx_n = lim_{n\to\infty}Bx_n = lim_{n\to\infty}Fy_n = lim_{n\to\infty}Ay_n = z,$$

for some $z \in$X.$Supposethat$A(X)$isaclosedsubspaceof$X, $thenthereexistssome$u $\in X$ such that $z = Au$.
Now we show that, $Fu = z$. Using condition (II) with $x = u$ and $y = x_n$, we get

$$\phi(M(Fu, Gx_n, kt), M(Au, Bx_n, t), M(Fu, Au, t), M(Gx_n, Bx_n, kt)) \geq 0.$$

Taking limit as $n \to \infty$,

$$\phi(M(Fu, z, kt), M(z, z, t), M(Fu, Au, t), M(z, z, kt)) \geq 0$$

or

$$\phi(M(Fu, z, kt), 1, M(Fu, Au, t), 1) \geq 0.$$

Since ϕ is non-decreasing, therefore

$$\phi(M(Fu, z, t), 1, M(Fu, Au, t), 1) \geq 0.$$

From the definition 3.1,

$$M(Fu, z, kt) \geq 1$$

Using condition (III) with $x = u$ and $y = x_n$.

$$\phi(N(Fu, Gx_n, kt), N(Au, Bx_n, t), N(Fu, Au, t), N(Gx_n, Bx_n, kt)) \leq 1.$$

Taking limit as $n \to \infty$.

$$\phi(N(Fu, z, kt), N(z, z, t), N(Fu, Au, t), N(z, z, kt)) \leq 1$$

or

$$\phi(N(Fu, z, kt), 0, N(Fu, Au, t), 0) \leq 1.$$

Since ϕ is non-decreasing, therefore

$$\phi(N(Fu, z, t), 1, N(Fu, Au, t), 1) \leq 1.$$

From the definition 3.1,

$$N(Fu, z, kt) \leq 0.$$

Therefore $Fu = z = Au$. Since $F \subseteq B$, there exists some $v \in X$, such that $Fu = z = v$.

Now we show that $Gv = z$. Using condition (II) with $y = v$ and $x = y_n$, we have

$$\phi(M(Fy_n, Gv, kt), M(Ay_n, Bv, t), M(Fy_n, Ay_n, t), M(Gv, Bv, kt)) \geq 0.$$

Taking limit as $n \to \infty$,

$$\phi(M(z, Gv, kt), M(z, z, t), M(z, z, t), M(Gv, z, kt)) \geq 0.$$

Since ϕ is non-decreasing, therefore

$$M(Fv, z, kt) \geq 1.$$

Using condition (III) with $y = v$ and $x = y_n$.

$$\phi(N(Fy_n, Gv, kt), N(Ay_n, Bv, t), N(Fy_n, Ay_n, t), N(Gv, Bv, kt)) \leq 1.$$

Taking limit as $n \to \infty$, we have

$$\phi(N(z, Gv, kt), N(z, z, t), N(z, z, t), N(Gv, z, kt)) \leq 1.$$

Since ϕ is non-decreasing, therefore

$$N(Fv, z, kt) \leq 0.$$

Therefore $Gv = z = Bv$. Hence $Fu = Au = Gv = Bv = z$.

Since F and A are R-weakly commutting, there exists $R > 0$ such that,

$$M(FAu, AFu, t) \geq M(Fu, Au, \frac{t}{R}) = 1$$

and

$$N(FAu, AFu, t) \leq N(Fu, Au, \frac{t}{R}) = 0,$$

that is $FAu = AFu = FFu = AAu$. Similarly G and B are R-weakly commutting, there exists $R > 0$ such that,

$$M(GBv, BGv, t)M(Gv, Bv, \frac{t}{R}) = 1$$

and

$$N(GBv, BGv, t)N(Gv, Bv, \frac{t}{R}) = 0,$$

that is

$$GBv = BGv = GGv = BBv.$$

Using condition (II) with $y = v$ and $x = F_i u$,

$$\phi(M(FFu, Gv, kt), M(AFu, Bv, t), M(FFu, AFu, t), M(Gv, Bv, kt)) \geq 0$$

or

$$\phi(M(FFu, Fu, kt), M(FFu, Fu, t), 1, 1) \geq 0.$$

Since ϕ is non-decreasing, therefore

$$\phi(M(FFu, Fu, t), M(FFu, Fu, t), 1, 1) \geq 0.$$

From the definition 3.1,

$$M(FFu, Fu, t) \geq 1.$$

Using condition (III) with $y = v$ and $x = F_i u$,

$$\phi(N(FFu, Gv, kt), N(AFu, Bv, t), N(FFu, AFu, t), N(Gv, Bv, kt)) \leq 1$$

or

$$\phi(N(FFu, Fu, kt), N(FFu, Fu, t), 0, 0) \leq 1.$$

Since ϕ is non-decreasing, therefore

$$\phi(N(FFu, Fu, t), N(FFu, Fu, t), 0, 0) \leq 1.$$

From the definition 3.1,

$$N(FFu, Fu, t) \leq 0.$$

It is possible only when, $FFu = Fu$. Therefore $Fz = z$. Hence $Fz = z = Az$. Thus z is a fixed point of F and A.
Now using condition (II) with $y = Gv$ and $x = u$,

$$\phi(M(Fu, GGv, kt), M(Au, BGv, t), M(Fu, Au, t), M(GGv, BGv, kt)) \geq 0$$

or

$$\phi(M(GGv, Gv, kt), M(GGv, Gv, t), 1, 1) \geq 0.$$

Since ϕ is non-decreasing, therefore

$$\phi(M(GGv, Gv, t), M(GGv, Gv, t), 1, 1) \geq 0.$$

From the definition 3.1,

$$M(GGv, Gv, t) \geq 1.$$

Using condition (III) with $y = Gv$ and $x = u$,

$$\phi(N(Fu, GGv, kt), N(Au, BGv, t), N(Fu, Au, t), N(GGv, BGv, kt)) \leq 1$$

or

$$\phi(N(GGv, Gv, kt), N(GGv, Gv, t), 1, 1) \leq 1.$$

Since ϕ is non-decreasing, therefore

$$\phi(N(GGv, Gv, t), N(GGv, Gv, t), 1, 1) \leq 1.$$

From the definition 3.1,

$$N(GGv, Gv, t) \leq 0.$$

It is possible only when, $Gv = GGv = z$. Therefore $Gz = z$. Hence $Gz = Bz = z$. Thus z is a fixed point of G and B. i. e. $Fz = Az = Gz = Bz = z$. Hence z is a common fixed point of F, G, A and B.
Now to show that z is unique common fixed point of F, G, A and B. Suppose z_1 is another common fixed point of F, G, A and B.
Now using condition (II) with $y = z_1$ and $x = z$,

$$\phi(M(Fz, Gz_1, kt), M(Az, Bz_1, t), M(Fz, Az, t), M(Fz_1, Bz_1, kt)) \geq 0$$

or

$$\phi(M(z, z_1, kt), M(z, z_1, t), 1, 1) \geq 0.$$

Since ϕ is non-decreasing, therefore

$$\phi(M(z, z_1, t), M(z, z_1, t), 1, 1) \geq 0.$$

From the definition 3.1,

$$M(z, z_1, t) \geq 1.$$

Using condition (III) with $y = z_1$ and $x = z$,

$$\phi(N(Fz, Gz_1, kt), N(Az, Bz_1, t), N(Fz, Az, t), N(Fz_1, Bz_1, kt)) \leq 1$$

or

$$\phi(N(z, z_1, kt), N(z, z_1, t), 0, 0) \leq 1.$$

Since ϕ is non-decreasing, therefore

$$\phi(N(z, z_1, t), N(z, z_1, t), 0, 0) \leq 1.$$

From the definition 3.1,

$$N(z, z_1, t) \leq 0.$$

It is possible only when, $z = z_1$. Hence z is unique common fixed point of F, G, A and B. □

Corollary 3.4. *Let A, B and $< F_i >$ where $i \in N \cup \{0\}$, be self-maps of intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$, satisfying the following conditions:*
(I) $F_i \subseteq B$ and $F_0 \subseteq A$;
(II) $\phi(M(F_ix, F_0y, kt), M(Ax, By, t), M(F_ix, Ax, t), M(F_0y, By, kt)) \geq 0$;
(III) $\phi(N(F_ix, F_0y, kt), N(Ax, By, t), N(F_ix, Ax, t), N(F_0y, By, kt)) \leq 1$;
(IV) The pair (F_0, B) satisfies property $(E.A)$;
(V) The pairs (F_i, A) and (F_0, B) are R- weakly commuting,
for all $x, y \in X$, $t > 0$, $k \in (0, 1)$. If range of one of A and B is closed subspace of X, then $< F_i >$ where $i \in N \cup \{0\}$, A and B have a unique common fixed point.

Proof. Since the pair (F_0, B) share the common property $(E.A)$, there exist a sequence $\{x_n\}$ such that,

$$lim_{n\to\infty} F_0x_n = lim_{n\to\infty} Bx_n = z, \ for \ some \ z \in X.$$

Since $F_0(X) \subseteq A(X)$, there exists $\{y_n\}$ in X such that, $F_0x_n = Ay_n$ and $lim_{n\to\infty} Ay_n = z$.
Now we show that, $lim_{n\to\infty} F_iy_n = z$.
Using condition (II) with $x = y_n$ and $y = x_n$, we have

$$\phi(M(F_iy_n, F_0x_n, kt), M(Ay_n, Bx_n, t), M(F_iy_n, Ay_n, t), M(F_0x_n, Bx_n, kt)) \geq 0.$$

Taking limit as $n \to \infty$, we get

$$\phi(M(F_iy_n, z, kt), M(z, z, t), M(F_iy_n, z, t), M(z, z, kt)) \geq 0$$

or

$$\phi(M(F_iy_n, z, kt), 1, M(F_iy_n, z, t), 1) \geq 0.$$

Since ϕ is non-decreasing, therefore

$$\phi(M(F_iy_n, z, t), 1, M(F_iy_n, z, t), 1) \geq 0.$$

From the definition 3.1,

$$M(F_i y_n, z, t) \geq 1,$$

using condition (III) with $x = y_n$ and $y = x_n$.

$$\phi(N(F_i y_n, F_0 x_n, kt), N(Ay_n, Bx_n, t), N(F_i y_n, Ay_n, t), N(F_0 x_n, Bx_n, kt)) \leq 1.$$

Taking limit as $n \to \infty$, we have

$$\phi(N(F_i y_n, z, kt), N(z, z, t), N(F_i y_n, z, t), N(z, z, kt)) \leq 1$$

or

$$\phi(N(F_i y_n, z, kt), 0, N(F_i y_n, z, t), 0) \leq 1.$$

Since ϕ is non-decreasing, therefore

$$\phi(N(F_i y_n, z, t), 0, N(F_i y_n, z, t), 0) \leq 1.$$

From the definition 3.1,

$$N(F_i y_n, z, t) \leq 0.$$

It is possible only when $lim_{n\to\infty} F_i y_n = z$. Therefore,

$$lim_{n\to\infty} F_0 x_n = lim_{n\to\infty} Bx_n = lim_{n\to\infty} F_i y_n = lim_{n\to\infty} Ay_n = z.$$

Hence the pairs (F_i, A) and (F_0, B) share the common property $(E.A)$.
Thus all the conditions of Theorem 3.2 are satisfied, hence $< F_i >$ where $i \in N \cup \{0\}$, A and B have a unique common fixed point. □

Corollary 3.5. *Let A and B be self mappings of fuzzy intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$, satisfying the following conditions:*
(I) $\phi(M(Ax, Ay, kt), M(Bx, By, t), M(Ax, Bx, t), M(Ay, By, kt)) \geq 0$;
(I) $\phi(N(Ax, Ay, kt), N(Bx, By, t), N(Ax, Bx, t), N(Ay, By, kt)) \leq 1$;
(III) The pair (A, B) satisfies property $(E.A)$;
(IV) The pair A and B are R-weakly commuting;
(V) Range of B is closed subspace of X,
for all $x, y \in X$, $t > 0$, $k \in (0, 1)$, then A and B have unique common fixed point in X.

Proof. The proof can be obtained by putting $F_i = F_0 = A$ and $A = B$ in Theorem 3.2. □

Corollary 3.6. *Let A and the identity mapping I be self mappings of intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$, satisfying the following conditions:*
(I) $\phi(M(Ax, Ay, kt), M(x, y, t), M(Ax, x, t), M(Ay, y, kt)) \geq 0$;
(II) $\phi(N(Ax, Ay, kt), N(x, y, t), N(Ax, x, t), N(Ay, y, kt)) \leq 1$;
(III) The pair (A, I) satisfies property $(E.A)$,
for all $x, y \in X$, $t > 0$, $k \in (0, 1)$, then A has unique common fixed point in X.

Proof. The proof can be obtained by putting $F_i = F_0 = A$ and $A = B = I$ in Theorem 3.2. □

4. *Conclusion*

In this paper, we established some common fixed point theorems in intuitionistic fuzzy metric spaces for sequence of self mappings using an implicit relation and common properties (E. A). The obtained results are true for both compatible and non-compatible mappings. Our results generalize and improve several results of metric spaces and fuzzy metric spaces to intuitionistic fuzzy metric spaces. The paper has scope to generalize the results in modified intuitionistic fuzzy metric space and modified intuitionistic M- fuzzy metric space.

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