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3-dimensional Visualization of Particle Trajectories in a Magnetic Bottle

Chandra Prayaga
Department of Physics
University of West Florida
Pensacola, FL 32514, USA
cprayaga@uwf.edu

Abstract

This paper reports the Simulation and 3-d visualization of the magnetic field and particle trajectories in a magnetic bottle. The magnetic fields are calculated by simple numerical integration of the Biot-Savart law, and the particle trajectories are calculated using a 4th order Runge-Kutta method. The program is implemented in VPython, which enables visualization of the fields and trajectories.

Keywords—magnetic bottle; magnetic field; charged particle; trajectories; simulation; modeling; visualization

I. INTRODUCTION

A magnetic bottle is a device that uses spatial gradients of the magnetic field to confine the hot, electrically charged plasma inside a fusion reactor. The simplest design of a magnetic bottle consists of two magnetic mirrors, each a circular coil of current, configured as Helmholtz coils, with parallel currents. A charged particle between the two mirrors is confined, each mirror reflecting the particle back into the bottle region.

The magnetic field of a single circular coil, at any point on the axis of the coil, is derived and discussed in introductory physics textbooks [1]. The off-axis field cannot be expressed in closed form, but can be written in terms of elliptic integrals, as discussed by Jackson [2] in his monumental graduate level textbook. A single circular coil of current acts as a magnetic mirror, the strong field near the coil reflecting a charged particle back into the region of weaker field away from the coil [3].

The magnetic field and the trajectory of a charged particle in a magnetic bottle is calculated and discussed in several publications [4, 5]. Of importance, both in the case of the single mirror and the bottle, are the adiabatic invariants, the energy and the magnetic dipole moment of the particle, both of which remain invariant during motion.

This paper reports the simulation and 3-dimensional visualization of the motion of a charged particle, near a magnetic mirror and confined in a magnetic bottle consisting of two parallel magnetic mirrors. The calculation of the magnetic field, in both cases, is performed by simple numeric integration of the Biot-Savart law [1], thus avoiding the use of elliptic integrals, to make the concept of the off-axis field accessible to undergraduates. The equation of motion is solved numerically, using the fourth order Runge-Kutta method. The entire simulation is implemented in VPython, which is a visual add-on to Python. The simulation enables the user to visualize and manipulate, in 3-dimensions, the magnetic field and the particle trajectories.

II. MAGNETIC FIELD OF A CURRENT

The magnetic field $d\vec{B}$ of an infinitesimal segment $d\vec{s}$ of a wire carrying current I, at a position $\vec{r}$ with respect to the segment of wire, is given by the Biot-Savart law [1, 2]:

$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{s} \times \vec{r}}{r^3} \tag{1}$$

The magnetic field of any shape of a wire carrying a current is obtained, in principle, by integrating (1). In the case of a circular coil, which acts as a magnetic mirror, the integration for the field $\vec{B}$at any point on the axis of the coil is simple, and yields the result [1]:

$$\vec{B} = \frac{\mu_0 I}{2} \frac{R^2}{(z^2 + R^2)^{3/2}} \hat{z} \tag{2}$$

R is the radius of the coil and *z* is the distance from the center of the coil to the field point, anywhere along the axis of the coil. The direction of the field and the direction of the current in the coil are related by the standard right-hand rule [1]. In (2), it has been assumed that the coil is in the xy plane, resulting in the magnetic field along the z-axis. The off-axis field cannot be expressed in closed form, and is given in terms of elliptic integrals [2].

In the current work, use of elliptic integrals is avoided by numerically integrating expression (1). The circular coil is divided into small arc segments, the magnetic field of each segment at any position is calculated using (1), and the contributions from all the segments are added:

$$\vec{B} = \frac{\mu_0}{4\pi} I \sum_i \frac{d\vec{s}_i \times \vec{r}_i}{{r_i}^3} \tag{3}$$

The summation is over all the elements of the coil.

III. MOTION OF A CHARGED PARTICLE IN A MAGNETIC FIELD

The force on a charge *q* moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$is given by the Lorentz force law:

$$\vec{F} = q(\vec{v} \times \vec{B}) \tag{4}$$

Integration of the force law, twice, yields the path or trajectory of the particle. The integration is simple for a uniform magnetic field, and yields a helical or a circular path depending on the initial orientation of the magnetic field and the velocity [2]. For a non-uniform field, as is the case for a circular coil, the trajectory is obtained by using numerical procedures. In this investigation, we have used a fourth order Runge-Kutta algorithm [6]. In this method, generally known as RK4, the total time of the trajectory is divided into small segments *dt*. Starting from known initial conditions, at time $t = 0$, the velocity and position after each time segment *dt*are successively estimated.

IV. MAGNETIC MIRROR

A. The magnetic field of a circular coil

The field at several points near a circular coil of current is estimated using the numerical procedure described in II. Figure 1 shows the magnetic field arrows plotted. The cylindrical symmetry of the field is made obvious by rotating the camera view on the computer screen, a 3-dimensional viewing facility available in VPython, which was the language used for the implementation of this simulation. In the figure, the green ring represents the circular coil of current, the black arrows are the magnetic field vectors and the blue lines are the coordinate axes, with the z-axis along the axis of the coil, which is in the xy-plane.

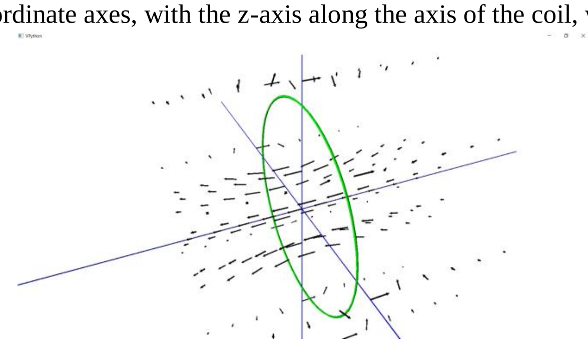

Fig. 1. Magnetic field of a circular coil.

B. Charged particle trajectory

The trajectory of a charged particle near the circular coil is also estimated using the RK4 algorithm. The program allows viewing the particle, live, in 3-dimensions as it moves. Figure 2a and 2b show two camera views of the trajectory of the

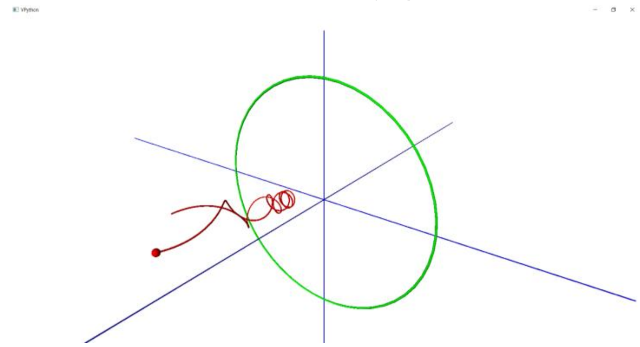

Fig. 2a Trajectory of a charged particle near a magnetic mirror

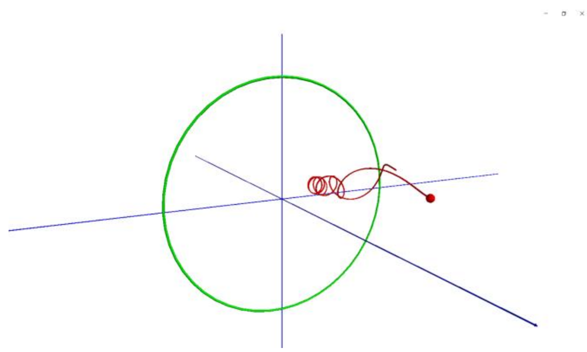

Fig 2b. Particle trajectory near a magnetic mirror, alternate view

particle. Note the reversal of the helical path when the particle approaches the coil – hence the name, magnetic mirror. In the figure, the trajectory of the particle is shown in red. Note that the radius of the helical trajectory reduces as the particle approaches the strong field region near the mirror.

V. MAGNETIC BOTTLE

A. The magnetic field

The magnetic field of a magnetic bottle made of two parallel magnetic mirrors is calculated in similar fashion. At any point in space, the total magnetic field is the vector sum of the fields due to each mirror. The field configuration is shown in figure 3. The two white rings represent the parallel circular coils of current. Here again the cylindrical symmetry is evident.

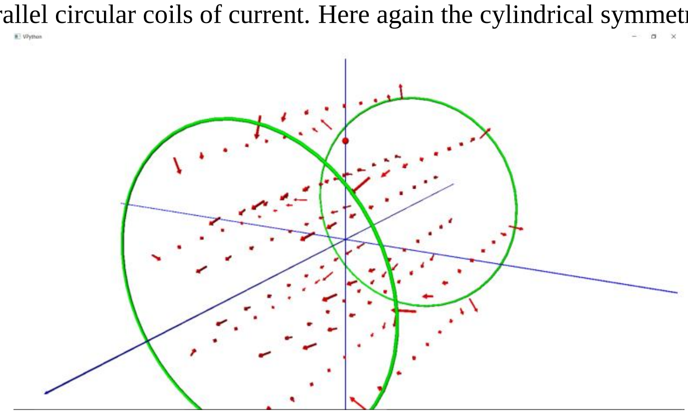

Fig 3. Magnetic field of a magnetic bottle. The red dot is the initial position of a charged particle

B. Charged particle trajectory

The trajectory of a charged particle, calculated using RK4, is shown here, in Figure 4a and 4b. It is evident that the particle is trapped between the two mirrors, as it keeps getting reflected from the strong field region near either mirror into the weak field region between the mirrors. The radius of the helix is larger in the weak field region in the central part of the bottle, and smaller in the strong field regions near the mirrors. Note the cylindrical symmetry of the trajectory.

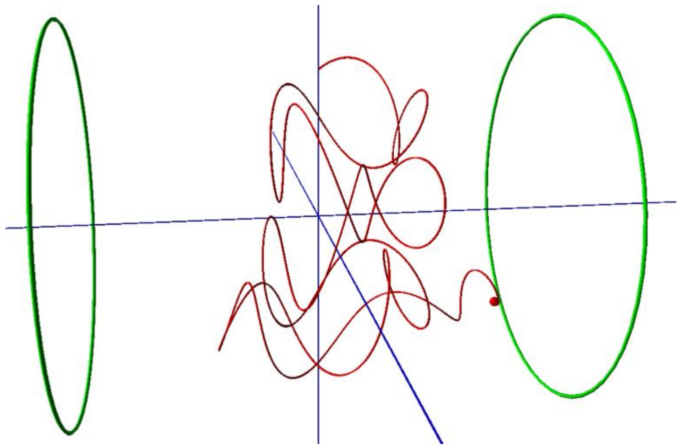

Fig. 4a Trajectory of a charged particle in a magnetic mirror

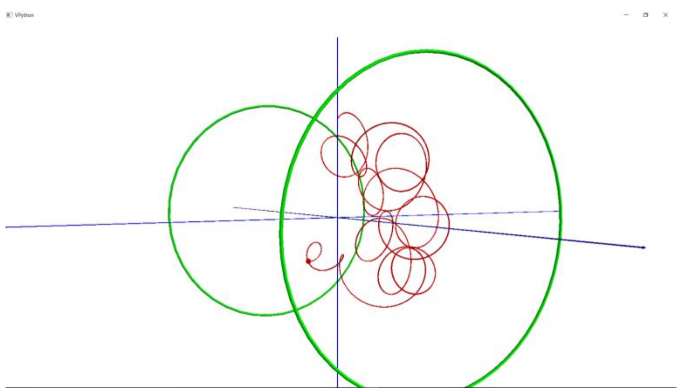

Fig. 4b Trajectory of a charged particle in a magnetic mirror, alternate view

CONCLUSIONS

A simple simulation of a magnetic mirror and magnetic bottle for plasma confinement is designed using numerical procedures for calculating magnetic fields and particle trajectories. The simulation, implemented in VPython, provides 3-d viewing of fields and trajectories. It can be used for quantitative calculation of reflection points for different velocities, and break-out velocities. Generalization to more elaborate confinement schemes, for example, with four magnetic mirrors, are possible with easy modifications. The simplicity of the numerical method also makes the program a pedagogical tool for teaching magnetism at the undergraduate level.

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