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On-Line Computing and Control for Decoupling Multivariable Processes with Gain and Phase Specifications

Tain-Sou Tsay*

Abstract: In this literature, an on-line computing and control method is proposed for multivariable processes. Systematic computing procedures and tuning rules are developed for finding pre-compensators and proportional plus integration with lead compensators. The performance of the compensated systems can be evaluated by keeping wanted bandwidth, phase margins and decoupling characteristics. No iteration for computing parameters is needed. Parameters of PI/Lead compensator can be retuning if they are necessary. Three numerical examples are provided to illustrate the proposed method giving better performance and robustness than those of other on-line computing and control methods.

Keywords: On-line computing and control, Multivariable processes

1. INTRODUCTION

The proportional-integration-derivative (PID) controllers have been used widely in industry due to robustness and simplicity. It is well-known that PID controllers have dominated applications for 60 years, though there has been a lot of interest in research into and implementation of advanced controllers. On-line computing and tuning controls are generally applicable for slow industry processes and can be retuning. Aström and Hägglund [1-3] identified the ultimate process information from a relay feedback where the process is activated by the relay. Based on the identified information, parameters of PID controllers can be tuned by many developed techniques [4-7]. The choice of studying PI controllers with Lead compensations and not PID controller in this work evolves from the fact that approximately 90% of industrial PID controllers have the derivative action turn off [8-10]. This is because of noisy measurement environment while faster system responses are wanted; i.e., very low frequency low-pass filter for noise rejection is not allowed. PI controllers usually suffer from wanted faster responses with high performance. Lead compensation can solve this problem. Therefore, lead compensators replace derivative parts of PID controllers.

In this work a pre-compensator described by first order dynamic plus time delay model (FODPT) is first estimated and applied to decouple the multivariable processes, and then parameters of PI /Lead compensator are determined by gain/phase crossover frequency ($\omega_{CRG} / \omega_{CRP}$) and phase/gain margin (*PM / GM*). Pre-compensator and PI/Lead compensator are basing on the information from identified process, and can be recomputed if they are necessary. Since the gain-crossover frequency of the system is closely related to the bandwidth of the closed-loop system, and the phase-margin is closely related to the performance. Therefore, the proposed method pays much more attentions about response time and performance than those of conventional on-line computing and control methods [1-10]. Fig.1 shows the on-line computing and control configuration of the multivariable process. P(s) is the pre-compensator, K(s) is the PI/Lead compensator, and relays, delays and real time computing block are used to force the closed-loop system to produce sustain oscillations and give information for computing $\omega_{CRG} / \omega_{CRP}$ and *PM / GM.*

* Department of Aeronautical Engineering, National Formosa University, 64, Wen Hua Road, Huwei, Yunlin 63208, Taiwan, *E-mail: tstsay@nfu.edu.tw*

This literature is organized as follows. In Section II, on-line computing and controlling algorithm for decoupling the multivariable process is developed. In Section III, the developed method is applied to three multivariable examples with comparisons of other famous design methods [5,11]. It will be seen the proposed method gives better performance, robustness, and decoupling properties than those of other mentioned on-line autotuning methods.

2. THE PROPOSED METHOD

Consider a general $m \times m$ multivariable process $G(s)$ of a feedback control system shown in Fig. 1 and given a pre-compensator $P(s)$ defined as

$$\begin{aligned} P(s) &= G(s)^{-1}\det(G(s))/\det(G(0) \\ &= adj(G(s))/\det(G(0)) \end{aligned} \tag{1}$$

Then the multivariable system $G(s)$ is decoupled ideally with $P(s)$ for

$$\begin{aligned} G(s)P(s) &= G(s)G(s)^{-1}\det(G(s))/\det(G(0) \\ &= I_m \det(G(s))/\det(G(0)) \end{aligned} \tag{2}$$

Since $G(s)$ $P(s)$ is a diagonal matrix, the multivariable design problem becomes one single-input single-output (SISO) design problem. The compensator $K(s)$ shown in Fig. 1 can be a diagonal matrix. Furthermore $P(s)$ and $G(s)P(s)$ are all stable for stable $G(s)$ is considered. This implies $P(s)$ is realizable. The transfer function matrix of the closed-loop system with $K(s)$ and $P(s)$ is

$$T(s) = \left[I_m + G(s)P(s)K(s))\right]^{-1} G(s)P(s)K(s) \tag{3}$$

The characteristic equation of the closed-loop system is

$$\Delta(s) \equiv \det\{I + G(s)P(s)K(s)\} \tag{4}$$

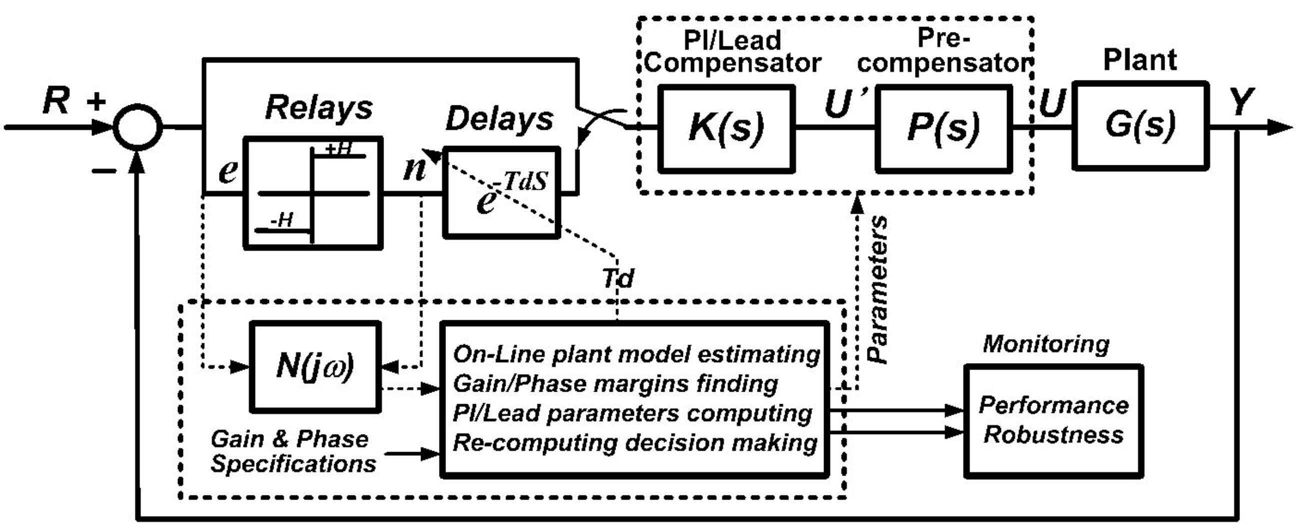


Fig. 1: The On-line Computing and Control Structure

and

$$\Delta(s)=\left[1+\frac{\det(G(s))}{\det(G(0))}\left(\frac{K_p s+K_i}{s}\right)\left(\frac{T_N s+1}{T_D s+1}\right)\right]^m \equiv [\Delta_1(s)]^m \tag{5}$$

for $K(s)\equiv I_m(K_P+K_i/s)(T_N s+1)/(T_D s+1)$. Then, one can solve $\Delta_1(s)\equiv 0$ for finding parameters K_P, K_i, T_N and T_D to give wanted performance with analyzing the root locations on polar plane. It is complicated and cannot be applied to on-line computing and control process. Gain margin (*GM*) and phase margin (*PM*: in degree) are two performance indices to describe the performance of the compensated system. They can be applied easily to on-line tuning process. The values of *PM and GM* are maximal extra phase lag p_m and maximal gain multiplication g_m in loops to destabilize the closed-loop system. The characteristic equation is modified with p_m and g_m. It is

$$\Delta_1(s)=\left[1+g_m e^{-j\pi P_m/180}\frac{det(G(s))}{det(G(0))}\left(\frac{K_p s+K_i}{s}\right)\left(\frac{T_N s+1}{T_D s+1}\right)\right] \tag{6}$$

for $g_m e^{-j\pi p_m/180}$ added to the on-diagonal loops. The phase margin is *PM* for one root of Equation (6) just crossing the imaginary axis with phase lag p_m added in loop and $g_m = 1.0$. The magnitude of the crossing point in the imaginary axis is the gain-crossover frequency ω_{CRG}. The found gain margin is *GM* for two complementary roots just crossing the imaginary axis with gain multiplication g_m added in loop and $p_m = 0°$. The magnitude of the crossing points in the imaginary axis is the phase-crossover frequency ω_{CRP}.

The pure phase lag is impossible to be implemented in time response for on-line computing and control. The time delay $e^{-T_d s}$ can be used to replace phase lag $e^{-j\pi PM/180}$ in Equation (6). The relay in loop always gives sustaining oscillation for stable process [1-3]. It gives the oscillating frequency ω_{CR} for finding frequency responses of relays and *G*(*s*). The relation between phase margin *PM* and time delay T_d is $PM\times\pi/180=T_d\omega_{CR}$ Based on this discussion, the proposed on-line computer and control structure is shown in Fig.1, and the characteristic equation of the decoupled closed loop system *G*(*s*) *P*(*s*) with relays and time delays is in the form of

$$\Delta_i(j\omega_{CR})=\left[1+N_i(j\omega_{CR})e^{-jT_d\omega_{CR}}\frac{det(G(j\omega_{CR}))}{det(G(0))}\left(\frac{jK_p\omega_{CR}+K_i}{j\omega_{CR}}\right)\left(\frac{jT_N\omega_{CR}+1}{jT_D\omega_{CR}+1}\right)\right] \tag{7}$$

where $s=j\omega_{CR}, i=1,2,...,m$ and frequency response of relays are

$$N_i(j\omega_{CR})=\int_0^T n_i(t)e^{-j2\pi/Tt}dt/(\int_0^T e_i(t)e^{-j2\pi/Tt}dt), i=1,2,..,m \tag{8}$$

where $e_i(t)$ and $n_i(t)$ are input and output of the relay *i* respectively, *T* is the oscillating period and $\omega_{CR}=2\pi/T$. Consider the closed-loop system with relays in loops only; i.e., $K(s)=I_m$. The frequency responses of the relays and *G*(*s*) *P*(*s*) shown in Fig. 1 can be written by following equations:

$$N_i(j\omega_{CR}) = \int_0^T u_i(t)e^{-j2\pi/Tt}dt/(-\int_0^T y_i(t)e^{-j2\pi/Tt}dt), i = 1,2,..,m \tag{9a}$$

and

$$\det G(j\omega_{CR})/\det G(0) = \int_0^T y_i(t)e^{-j2\pi/Tt}dt\int_0^T u_i(t)e^{-j2\pi/Tt}dt, i = 1,2,..,m \tag{9b}$$

for $R = 0I_m$, $K(s) = I_m$, $e_i(t) = -y_i(t)$ and $u_i(t) = n_i(t)$. Equations (9a) and (9b) give $N_i(j\omega_{CRP})\det(G(j\omega_{CRP}))/\det(G(0)) = -1$ for sustain oscillation. The oscillating frequency is the phase crossover frequency ω_{CRP} (i.e., cross -180°). This is the oscillating condition of the closed- loop system. The characteristic equation of the closed-loop system is in the form of

$$\Delta_i(j\omega_{CRP}) = \left[1 + N_i(j\omega_{CRP})\frac{det(G(j\omega_{CRP}))}{det(G(0))}\left(\frac{jK_p\omega_{CRP} + K_i}{j\omega_{CRP}}\right)\left(\frac{jT_N\omega_{CRP} + 1}{jT_D\omega_{CRP} + 1}\right)\right] \tag{10}$$

The gain margin of the system can be evaluated as $GM \equiv |N_i(j\omega_{CRP})|$ or $GM \equiv |\det(G(0)/\det(G(j\omega_{CRP}))|$. Eliminating the term $N_i(j\omega_{CRP})$ in Equation (10), the system gives gain margin $GM \equiv |N_i(j\omega_{CRP})|$. The closed-loop system is stable for $GM > 1$ and unstable for $GM \leq 1$. For example, one must reduce the loop gain (e.g., $K_p < 1$) to get stable system for $GM \leq 1$. Equation (7) gives the gain margin $GM \equiv |N(j\omega_{CR})|$ will be degraded for extra phase lag is added. The corresponding extra phase lag for $GM \equiv 1$ gives the phase margin PM. The oscillating frequency at $GM \equiv 1$ is called the gain crossover frequency ω_{CRG}. Now, the characteristic equation becomes

$$\Delta_i(j\omega_{CRG}) = \left[1 + e^{-j\pi PM/180}\frac{det(G(j\omega_{CRG}))}{det(G(0))}\left(\frac{jK_p\omega_{CRG} + K_i}{j\omega_{CRG}}\right)\left(\frac{jT_N\omega_{CRG} + 1}{jT_D\omega_{CRG} + 1}\right)\right] \tag{11}$$

Therefore, gain/phase margins and phase/gain crossover frequencies can be found by introducing relays and time delays, and they can be used for on-line computing and control process. Taking the real and imaginary parts of Equations (10) and (11), four stability equations [17] are ready for finding four parameters K_p, K_i, T_N and T_D mathematically. The solution of them may be complex number. Complex number represents compensator cannot be realized. In this work, two tracking laws in real-time manner for finding parameters are developed and further simplification is made to reduce four parameters K_p, K_i, T_N and T_D to be two parameters (K_p and K_i) and avoid not real solutions of them.

Note that the plant model $G(s)$ is generally not known exactly and may vary from producing environment changed (e.g. temperature). Therefore, real-time computing process to find estimated model $\hat{G}(s)$ for computing and control is usually required. Once the estimated model $\hat{G}(s)$ found, the method stated above for finding pre-compensator $P(s)$ and PI/Lead compensator can be applied. The computing and control configuration shown in Fig. 1 including relays and delays. It will result in sustaining oscillation for stable $G(s)$[8]. Then, the Laplacetransformation of $G(s)$ can be derived from computing Fourier integrations of input/output signals of relays [11-15]. The estimating process is given in the next paragraph.

Consider a $m \times m$ multivariable process in frequency domain as

$$\begin{bmatrix} y_1(j\omega) \\ \vdots \\ y_m(j\omega) \end{bmatrix} = \begin{bmatrix} g_{11}(j\omega) & \cdots & g_{1m}(j\omega) \\ \vdots & \ddots & \vdots \\ g_{m1}(j\omega) & \cdots & g_{mm}(j\omega) \end{bmatrix} \begin{bmatrix} u_1(j\omega) \\ \vdots \\ u_m(j\omega) \end{bmatrix} \tag{12}$$

The $(i, j)th$ elements of the process are described by

$$g_{ij}(j\omega) = y_i(j\omega) / u_j(j\omega)\Big|_{u_k=0,k\neq j} \tag{13}$$

The above equation represents it needs m individual procedures for identifying whole elements of a $m \times m$ multivariable process. Wang [11] proposed a decentralized relay- feedback identification method. Although all relays are used simultaneously, but it still needs m procedures with perturbing the amplitude of relays (H). If the oscillations in m loops have common frequency ω_c, then the direct-current components and the first harmonics of these periodic waves are extracted as

$$\hat{g}_{ij}(0) = \left.\frac{\int_0^T y_i(t)dt}{\int_0^T u_j(t)dt}\right|_{u_k=0,k\neq j} \tag{14}$$

and

$$\hat{g}_{ij}(j\omega_c) = \left.\frac{\int_0^T y_i(t)e^{-j\omega_c t}dt}{\int_0^T u_j(t)e^{-j\omega_c t}dt}\right|_{u_k=0,k\neq j} \tag{15}$$

$\hat{G}(0)$ and $\hat{G}(j\omega_c)$ can be found by Equations (14) and (15). For industry process, first order dynamic models with time delays (FODPT) are usually used for modeling and control [8-13]. The (i, j)th elements of he estimated model $\hat{G}(s)$ can be defined as

$$\hat{g}_{ij}(s) \equiv \hat{g}_{ij}(0)e^{-L_{ij}s} / T_{ij}s + 1, \tag{16}$$

and values of L_{ij} and T_{ij} are evaluated from $\hat{g}_{ij}(j\omega_c)$. The pre-compensator can be found by $\hat{P}(s) \equiv adj(\hat{G}(s)) / \det(\hat{G}(0))$. Further on-line computing processes are developed to find parameters of PI/ Lead compensator.

The completely on-line procedure for finding parameters of the PI/Lead compensator is partition into five steps and discussed in detail. They are given below:

Step 1: Find Gain Margin *GM*(0) and Phase Crossover Frequency ω_{CRPO}

The compensator *K(s)* is first set be identity matrix and introducing the relays in loop only. Finding the oscillating period $T(0)$, $\omega_{CRP0} = 2\pi / T(0)$ and calculating the equivalent gain $N_i(j\omega_{CRP0})$ of the relay described by Equation (8). The gain margin found is defined as $GM(0) \equiv \sum_{i=1}^{m} \left| N_i(j\omega_{CRP0}) \right| / m$. Averages of $\left| N_i(j\omega_{CRP}) \right|, i = 1,2,..,m$ is used in computing and control processes for responses of the plants have been balanced by the pre-compensators. Note that all parameters of P/PI/PID compensators by Ziegler-Nichols rules [15, 16] are given in this step. They need not sufficient process information but may give poor performance and robustness. They are

ZN(P) : $K_p = GM(0)/2.0$;

ZN(PI) : $K_p = GM(0)/2.2$ and $K_i = 1.2/T(0)$;

ZN(PID) : $K_p = GM(0)*0.60, K_i = 2.0/T(0), K_D = T(0)/8$.

They are developed from large number of simulations of different processes. Further tuning step gives below will give a robust result for PI/Lead compensators with gain and phase specifications.

Step 2: Find the Proportional Gain K_p for Wanted Gain Margin is *GM*(1)

Using the gain margin found in step 1, the proportional gain $K_P \equiv GM(0)/GM(1)$. The gain margin will become to be *GM*(1). The compensator in this step is $K(s) = K_P I_m$.

Step 3: Find the Integration Gain K_i for Wanted Gain Margin is *GM*(2) with K_p

The selected value of *GM*(2) is always less than that of *GM*(1) for integration K_i / s in loop will destabilize the system. The tracking law for finding K_i is given below:

$$G_n(kT+T) = G_n(kT) \times [\,\alpha\, GM / GM(2) + (1-\alpha)\,]; \tag{17a}$$

$$K_i = G_n(kT+T); \tag{17b}$$

$$GM = \sum_{i=1}^{m} \left| \int_0^T u_i(t) e^{-j2\pi/T_u t} dt \Big/ \int_0^T y_i(t) e^{-j2\pi/Tt} dt \right| / m \tag{17c}$$

where T is the oscillating period, $G_n(kT)$ is a time series and α is the adjustable parameter for tracking speed. The initial value $G_n(0)$ is set to be $K_p/10.0$ and value of α is greater than zero and less than one. Equation (17a) gives $G_n(kT+T) = G_n(kT)$ after $GM \equiv \sum_{i=1}^{m} \left| N_i(j\omega_{CRP}) \right| / m = GM(2)$ and $\omega_{CRP} = 2\pi / T$. The tracking law is a negative feedback loop for small *GM*/*GM*(2) ratio gives small value of K_i introduced into loop and large gain margin *GM* will be and large *GM*/*GM*(2) ratio gives large value of K_i introduced into loop and small gain margin *GM* will be. Therefore, Equation (17a) will give a steady-state value $G_n(kT)$ (i.e., K_i) after $GM = GM(2)$.

Note the gain margin is the absolute value of $N_i(j\omega_{CRP})$ described by Equation (8). The found compensator is $K(s) = (K_P + K_i / s)I_m$ in this step.

Step 4: Finding T_N and T_D from found phase margin *PM*(2) and gain crossover frequency ω_{CRG} with $(K_p + K_i / s)I_m$

The time lag T_d in loop is used for finding phase margin *PM*(2). The relationship of *PM*(2) and T_d is $PM(2) = 180T_d\omega_{CRG} / \pi$. The tracking law for finding corresponding maximal time lag T_d for *PM*(2) is

$$P_n(kT + T) = P_n(kT) \times [\beta\, GM / 1.0 + (1 - \beta)]; \tag{18a}$$

$$T_d = P_n(kT + T); \tag{18b}$$

$$PM(2) = 360T_d / T; \tag{18c}$$

$$GM = \sum_{i=1}^{m} \left| \int_0^T u_i(t)e^{-j2\pi/T_u t}dt \Big/ \int_0^T y_i(t)e^{-j2\pi/Tt}dt \right| / m \tag{18d}$$

where T is the oscillating period, $P_n(kT)$ is a time series and β is a parameter to adjust the tracking speed. The value of β is greater than zero and less than one. Equation (18a) gives $P_n(kT + T) = P_n(kT)$ after the wanted time lag $T_d \equiv T \times PM / 360$ is found; i.e., $GM = 1$. Similar to the tracking law for finding the integration gain K_i, the tracking law is a negative feedback loop also for small *GM*/1.0 ratio gives small value of T_d introduced into loop and large gain margin *GM* will be and large *GM*/1.0 ratio gives large value of T_d introduced into loop and small gain margin *GM* will be. Therefore, the tracking law will give a steady- state value of T_d after *GM* converged to be one and gives gain crossover frequency $\omega_{CRG} = 2\pi / T$.

Consider phase and gain of a lead compensator $Lead(s) = (T_N s + 1)/(T_D s + 1)$ with respect to the ratio of $(1/T_N)/\omega_{CRG}$ and $T_D = T_N / 100$. $1/T_N$ is the corner frequency of the lead compensator. It gets maximal leading phase 26.28° and gain 1.118 (0.97dB) for $T_N = 1/2\omega_{CRG}$, and leading phase 13.89° and gain 1.036(0.264dB) for $T_N = 1/4\omega_{CRG}$. In this work, lead compensation with corner frequency equal to or greater than two will be added if the found *PM*(2) is not satisfactory. The overall compensator becomes $K(s) = [(K_P + K_i / s)(T_N s + 1)/(T_D s + 1)]I_m$. Note that found gain/phase margins will be greater than those of *GM*(2) and *PM*(2).

Step 5: Step Response Verifications for found PI/Lead Compensators

The tracking laws given by Equations (17) and (18) are used for finding the gain/phase margins (*GM*(3)/ *PM*(3) and phase/gain crossover frequencies of the controlled system with found parameters: K_p, K_i, T_N and T_D. If the wanted *GM*(3) and *PM*(3) are not satisfactory, then go back to Step 1 to restart a new computing process. If it is satisfactory, then switches back to normal operation process. The flow chart of the overall on-line computing and control algorithm is illustrated by Fig. 2 and applied to three numerical examples. The first example is described by first order dynamic plus time delay model (FODPT). The second example is described by high-order dynamic model. The third example is a system with large different response times between channels. Precompensators will be all realized by FODPT models.

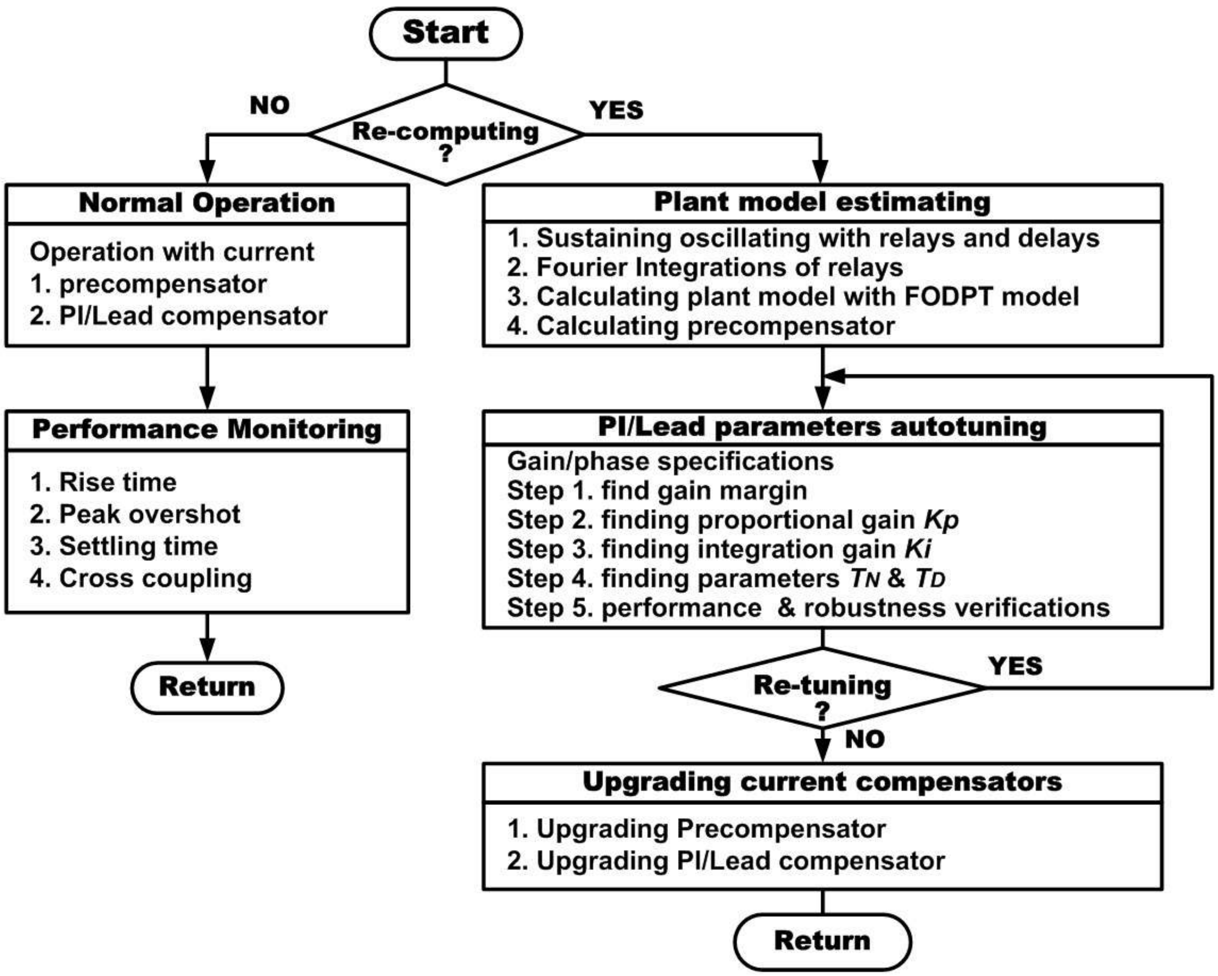


Fig. 2: The Flow-chart of the On-line Computing and Control

3. NUMERICAL EXAMPLES

Example 1: Consider the Wood and Berry binary distillation column plant [18]:

$$G(s) = \begin{bmatrix} \dfrac{12.8}{16.7s+1}e^{-s} & \dfrac{-18.9}{21s+1}e^{-3s} \\ \dfrac{6.6}{10.9s+1}e^{-7s} & \dfrac{-19.4}{14.4s+1}e^{-3s} \end{bmatrix} \tag{19}$$

Estimating processes with relays (H = 0.5) are shown in Fig. 3. The sustain oscillation frequency $\omega_{c1} = 0.439 rad/s$ for loop 1 closed with relay and time lag is 3.0seconds. The time delay is used to slow down the oscillating frequency [1-3] for reducing computing efforts of Equations (14) and (15). Using Equations (14) and (15), the found $\hat{G}_{j1}(0)$, and $\hat{G}_{j1}(j\omega_{c1})$ are

$$\hat{G}_{j1}(0) = \begin{bmatrix} 12.8 \\ 6.6 \end{bmatrix} \tag{20a}$$

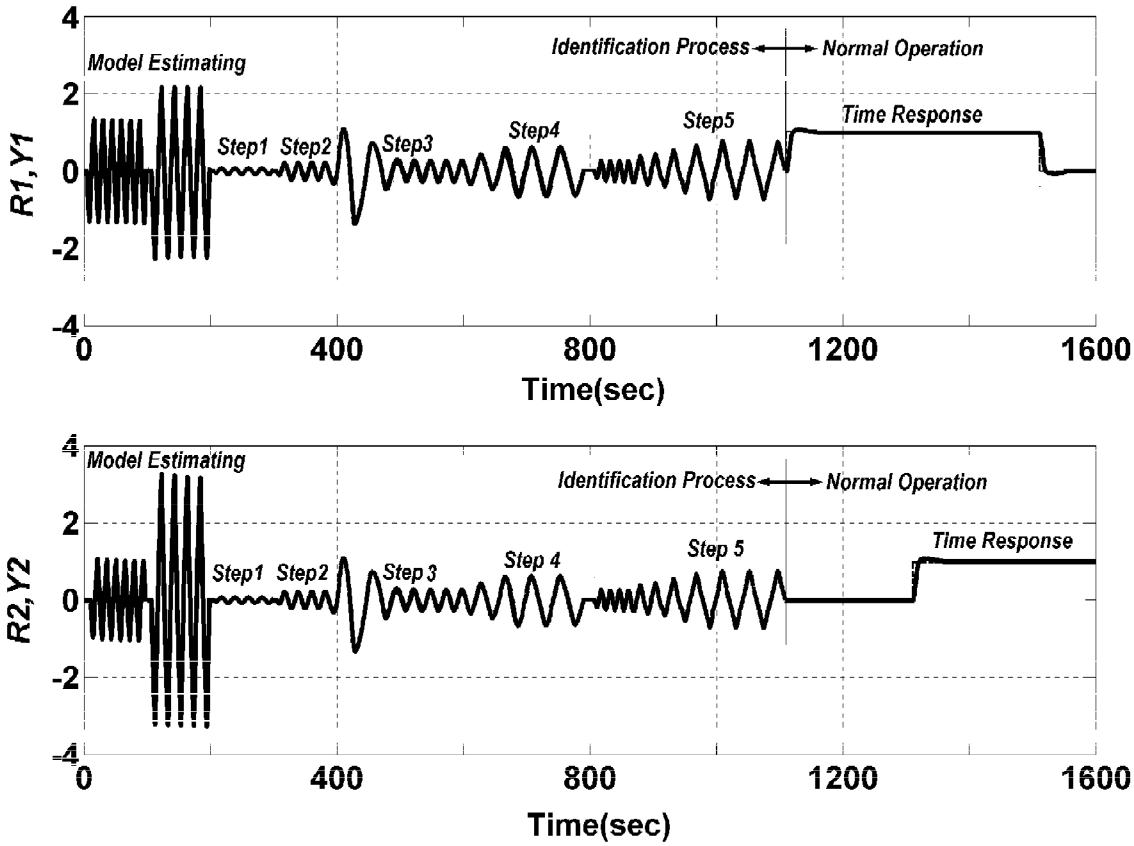


Fig. 3: Outputs of On-line Computing Process of Example 1

and

$$e^{-1.2987j}\hat{G}(0.4329j) = \begin{bmatrix} e^{-0.02837j} / 0.570 \\ e^{3.730j} / 0.7105 \end{bmatrix} \tag{20b}$$

The sustain oscillation frequency $\omega_{c2}= 0.3066 rad/s$ for loop 2 closed with relay and time lag 3.0s. Using Equations (14) and (15), the found $\hat{G}_{j2}(0)$, and $\hat{G}_{j2}(j\omega_{c2})$ are

$$\hat{G}_{j2}(0) = \begin{bmatrix} -18.9 \\ -19.4 \end{bmatrix} \tag{21a}$$

and

$$e^{-0.9199j}\hat{G}(0.3066j) = \begin{bmatrix} e^{-0.1201j} / 0.3450 \\ e^{-0.05147j} / 0.2335 \end{bmatrix} \tag{21b}$$

Using FODPT dynamic models given in Equation (16), we have

$$\hat{G}(s)=\begin{bmatrix}\frac{12.8}{16.695s+1}e^{-1.012s} & \frac{-18.9}{21.014s+1}e^{-3.017s}\\ \frac{6.6}{10.895+1}e^{-7.009s} & \frac{-19.4}{14.410s+1}e^{-3.017s}\end{bmatrix} \tag{22}$$

and

$$\hat{P}(s)=\frac{-1}{123.58}\begin{bmatrix}\frac{-19.4}{14.410s+1}e^{-2.005s} & \frac{+18.9}{21.014s+1}e^{-2.005s}\\ \frac{-6.6}{10.895s+1}e^{-5.997s} & \frac{12.8}{16.695s+1}\end{bmatrix} \tag{23}$$

The on-line computing for finding parameters of PI/Lead compensator is given below:

Step 1: Oscillating period $T(0)=21.41s$ (i.e., $\omega_{CRP0}=0.293rad/s$) and gain margin $GM(0)=9.04$ are found with relays added.

Step 2: Proportional gain K_p = 3.013 is found for the wanted gain margin GM(1) = 3.0 (9.54dBs).

Step 3: Integration gain K_i = 0.172 and phase crossover frequency ω_{CRP} = 0.21*rad*/*s* are found for the wanted the gain margin *GM*(2) = 2.5(~8*dBs*).

Step 4: Phase margin $PM(2)=27.99°$ *and* $\omega_{CRG}=0.123rad/s$ are found for $(K_p,K_i)=(3.013,0.172)$ and parameters of the lead compensator $T_N=1/2\omega_{CRG}=4.08$ *and* $T_D=0.00408$ are evaluated.

Step 5: Final results are $PM(3)=60.1°$, $\omega_{CRG}=0.130rad/s$, $GM(3)=2.65(8.46dBs)$ and $\omega_{CRP}=0.439rad/s$.

Final results and three other methods are presented for comparisons and showing the merit of the proposed method. They are Ziegler-Nichols method [4] for finding PI and PID compensators, and Wang's method [11] for finding cross multivariable PID compensator. Parameters of four found compensators are given below:

Proposed : $K_p=3.0132,\ K_i=0.172, T_N=4.08$ *and* $T_D=0.0408$;

ZN(PI) : $K_p=GM(0)/2.2=4.109$ *and* $K_i=1.2/T(0)=0.056$

ZN(PID) : $K_p=GM(0)*0.60=5.424, K_i=2.0/T(0)=0.034$ *and* $K_D=T(0)/8=2.676$

Wang's :

$$K(s)=\begin{bmatrix}0.184(1+1/3.92s) & -0.0102(1+1/0.445s-0.804s)\\ -0.0674(1-1/4.23s+0.796s) & -0.0660(1+1/4.25s)\end{bmatrix}$$

The ZN(PI) and ZN(PID) compensators are found after pre-compensator $\hat{P}(s)$ has been added to the system. Wang's compensator is applied to the system without $\hat{P}(s)$. The time responses of them are shown in Fig. 4. It shows the proposed method gives better performance and decoupling results than those of other three methods. Wang's method gives larger cross couplings than that of the proposed method. This is due to the phase crossover frequency was used for evaluating the cross compensator. Gain crossover frequency for decoupling is usually preferred.

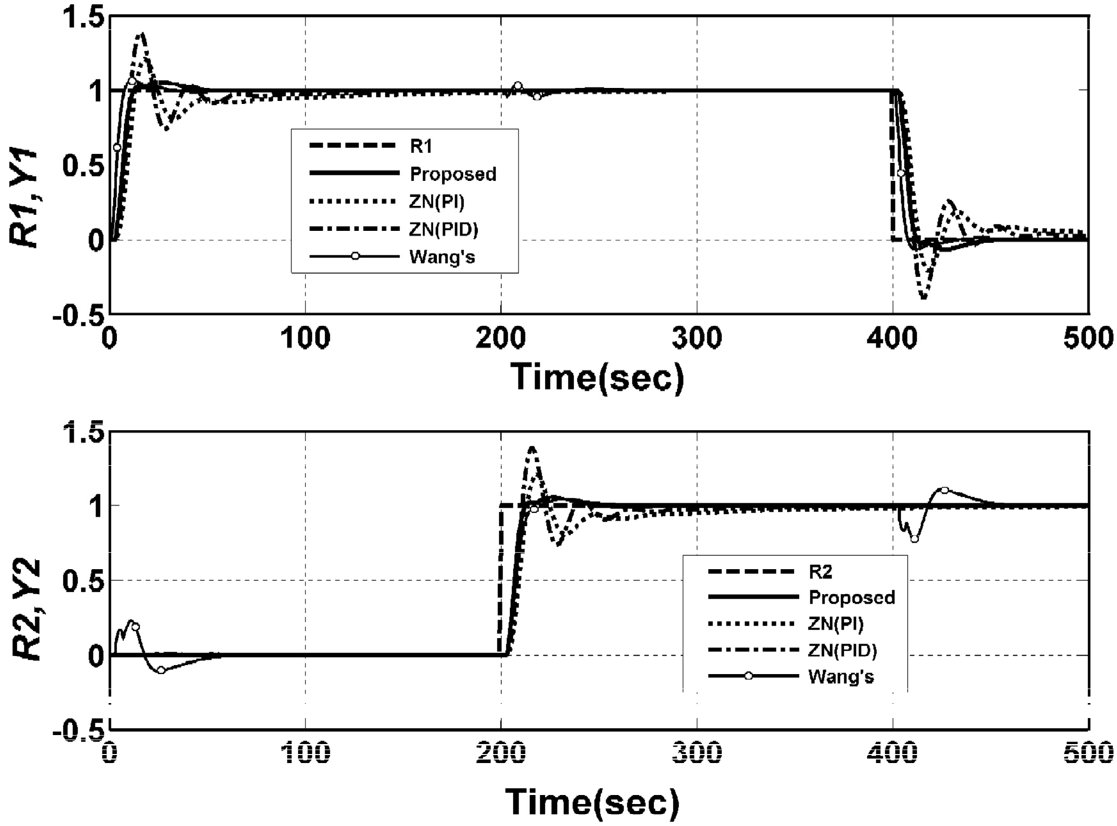


Fig. 4: Time Responses of Example 1

Example 2. Consider the multivariable process given by Palmor *et al.* [**19**] as

$$G(s)=\begin{bmatrix} \dfrac{0.5}{(0.1s+1)^2(0.2s+1)^2} & \dfrac{-1}{(0.1s+1)(0.2s+1)^2} \\ \dfrac{1}{(0.1s+1)(0.2s+1)^2} & \dfrac{2.4}{(0.1s+1)(0.2s+1)^2(0.5s+1)} \end{bmatrix} \tag{24}$$

The elements of plant $G(s)$ are described by four high-order dynamic models. Estimating processes with relays (H = 0.5) are performed. The oscillation frequency $\omega_{c1}= 0.8878 rad/s$ for loop 1 closed with relay (H = 0.5) and time lag 3.0s. Using Equations (14) and (15), the found $\hat{G}_{j1}(0)$, and $\hat{G}_{j1}(j\omega_{c1})$ are

$$\hat{G}_{j1}(0)=\begin{bmatrix} 0.5 \\ 1.0 \end{bmatrix} \tag{25a}$$

and

$$e^{-2.6634j}\hat{G}(0.8878j) = \begin{bmatrix} e^{-0.04617j}/2.079 \\ e^{0.03939j}/1.0360 \end{bmatrix} \tag{25b}$$

The oscillation frequency ω_{c2}= $0.8124rad/s$ for loop 2 closed with relay and time lag 3.0s. The found $\hat{G}_{j2}(0)$, and $\hat{G}_{j2}(j\omega_{c2})$ are

$$\hat{G}_{j2}(0) = \begin{bmatrix} -1.0 \\ 2.4 \end{bmatrix} \tag{26a}$$

and

$$e^{-2.4372j}\hat{G}(0.8124j) = \begin{bmatrix} e^{-2.838j}/1.030 \\ e^{-0.07952j}/0.463 \end{bmatrix} \tag{26b}$$

The estimated model of the plant $\hat{G}(s)$ is

$$\hat{G}(s) = \begin{bmatrix} \dfrac{0.5}{0.32s+1}e^{-0.279s} & \dfrac{-1.0}{0.304s+1}e^{-0.196s} \\ \dfrac{1.0}{0.305s+1}e^{-0.196s} & \dfrac{2.4}{0.597s+1}e^{-0.409s} \end{bmatrix} \tag{27}$$

The pre-compensator $\hat{P}(s)$ is

$$\hat{P}(s) = \frac{1}{2.2}\begin{bmatrix} \dfrac{2.4}{0.597s+1}e^{-0.409s} & \dfrac{+1.0}{0.304s+1}e^{-0.196s} \\ \dfrac{-1.0}{0.305s+1}e^{-0.196s} & \dfrac{0.5}{0.32s+1}e^{-0.279s} \end{bmatrix} \tag{28}$$

The results of on-line computing procedures for finding parameters of the PI/Lead compensator are given below:

Step 1: Oscillating period $T(0) = 1.6197s$ (i.e., $\omega_{CRP0} = 3.879rad/s$) and gain margin $GM(0) = 4.25$ are found with relays added.

Step 2: Proportional gain $K_p = 4.251/3.0 = 1.417$ is found for the wanted gain margin $GM(1) = 3.0(9.54dBs)$.

Step 3: Integral gain $K_i = 1.7808$ and phase crossover frequency ω_{CRP} = $3.174rad/s$ are found for the wanted gain margin $GM(2) = 2.5(\sim 8dBs)$.

Step 4: Phase margin $PM(2) = 45.9°$ *and* $\omega_{CRG} = 1.727rad/s$ are found for $(K_p, K_i) = (1.417, 1.7808)$ and parameters of the lead compensator $T_N = 1/1.414\omega_{CRG} = 0.4095$ *and* $T_D = 0.004095$ are evaluated.

Step 5: Final results are $PM(3) = 63.8°, \omega_{CRG} = 1.965 rad/s$, $GM(3) = 3.82(11.6dBs)$ and $\omega_{CRP} = 5.51 rad/s.$

Final results and three other methods are presented for comparisons and showing the merit of the proposed method. They are Ziegler-Nichols method [4] for finding PI and PID compensators, and Wang's method [11] for finding cross PID compensator. Parameters of four found compensators are given below:

Proposed : $K_p = 1.417, K_i = 1.7808, T_N = 0.4095$ *and* $T_D = 0.004095$

ZN(PI) : $K_p = 1.932$ *and* $K_i = 0.7409$

ZN(PID) : $K_p = 2.551, K_i = 1.235$ *and* $K_D = 0.203$

Wang's:

$$K(s) = \begin{bmatrix} 2.83(1+1/0.285s) & 1.51(1+1/0.865s+0.0911s) \\ -3.25(1+1/0.785s+0.182s) & 0.667(1+1/0.776s) \end{bmatrix}$$

The time responses of them are shown in Fig.5. It shows the proposed method gives better performance and decoupling results than those of other three on-line computing and control methods.

Example 3: Consider the solid-fuel boiler plant given by Johansson [11]

$$G(s) = \begin{bmatrix} \frac{-1}{10.0s+1}e^{-2s} & \frac{-1}{10s+1} \\ 0 & \frac{1}{60s+1}e^{-10s} \end{bmatrix} \tag{29}$$

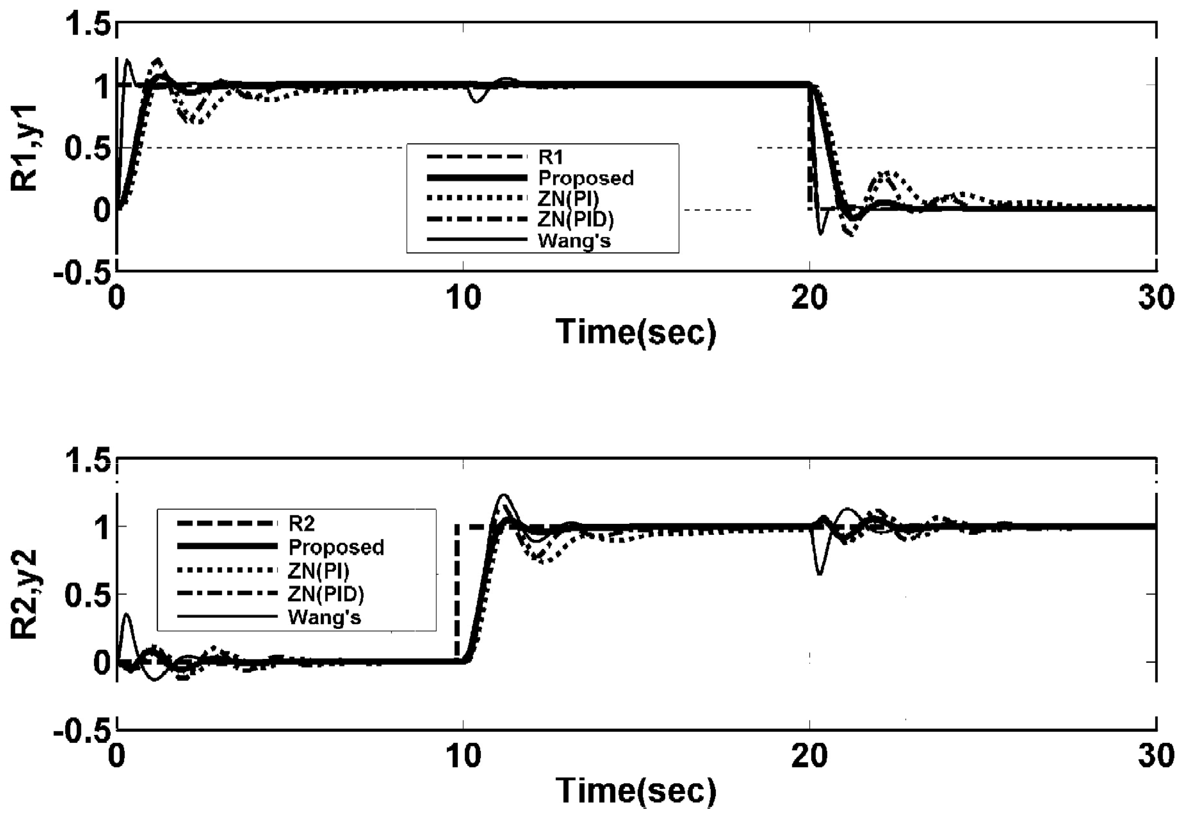


Fig. 5: Time Responses of Example 2

Estimating processes with relays (H = 0.5) are performed. The oscillation frequency $\omega_{c1} = 0.377 rad/s$ for loop 1 closed with relay and time lag 3.0s. Using Equations (14) and (15), the found $\hat{G}_{j1}(0)$ and $\hat{G}_{j1}(j\omega_{c1})$ are

$$\hat{G}_{j1}(0) = \begin{bmatrix} -1.0 \\ 0.0 \end{bmatrix} \tag{30a}$$

and

$$e^{-1.131j}\hat{G}(0.377j) = \begin{bmatrix} e^{-0.05819j}/3.887 \\ 0 \end{bmatrix} \tag{30b}$$

The oscillation frequency $\omega_{c2} = 0.133 rad/s$ for loop 2 closed with relay and time lag 3.0s. The found $\hat{G}_{j2}(0)$ and $\hat{G}_{j2}(j\omega_{c2})$ are

$$\hat{G}_{j2}(0) = \begin{bmatrix} -1.0 \\ +1.0 \end{bmatrix} \tag{31a}$$

and

$$e^{-0.3978j}\hat{G}(0.1326j) = \begin{bmatrix} e^{-1.324j}/1.660 \\ e^{-0.02913j}/8.016 \end{bmatrix} \tag{31b}$$

The found $\hat{G}(s)$ and $\hat{P}(s)$ are

$$\hat{G}(s) = \begin{bmatrix} \dfrac{-1.0}{9.963s+1}e^{-2.011s} & \dfrac{-1.0}{9.992s+1} \\ 0 & \dfrac{1}{59.984s+1}e^{-10.009s} \end{bmatrix} \tag{32}$$

and

$$\hat{P}(s) = \begin{bmatrix} \dfrac{1}{59.984s+1}e^{-10.009s} & \dfrac{+1.0}{9.992s+1} \\ 0 & \dfrac{-1.0}{9.963s+1}e^{-2.011s} \end{bmatrix} \tag{33}$$

The results of on-line computing procedures for finding parameters of PI/Lead compensator are given below:

Step 1: Oscillating period $T(0) = 72.99s$ (i.e., $\omega_{CRP0} = 0.0861 rad/s$) and gain margin $GM(0) = 6.94$ are found with relays added.

Step 2: Proportional gain $K_p = 2.314$ is found for the wanted gain margin $GM(1) = 3.0 (9.54 dBs)$.

Step 3: Integral gain $K_i = 0.0615$ and phase crossover frequency $\omega_{CRP} = 0.0666 rad/s$ are found for the wanted gain margin $GM(2) = 2.5 (\sim 8 dBs)$.

Step 4: Phase margin $PM(2) = 26.76°$ *and* $\omega_{CRG} = 0.0408 rad/s$ are found for $(K_p, K_i) = (2.314, 0.0615)$ and parameters of the lead compensator $T_N = 1/1.414\omega_{CRG} = 17.33$ *and* $T_D = 0.1733$ are evaluated.

Step 5: Final results are $PM(3) = 59.3°, \omega_{CRG} = 0.049 rad/s$, $GM(3) = 2.505(7.988 dBs)$ and $\omega_{CRP} = 0.143 rad/s$.

Final results and three other methods are presented for comparisons and show the merit of the proposed method. They are Ziegler-Nichols method [4] for finding PI and PID compensators, and Wang's method [11] for finding cross PID compensator. Parameters of four found compensators are given below:

Proposed : $K_P \equiv 2.314, K_i = 0.0615$, $T_N = 17.33$ *and* $T_D = 0.1733$;

ZN(PI) : $K_P \equiv 3.155, K_i = 0.0163$;

ZN(PID) : $K_P \equiv 4.165, K_i = 0.0274$ *and* $K_D = 0.00685$;

Wang's :

$$K(s) = \begin{bmatrix} 2.61(1 + 1/10s) & 3.08(1 + 1/58.8s + 2.04s) \\ 0 & 3.14(1 + 1/60s) \end{bmatrix}$$

Simulation results are shown in Fig. 6. It shows the proposed method give better results than those of other methods. From Figs. 4 and 6, one can see that the ZN(PI) and ZN(PID) gave bad performance and robustness, Wang's method gave bad results for decoupling characteristics, and the proposed method gave better performance and decoupling characteristics simultaneously.

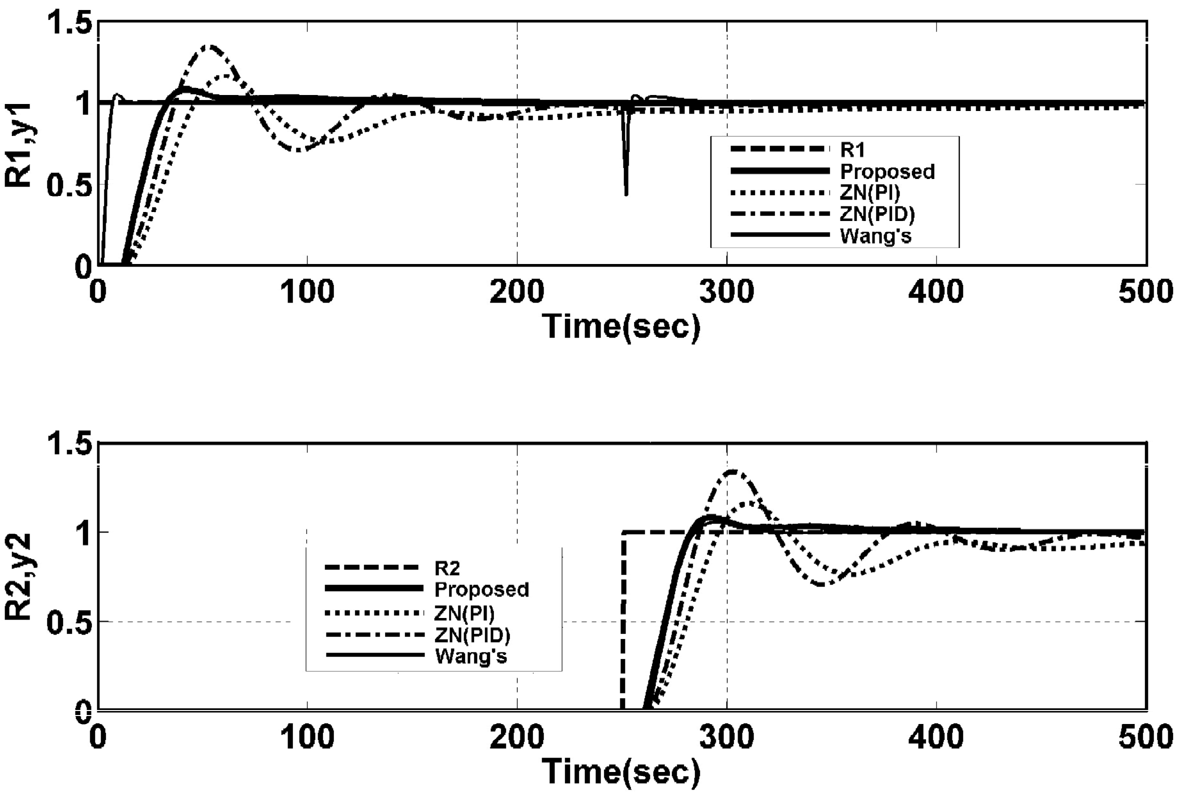
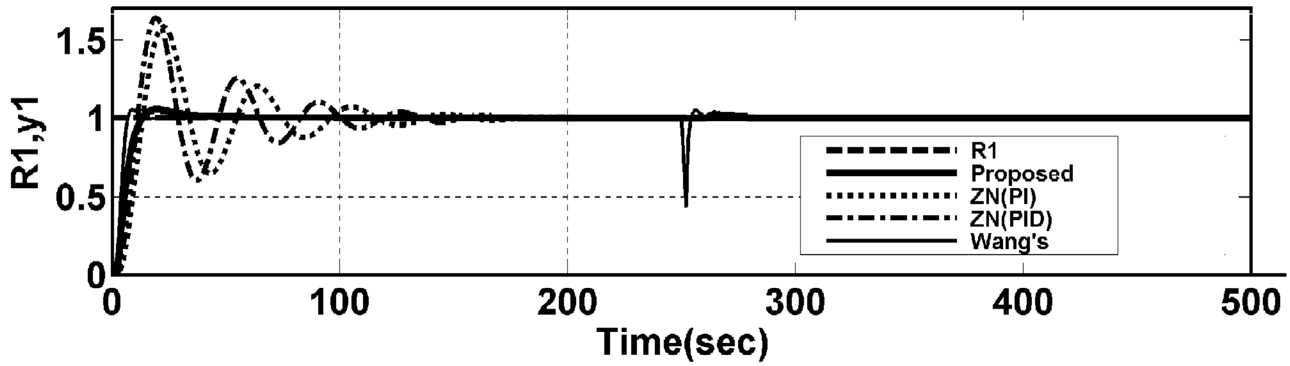


Fig. 6: Time Responses of Example 3



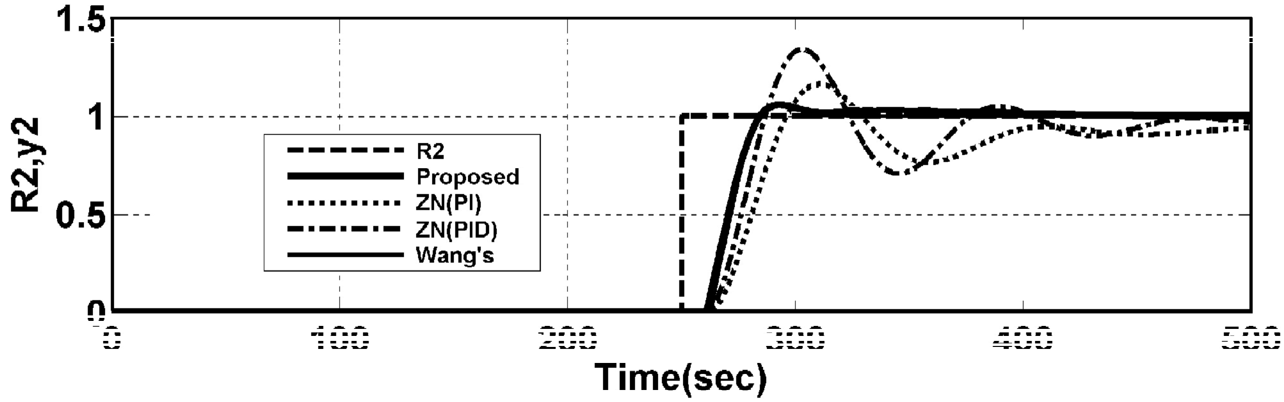


Fig. 7: Time Responses of Example 3 with two On-line Processes

4. DISCUSSIONS

The pre-compensator described by Equation(1) decouples and balances the response times of each channels. Figs. 4, 5 and 6 show the conclusions. Example 3 gives response of channel 1 is much faster than that of channel 2 of the uncompensated system. This is due to pre-compensator described by Equation (33) balances response times of two channels; i.e., introducing time lag to channel 1 to slow down time response. One can eliminate common delays (or dynamics) of each column of $\hat{P}(s)$, then the characteristics of the compensated system becomes

$$\Delta(s) = \prod_{k=1}^{m} \left\{ 1 + [G(s)\hat{P}(s)]_{kk} \left(\frac{K_{p,k}s + K_{i,k}}{s} \right) \left(\frac{T_{N,k}s + 1}{T_{D,k}s + 1} \right) \right\} \tag{34}$$

That is, it becomes m SISO system design problems. The design steps given in Section II can be applied. The pre-compensator $\hat{P}(s)$ of Example 3 can be reformulated as

$$\hat{P}(s) = \begin{bmatrix} \dfrac{1}{59.984s+1} & \dfrac{+1.0}{9.992s+1} \\ 0 & \dfrac{-1.0}{9.963s+1} e^{-2.011s} \end{bmatrix} \tag{35}$$

The identification process for channels 1 and 2 give

$$K_{P,1} \equiv 2.167, K_{i,1} = 0.0420,\ T_{N,1} = 8.38 \ \textit{and} \ T_{D,1} = 0.0838$$

and

$$K_{P,2} \equiv 2.320, K_{i,1} = 0.0572,\ T_{N,2} = 16.8 \ \textit{and} \ T_{D,2} = 0.168$$

The simulation results are shown in Fig. 7. ZN(PI), ZN(PID) and Wang's method are presented for comparison also. Fig.7 shows Equation (35) speed up response time of channel while keeping decoupling characteristics, ZN(PI) and ZN(PID) give bad performance, and Wang's method give bad decoupling characteristics.

5. CONCLUSIONS

In this literature, a systematic on-line computing and control algorithm has been developed for analyses and designs of multivariable processes with multiple time delays. Identifications were performed by introducing relays and delays in loops. The plant dynamics was identified for finding pre-compensators, and parameters of PI/Lead compensators are tuned with wanted gain/phase margins. The compensated systems gave wanted gain/phase margins and better decoupling characteristics than those of other on-line computing and control methods. Three numerical examples have illustrated effectives of the proposed method.

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