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Existence and Stability of Positive Periodic Solutions to a Periodic Integro-differential Competition System with Infinite Delays

Fengde Chen[a], Yuming Chen[b,*], Shangjiang Guo[c]

[a] *College of Mathematics and Computer Science, Fuzhou University, Fuzhou, Fujian 350002, P. R. China*
[b] *Department of Mathematics, Wilfrid Laurier University, 75 University Avenue West, Waterloo, Ontario, N2L 3C5, Canada*
[c] *College of Mathematics and Econometrics, Hunan University, Changsha, Hunan 410082, P. R. China*

Abstract. We concern with the existence and global asymptotic stability of positive (componentwise) periodic solutions to a periodic integro-differential competition system with infinite delays. The purpose of this paper is two-fold. One is to point out that some existing results are/may be wrong. The other is to establish new sets of sufficient conditions on the existence and global asymptotic stability of positive periodic solutions by using the continuation theorem based on the coincidence degree theory and Lyapunov functional. The obtained results greatly improve some existing ones.

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1. Introduction

One of the celebrated population dynamics models on species interaction is the Lotka-Volterra competition system, which is originally described by a system of autonomous

E-mail addresses: fdchen@fzu.edu.cn (F. Chen), ychen@wlu.ca (Y. Chen), shangjguo@hnu.cn (S. Guo)
*Corresponding author

ordinary differential equations. However, due to fluctuations of the environment and delays arising from factors such as gestation and consumption of biomass, it is quite natural to study nonautonomous Lotka-Volterra systems with delays, especially those with periodic coefficients. Because of their theoretical and practical significance, these systems have been studied extensively. One of the main concern is the existence and stability of periodic solutions (see, for example, [1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14] and the references cited therein).

In this paper, we study the periodic nonautonomous integro-differential competition system with infinite delays,

$$\frac{dy_i(t)}{dt} = y_i(t)\left\{r_i(t) - \sum_{j=1}^{n}\left(a_{ij}(t)y_j(t) + b_{ij}(t)\int_{-\infty}^{t} K_{ij}(t-u)y_j(u)du\right)\right\}, \quad (1.1)$$

$i = 1, 2, \ldots, n$, where a_{ij}, b_{ij}, r_i are continuous ω-periodic functions with $a_{ij}(t) \geq 0$, $b_{ij}(t) \geq 0$, $\int_0^\omega r_i(t)dt > 0$, and $K_{ij} : [0,\infty) \to [0,\infty)$ are measurable, ω-periodic, normalized functions such that $\int_0^\infty K_{ij}(t)dt = 1$ and $\int_0^\infty tK_{ij}(t)dt < \infty$, i, $j = 1$, $2, \ldots, n$. From the point of mathematical biology, each individual competes with all others for common resources and the intra- and inter-species competitions involve time delays extending over the entire past as denoted by K_{ij} in (1.1). Obviously, system (1.1) includes most of the commonly studied Lotka-Volterra competition systems as special cases.

System (1.1) was studied by Fan and Wang [5] and later by Chen [4]. To state their main results, which are on existence of ω-periodic solution with strictly positive components (hereafter referred to as positive periodic solution), we introduce some notations. Let $g : \mathbb{R} \to \mathbb{R}$ be a continuous and bounded function. Then $g^l = \inf_{t\in\mathbb{R}} g(t)$ and $g^u = \sup_{t\in\mathbb{R}} g(t)$. In addition, if g is also ω-periodic, then $\overline{g} = \frac{1}{\omega}\int_0^\omega g(t)dt$.

Theorem A (Theorem 2.1 of Fan and Wang [5]) *If* $\overline{a_{ii}} + \overline{b_{ii}} > 0$ *and*

$$\overline{r_i} > \sum_{j=1, j\neq i}^{n} \frac{(\overline{a_{ij}} + \overline{b_{ij}})\overline{r_j}}{\overline{a_{jj}} + \overline{b_{jj}}} \exp\{(\overline{r_j} + \overline{|r_j|})\omega\}, \qquad i = 1, 2, \ldots, n, \quad (1.2)$$

then (1.1) *has at least one positive periodic solution.*

Definition 1.1. (Definition 3.1 of Fan and Wang [5]) A positive periodic solution $y^*(t) = (y_1^*(t), y_2^*(t), \ldots, y_n^*(t))^T$ of (1.1) is said to be globally asymptotically stable if any other solution $y(t) = (y_1(t), y_2(t), \ldots, y_n(t))^T$ of (1.1) together with the initial condition

$$y_i(s) = \varphi_i(s), \quad s \in (-\infty, 0]; \quad \varphi_i(0) > 0; \quad \sup_{s\leq 0} \varphi_i(s) < \infty, \quad i = 1, 2, \ldots, n,$$

has the property

$$\lim_{t\to\infty} \sum_{i=1}^{n} |y_i(t) - y_i^*(t)| = 0,$$

where $\varphi_i \in C((-\infty, 0], [0, \infty))$.

Theorem B (Theorem 3.1 of Fan and Wang [5]) *Assume that the conditions in Theorem A hold. Furthermore, suppose that*

$$\sum_{i=1}^{n} \min_{t\in[0,\omega]} a_{ij}(t) > \sum_{i=1}^{n} \max_{t\in[0,\omega]} b_{ij}(t), \qquad j = 1, 2, \dots, n. \tag{1.3}$$

Then there exists a unique positive periodic solution of (1.1) *which is globally asymptotically stable.*

Note that the explicit presence of ω in (1.2) may impose a very strict constraint on the coefficients of (1.1). This inspired the following result of Chen [4], which was also obtained by employing the continuation theorem based on the coincidence degree theory. For comparison of this result with Theorem A, we refer the readers to Chen [4] for detail.

Theorem C (Theorem 3 of Chen [4]) *Assume that $r_i(t) > 0$ and $a_{ii}(t) > 0$. Furthermore, suppose that*

$$r_i^l > \sum_{j=1, j\neq i}^{n} \frac{a_{ij}^u r_j^u}{a_{jj}^l} + \sum_{j=1}^{n} \frac{b_{ij}^u r_j^u}{a_{jj}^l}, \qquad i = 1, 2, \dots, n. \tag{1.4}$$

Then system (1.1) *has at least one positive periodic solution.*

Recently, Tang and Zou [11] considered the following special case of (1.1),

$$\dot{x}_i(t) = x_i(t)\left[r_i(t) - \sum_{j=1}^{n} a_{ij}(t)x_j(t - \tau_{ij}(t))\right], \qquad i = 1, 2, \dots, n. \tag{1.5}$$

Theorem D (Theorem 2.5 of Tang and Zou [11]) *Assume that the linear system*

$$\sum_{j=1}^{n} \overline{a_{ij}} x_j = \overline{r_i}, \qquad i = 1, 2, \dots, n, \tag{1.6}$$

has a positive solution. Then (1.5) *has at least one positive periodic solution.*

Now, let us give some remarks on the above results.

First, the condition in Theorem D is a very nice one as it reduces exactly to that on the existence of a positive equilibrium when (1.5) reduces to the autonomous system. However, Theorem D is too wonderful to believe. A careful check reveals a gap in Tang and Zou's arguments: To derive (2.15), the authors used $|x_i|_0 = (\sigma^2 A\omega)^{-1} x_i^*$ for all $i = 1, 2, \dots, n$. Unfortunately, this is true only for at least one i but not necessarily for all i. What worse is that even the gap can be filled Theorem D can not be established by using the Krasnoselskii's fixed point theorem. One can easily see that a particular positive solution to system (1.6) did not play any special role in their arguments. In fact, if $x^* = (x_1^*, x_2^*, \dots, x_n^*)^T$ with $x_i^* > 0$ is a vector such that

$\sum\limits_{j=1}^{n} \overline{a_{ij}} x_j^* > 0$, then their arguments can still go through with A and B being replaced with

$$A = \min \left\{ A_i \sum_{j=1}^{n} (\overline{a_{ij}} x_j^*) : i = 1, 2, \ldots, n \right\}$$

and

$$B = \max \left\{ B_i \sum_{j=1}^{n} (\overline{a_{ij}} x_j^*) : i = 1, 2, \ldots, n \right\},$$

respectively. In particular, the condition in Theorem D implies

$$\sum_{j=1}^{n} \overline{a_{ij}} > 0, \qquad i = 1, 2, \ldots, n, \tag{1.7}$$

and we can pick $x^* = (1, 1, \ldots, 1)^n$. Thus (1.5) has a positive periodic solution if (1.7) holds. This contradicts with Theorem 2.6 of Tang and Zou [11] as condition (1.7) will not necessarily guarantee system (1.6) has a positive solution. In other words, one can reasonably doubt Theorem D.

Second, in Theorem C, we require $r_i(t) > 0$ and $a_{ii}(t) > 0$. Though they are reasonable, due to random fluctuation of environment, r_i is unnecessary to remain positive and a_{ii} may be zero for some time. Also, condition (1.4) only involves the maxima and minima of the parameters, which may be undesirable for periodically varying situation. This calls for more natural sufficient conditions like (1.2) which involve averages of coefficients.

Finally, we mention that conditions (1.3) are not enough to guarantee the global asymptotic stability of the positive periodic solution. Consider the following two species competition system,

$$\begin{cases} \frac{dy_1}{dt} &= y_1(2 - y_1 - 3y_2), \\ \frac{dy_2}{dt} &= y_2(2 - 2y_1 - y_2). \end{cases} \tag{1.8}$$

It is easy to see that (1.2) is satisfied (with $\omega = 0$ or very small) and (1.8) admits a unique positive periodic solution (equilibrium) $(y_1, y_2) = (\frac{4}{5}, \frac{2}{5})$. Moreover, one can verify condition (1.3). However, the positive equilibrium is a saddle point which is unstable. We should point out that Fan and Wang's arguments are correct except that they need the conditions,

$$\sum_{i=1}^{n} \left(\min_{t \in [0,\omega]} a_{ij}(t) - \max_{t \in [0,\omega]} b_{ij}(t) \right) > 0, \qquad j = 1, 2, \ldots, n. \tag{1.9}$$

Theorem B′ *Assume that the conditions in Theorem A and* (1.9) *hold. Then there exists a unique positive periodic solution of* (1.1) *which is globally asymptotically stable.*

Motivated by the above remarks, we further seek improvements of Theorem A and Theorem C. To state our main results, we introduce another notation. For a function $g : \mathbb{R} \to \mathbb{R}$, we decompose g as $g = g^+ + g^-$, where $g^+(t) = \max\{g(t), 0\}$ and $g^-(t) = \min\{0, g(t)\}$ for $t \in \mathbb{R}$.

Theorem 1.1. *Suppose* $\overline{r_i} > 0$ *and* $\overline{a_{ii}} + \overline{b_{ii}} > 0$ *for* $i = 1, 2, \ldots, n$. *If*

$$\Delta_i \triangleq \overline{r_i} - \sum_{j=1, j\neq i}^{n} \frac{(\overline{a_{ij}} + \overline{b_{ij}})\overline{r_j}}{\overline{a_{jj}} + \overline{b_{jj}}} \exp\{\overline{r_j^+}\omega\} > 0, \qquad i = 1, 2, \ldots, n, \tag{1.10}$$

then (1.1) *has at least one positive periodic solution.*

Theorem 1.2. *Suppose* (1.1) *has a positive periodic solution. If we further suppose that there exist positive constants* c_j, $j = 1, 2, \ldots, n$, *such that*

$$c_j a_{jj}(t) > \sum_{i=1, i\neq j}^{n} c_i a_{ij}(t) + \sum_{i=1}^{n} c_i \int_0^{\infty} K_{ij}(s) b_{ij}(t+s)ds, \qquad j = 1, 2, \ldots, n. \tag{1.11}$$

Then this positive periodic solution is globally asymptotically stable.

Corollary 1.1. *Assume that* $r_i(t) > 0$ *and* $a_{ii}(t) > 0$. *If*

$$r_i^l > \sum_{j=1, j\neq i}^{n} a_{ij}^u \left(\frac{r_j}{a_{jj}}\right)^u + \sum_{j=1}^{n} b_{ij}^u \left(\frac{r_j}{a_{jj}}\right)^u, \qquad i = 1, 2, \ldots, n, \tag{1.12}$$

and (1.1) *admits a positive periodic solution then this positive periodic solution is globally asymptotically stable.*

Note that conditions (1.2) are, in fact,

$$\overline{r_i} > \sum_{j=1, j\neq i}^{n} \frac{(\overline{a_{ij}} + \overline{b_{ij}})\overline{r_j}}{\overline{a_{jj}} + \overline{b_{jj}}} \exp\{2\overline{r_j^+}\omega\}, \qquad i = 1, 2, \ldots, n.$$

Also note that $\left(\frac{r_j}{a_{jj}}\right)^u \leq \frac{r_j^u}{a_{jj}^l}$, $j = 1, 2, \ldots, n$. Obviously, Theorem 1.1, Theorem 1.2, and Corollary 1.1 greatly improve Theorem A, Theorem B′, and Theorem C, respectively. Nevertheless, there is still room to improve Theorem 1.1. But, to the best of the authors' knowledge, Theorem 1.1 is the best so far. Moreover, the technique in the proof of Proposition 2.1 might be used to improve some other results on existence of periodic solutions, which are obtained by employing the continuation theorem based on coincidence degree theory.

The main results will be proved in the coming section. To conclude this section, we just mention that Theorem 1.1 is proved again by using the continuation theorem while Theorem 1.2 is proved by constructing a suitable Lyapunov functional.

2. Proofs of the main results

Before proving Theorem 1.1, we first establish better estimations for bounds of positive periodic solutions of (1.1). Indeed, we consider the following system with parameter,

$$\frac{dy_i(t)}{dt} = \lambda y_i(t)\left\{r_i(t) - \sum_{j=1}^{n}\left(a_{ij}(t)y_j(t) + b_{ij}(t)\int_{-\infty}^{t} K_{ij}(t-u)y_j(u)du\right)\right\}, \quad (2.1)$$

$i = 1, 2, \ldots, n$, $\lambda \in (0,1]$. For $i = 1, 2, \ldots, n$, denote

$$U_i = \frac{\overline{r_i}}{\overline{a_{ii}} + \overline{b_{ii}}}\exp\{\overline{r_i^+}\omega\}$$

and

$$L_i = \frac{\Delta_i}{\overline{a_{ii}} + \overline{b_{ii}}}\exp\left\{\omega\left[\overline{r_i^-} - \sum_{j=1}^{n}(\overline{a_{ij}} + \overline{b_{ij}})U_j\right]\right\}.$$

The following result will pave the way for employing the continuation theorem.

Proposition 2.1. *Under the assumptions of Theorem 1.1, we have*

$$L_i \le y_i^*(t) \le U_i, \qquad i = 1, 2, \ldots, n, t \in \mathbb{R},$$

where $y^*(t) = (y_1^*(t), y_2^*(t), \ldots, y_n^*(t))^T$ *is any positive periodic solution of* (2.1).

Proof. Since $\lambda > 0$ and $y^*(t)$ is a positive periodic solution, for each i, we have

$$\frac{dy_i^*(t)}{dt} \le \lambda y_i^*(t)\{r_i(t) - [a_{ii}(t) + b_{ii}(t)](y_i^*)^l\},$$

or, equivalently,

$$\frac{d}{dt}\ln y_i^*(t) \le \lambda\{r_i(t) - [a_{ii}(t) + b_{ii}(t)](y_i^*)^l\}.$$

Integrating the above differential inequality over $[0, \omega]$ immediately gives us

$$0 \le \lambda\omega\{\overline{r_i} - [\overline{a_{ii}} + \overline{b_{ii}}](y_i^*)^l\},$$

which implies

$$(y_i^*)^l \le \frac{\overline{r_i}}{\overline{a_{ii}} + \overline{b_{ii}}}.$$

Choose $t_i^l \in [0, \omega)$ such that $y_i^*(t_i^l) = (y_i^*)^l$. Then, for $s \in [0, \omega]$, it follows from the differential inequality

$$\frac{dy_i^*(t)}{dt} \le \lambda y_i^*(t)r_i(t)$$

that

$$y_i^*(t_i^l + s) \quad \le \quad (y_i^*)^l \exp\left(\lambda \int_{t_i^l}^{t_i^l+s} r_i(u)du\right)$$

$$\begin{aligned} &\le (y_i^*)^l \exp\left(\int_{t_i^l}^{t_i^l+s} r_i^+(u)du\right) \\ &= (y_i^*)^l \exp(\overline{r_i^+}\omega) \\ &\le U_i. \end{aligned}$$

Here we used $\lambda \in (0,1]$. By periodicity, we conclude that

$$y_i^*(t) \le U_i, \qquad i = 1, 2, \ldots, n, t \in \mathbb{R}. \tag{2.2}$$

In the following we derive lower bounds for $y_i^*(t)$, $i = 1, 2, \ldots, n$. With the help of (2.2), we have

$$\frac{dy_i^*(t)}{dt} \ge \lambda y_i^*(t) \left\{ r_i(t) - [a_{ii}(t) + b_{ii}(t)](y_i^*)^u - \sum_{j=1, j\ne i}^{n} [a_{ij}(t) + b_{ij}(t)]U_j \right\}.$$

Similarly, integrating the above differential inequality over $[0, \omega]$ gives us

$$0 \ge \lambda\omega \left\{ \overline{r_i} - [\overline{a_{ii}} + \overline{b_{ii}}](y_i^*)^u - \sum_{j=1, j\ne i}^{n} [\overline{a_{ij}} + \overline{b_{ij}}]U_j \right\} = \lambda\omega\{\Delta_i - [\overline{a_{ii}} + \overline{b_{ii}}](y_i^*)^u\}.$$

Since $\Delta_i > 0$, we get

$$(y_i^*)^u \ge \frac{\Delta_i}{\overline{a_{ii}} + \overline{b_{ii}}}.$$

Then similar argument as that for the upper bound of $y_i^*(t)$ produces

$$L_i \le y_i^*(t), \qquad i = 1, 2, \ldots, n, t \in \mathbb{R}.$$

This completes the proof. $\square$

Proof of Theorem 1.1: Theorem 1.1 can be proved with slight modifications of the proof of Theorem 3 of Chen [4]. We just mention the differences here. The interesting readers can refer to [4] for the details. First, (1.10) also implies

$$\overline{r_i} > \sum_{j=1, j\ne i}^{n} \frac{(\overline{a_{ij}} + \overline{b_{ij}})\overline{r_j}}{\overline{a_{jj}} + \overline{b_{jj}}}.$$

Second, Proposition 2.1 tells us that if $x \in X$ is a solution to $Lx = \lambda Nx$, $\lambda \in (0,1)$, then

$$\max_{t\in[0,\omega]} |x_i(t)| \le \max\{|\ln L_i|, |\ln U_i|\} := H_i, \qquad i = 1, 2, \ldots, n.$$

This completes the proof. $\square$

Proof of Theorem 1.2: The proof is similar to that of Theorem 3.1 of Chen and Xie [2]. Because of periodicity, it follows from (1.11) that there exists a positive constant A such that

$$c_j a_{jj}(t) - \sum_{i=1, i\neq j}^{n} c_i a_{ij}(t) - \sum_{i=1}^{n} c_i \int_0^\infty K_{ij}(s) b_{ij}(t+s)ds > A,$$

$j = 1, 2, \ldots, n$. Let $y^*(t) = (y_1^*(t), y_2^*(t), \ldots, y_n^*(t))^T$ be a positive periodic solution of (1.1) and $y(t) = (y_1(t), y_2(t), \ldots, y_n(t))^T$ be any other solution of (1.1) with the initial condition specified in Definition 1.1. Construct a Lyapunov functional $V(t)$ as follows,

$$\begin{aligned} V(t) &= \sum_{i=1}^{n} \Big\{ c_i |\ln y_i(t) - \ln y_i^*(t)| \\ &\quad + c_i \sum_{j=1}^{n} \int_0^\infty K_{ij}(s) \Big(\int_{t-s}^{t} b_{ij}(s+u)|y_j(u) - y_j^*(u)|du \Big) ds \Big\} \end{aligned}$$

for all $t \geq 0$. Obviously,

$$\begin{aligned} V(0) &\leq \sum_{i=1}^{n} c_i |\ln y_i(0) - \ln y_i^*(0)| \\ &\quad + \sum_{i=1}^{n} \Big(\sum_{j=1}^{n} c_i \max_{t\in[0,\omega]} b_{ij}(t) \sup_{t\in(-\infty,0]} |\varphi_i(t) - y_i^*(t)| \int_0^\infty sK_{ij}(s)ds \Big) \\ &< \infty \end{aligned}$$

and

$$V(t) \geq V_1(t) = \sum_{i=1}^{n} c_i |\ln y_i(t) - \ln y_i^*(t)|, \qquad t \geq 0.$$

Then a direct computation gives us

$$\begin{aligned} D^+V(t) &\leq -\sum_{i=1}^{n} c_i a_{ii}(t)|y_i(t) - y_i^*(t)| + \sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} c_i a_{ij}(t)|y_j(t) - y_j^*(t)| \\ &\quad + \sum_{i=1}^{n} \sum_{j=1}^{n} c_i b_{ij}(t) \int_{-\infty}^{t} K_{ij}(t-u)|y_j(u) - y_j^*(u)|du \\ &\quad + \sum_{i=1}^{n} \sum_{j=1}^{n} c_i \int_0^\infty K_{ij}(s) b_{ij}(t+s)|y_j(t) - y_j^*(t)|d\,s \\ &\quad - \sum_{i=1}^{n} \sum_{j=1}^{n} c_i \int_0^\infty K_{ij}(s) b_{ij}(t)|y_j(t-s) - y_j^*(t-s)|d\,s \\ &\leq -\sum_{i=1}^{n} c_i a_{ii}(t)|y_i(t) - y_i^*(t)| + \sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} c_j a_{ji}(t)|y_i(t) - y_i^*(t)| \end{aligned}$$

$$
\begin{aligned}
& +\sum_{i=1}^{n}\sum_{j=1}^{n} c_j \int_0^{\infty} K_{ji}(s) b_{ji}(t+s)|y_i(t)-y_i^*(t)|ds \\
\leq \quad & -A\sum_{i=1}^{n}|y_i(t)-y_i^*(t)|.
\end{aligned}
$$

The proof is completed by using the above differential inequality and similar arguments as those to (3.8)–(3.13) of Fan and Wang [5]. □

Corollary 1.1 can be proved by adapting the ideas used by Ahmad and Lazer [1] and Tineo and Alvarez [12].

Proof of Corollary 1.1: It suffices to show that (1.12) implies (1.11). For $i = 1, 2, \ldots, n$, note that

$$
a_{ii}^l \Big(\frac{r_i}{a_{ii}}\Big)^u = \min_{t\in[0,\omega]} \Big(a_{ii}(t)\Big(\frac{r_i}{a_{ii}}\Big)^u\Big) \geq \min_{t\in[0,\omega]} \Big(a_{ii}(t)\frac{r_i(t)}{a_{ii}(t)}\Big) = r_i^l.
$$

It follows from (1.12) that

$$
a_{ii}^l \Big(\frac{r_i}{a_{ii}}\Big)^u > \sum_{j=1, j\neq i}^{n} a_{ij}^u \Big(\frac{r_j}{a_{jj}}\Big)^u + \sum_{j=1}^{n} b_{ij}^u \Big(\frac{r_j}{a_{jj}}\Big)^u. \tag{2.3}
$$

For i, $j = 1, 2, \ldots, n$, set

$$
m_{ij} = \begin{cases} \frac{a_{ij}^u + b_{ij}^u}{a_{ii}^l} & i \neq j \\ \frac{b_{ii}^u}{a_{ii}^l}, & i = j \end{cases}
$$

and

$$
\gamma_i = \Big(\frac{r_i}{a_{ii}}\Big)^u.
$$

Then (2.3) is equivalent to

$$
\sum_{j=1}^{n} m_{ij}\gamma_j < \gamma_i, \qquad i = 1, 2, \ldots, n.
$$

It follows that we can choose $p_{ij} > m_{ij}$, i, $j = 1, 2, \ldots, n$, such that

$$
\sum_{j=1}^{n} p_{ij}\gamma_j < \gamma_i, \qquad i = 1, 2, \ldots, n. \tag{2.4}
$$

Let $P = (p_{ij})_{n\times n}$. Since P^T is a strictly positive (entrywise) matrix, by the Perron-Frobenius theorem [8] there exist $\lambda > 0$, $z_i > 0$, $i = 1, 2, \ldots, n$, such that

$$
P^T(z_1, z_2, \ldots, z_n)^T = \lambda(z_1, z_2, \ldots, z_n)^T,
$$

i.e.,

$$\sum_{i=1}^{n} p_{ij} z_i = \lambda z_j, \qquad j = 1, 2, \ldots, n. \tag{2.5}$$

From (2.4) and (2.5), we have

$$\lambda \sum_{j=1}^{n} \gamma_j z_j = \sum_{j=1}^{n} \sum_{i=1}^{n} p_{ij} \gamma_j z_i = \sum_{i=1}^{n} \left(\sum_{j=1}^{n} p_{ij} \gamma_j \right) z_i < \sum_{i=1}^{n} \gamma_i z_i.$$

It follows that $\lambda < 1$ and hence

$$\sum_{i=1}^{n} p_{ij} z_i < z_j, \qquad j = 1, 2, \ldots, n.$$

This implies

$$\sum_{i=1}^{n} m_{ij} z_i < z_j, \qquad j = 1, 2, \ldots, n,$$

as $p_{ij} > m_{ij}$, i, $j = 1$, 2, $\ldots$, n. It follows from the definitions of m_{ij}'s that

$$\sum_{i=1, i \neq j}^{n} \frac{a_{ij}^u}{a_{ii}^l} z_i + \sum_{i=1}^{n} \frac{b_{ij}^u}{a_{ii}^l} z_i < z_j, \qquad j = 1, 2, \ldots, n. \tag{2.6}$$

Denote

$$c_j = \frac{z_j}{a_{jj}^l}, \qquad j = 1, 2, \ldots, n.$$

Then (2.6) can be rewritten as

$$c_j a_{jj}^l > \sum_{i=1, i \neq j}^{n} c_i a_{ij}^u + \sum_{i=1}^{n} c_i b_{ij}^u, \qquad j = 1, 2, \ldots, n.$$

Therefore, there exists $\delta > 0$ such that

$$c_j a_{jj}^l - \sum_{i=1, i \neq j}^{n} c_i a_{ij}^u - \sum_{i=1}^{n} c_i b_{ij}^u > \delta, \qquad j = 1, 2, \ldots, n,$$

from which one can easily see that (1.11) holds. This completes the proof. □

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