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Fractal Nano-hydrodynamics

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Abstract: *A fascinating phenomenon arises in nano-scale hydrodynamics that liquid flow through a carbon nanotube is 4 to 5 orders of magnitude faster than would be predicted from conventional fluid-flow theory. A theoretical analysis is given to explain the phenomenon.*

Keywords: *Nanohydrodynamics , fractal geometry, carbon nanotubes*

1. INTRODUCTION

According to El-Naschie's definition [1], nanotechnology is defined as a technology applied in the grey area between classical mechanics and quantum mechanics. Nanotechnology links both deterministic classic mechanics and chaotic quantum mechanics. The grey area sometimes behaves like a continuum, and sometimes have quantum-like properties or nano-effects [2]. Fractal nano-hydrodynamics is suggested to explain the nano-effects of nanoscale flows in carbon nanotubes. We argue that nanotubes at the nanoscale is not a continuum, and we stress the fact that a fractal integral [3] possesses the same cardinality of the continuum. Our theory can explain well why fluids in nanotubes flow extremely faster than would be predicted from continuum theory.

2. FRACTAL NANO-HYDRODYNAMICS

Majumder *et al.* [4] found that liquid flow through a membrane composed of an array of aligned carbon nanotubes is 4 to 5 orders of magnitude faster than would be predicted from conventional fluid-flow theory, similar phenomena were observed by other researchers [5, 6]. Why does the fluid in nano-tubes flow extraordinary fast? We will give hereby a heuristical explanation.

We write the conversation of mass in continuum media in the form

$$\oint \frac{1}{2}\rho u r dl = Q \tag{1}$$

where Q is the flow rate. For the continuum media, we have

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$$u = \frac{Q}{\pi r^2 \rho} \tag{2}$$

For nano scale hydrodynamics, for example, the flow in carbon nanotubes, the perimeter of a section is of fractal [2]:

$$l_{NH} = kl^D \tag{3}$$

where D is the fractal dimension of the perimeter.

We re-write Eq.(1) in the form

$$Q = \oint \frac{1}{2} \rho urdl_{NH} = \oint \frac{1}{2} \rho kur(dl)^D = \oint \frac{1}{2} \rho kur^{(1+D)} d\theta^D \tag{4}$$

According to the fractal integral [2], Eq.(4) can be approximately calculated as

$$Q \approx \frac{1}{2} \rho kur^{(1+D)} (2\pi)^D \tag{5}$$

So the velocity in discontinuous carbon nanotubes can be expressed as

$$u_{NH} = \frac{2Q}{\rho kr^{(1+D)} (2\pi)^D} \tag{6}$$

Comparing (6) with (2), we find

$$\frac{u_{NH}}{u} \propto r^{1-D} \tag{7}$$

We consider a case as illustrated in Fig. 1. The fractal dimension, D, can be calculated as

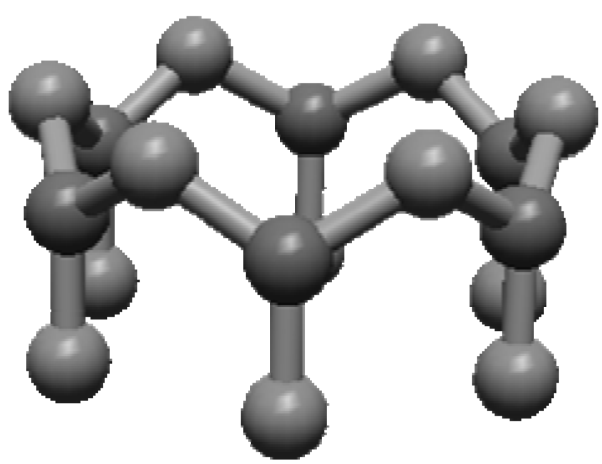

Figure 1: Fractal Boundary of Carbon Nano-tube.

$$D = \frac{\ln 2}{\ln(\sqrt{3})} = 1.26 \tag{8}$$

Considering a nanotube with a radius of 10 nm, we have

$$\frac{u_{NH}}{u} \propto (10\times10^{-9})^{-0.26} = 10^{4.18} \approx 1.5\times10^{4} \tag{9}$$

Now we consider the Hagen-Poiseuille equation in continuum media, which reads

$$\Delta P \propto 1/L \tag{10}$$

where ΔP is the press difference between the two ends, and L is the length of the continuous tube.

For nanohydrodynamics, we predict

$$\Delta P_{NH} \propto L^{-d} \tag{11}$$

where d is the fractal dimension of longitudinal length . For the carbon nanotubes, we consider a case

$$d = \frac{\ln 4}{\ln(\sqrt{3})} = 2.52 \tag{12}$$

We, therefore, predict that

$$\Delta P_{NH} \propto L^{-2.52} \tag{13}$$

Experimental verification of our predictions is very much needed.

3. CONCLUSION

We have proposed a fractal approach to the nanohydrodynamics dealing with for the first time a fascinating phenomenon of critical importance for nanoscience and nanotechnology. Of course the author understands that no matter how rigorous, some experimentally verification is strongly needed to validate the model.

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