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Validation of a Protocol to Characterize the Temporomandibular Joint kinematics

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This paper describes an experimental protocol to analyse the 3D kinematics of a human joint. The method (3D video analysis, computer-assisted recording and data processing) and materials (gutters, plates, and coordinate systems) which were elaborated and implemented for the application to the Temporomandibular Joint (TMJ) are presented. The main contribution consists in defining rigorously the measured displacements in a functional coordinate system attached to an anatomical reference: the Camper's plane. The protocol was then applied and validated for two mesodivergent volunteers and made it possible to quantify any displacement of the mandible with respect to the cranium.

The first results calculate the displacements of the inter-incisor point and those of the condyle centres. They point out that during the open-close movement the rotation and the translation of the mandible remain concomitant.

Keywords: *Joint biomechanics, mechanical modelling, 3D video-analysis, articular displacements, rotation and translation, prosthesis.*

INTRODUCTION

Occlusion disorders and dysfunctions of the temporomandibular joint (TMJ), worsened by possible disorders of a psychic nature, result in disc luxations, perforations of the meniscus or of permanent jaw constrictions. Traumatisms and tumoral pathologies multiply requiring invasive surgical treatments of the fractures or tumours. Such interventions frequently lead to functional after-effects of the TMJ.

The rebuilding of the mandible condyle can frequently reduce these after-effects or congenital hemi facial malformations [1]. In a significant number of cases, the forecast of return to a satisfying functioning of the TMJ remains unrealistic. In such circumstances, among curative solutions, actual TMJ prostheses seem to offer oral surgeons various options to treat joints diseases.

In several comparative studies, Speculand [2] and Wolford [3] [4] point out that placing TMJ prostheses the restored inter-incisors opening always reaches twenty-five millimetres while the occlusion corrections get to the patient a psychological and a physical comfort. On the other hand, if the propulsion movement is immediately restored, the prostheses currently do not regenerate every natural displacement and the lateral movement amplitudes remain hardly very limited.

These problems are complex and cannot be handled by just one medical discipline who still often describes the TMJ kinematics during an open-close movement by a rotation of the condylar head in the glenoïd fossa before the translation of the mandible condyle out of the cavity.

An application of the functional analysis to the human TMJ revealed the necessity to seek complementary data characterizing the natural joint kinematics [5]. The TMJ was assimilated to a bilateral mechanical joint between two rigid solid bodies: the cranium (temporal zones) and the mandible (two condyles). This approach pointed out the investigation directions and the experimental techniques that were associated in:

– kinematics and 3D motion analysis,
– mechanical actions and electromyography.

This paper presents a kinematic protocol using 3D video analysis to describe the 3D mandible movements with respect to the cranium.

Most of the systems used for human motion tracking are based on stereoscopy principles. Several systems are commercialized [6] [7]; some of them are used for the analysis of the mandible displacements [8] [9] [10].

Various equipments were developed specifically for jaw tracking and various acquisition principles are used: optoelectronic [11] [12] [13], electromagnetic [14] [15] [16], ultrasounds [17], accelerometer [16]. Some of these systems allow the measurement of the coordinates for only one point according to one or more directions [14] [17] [15] [16]. However, the majority of them allow the measurement of several points in 3D space. This last case corresponds to the minimum requirement for a 3D motion tracking taking into account translations and rotations.

The equipment used to develop our protocol satisfies this requirement. It is based on standard video cameras and, then, is a low-cost system. In spite of this, its accuracy is comparable to the accuracy of much more elaborated systems. The major disadvantage lies in a more complex implementation, without any consequence on the results.

The technical adaptation to the measurement of the mandible and TMJ displacements leads to design specific equipments and protocol. In this paper, the equipments are described and the protocol is detailed.

As in scientific studies analysing jaw motion, the coordinate system used to present the results is never defined, or is exotic, this paper attempts to present a functional coordinate system attached to an anatomical reference: the Camper's plane.

PROTOCOL AND MATERIAL

The protocol consists in measuring the volunteers' mandible displacements, with respect to the cranium. Movements such as jaw opening, protrusion ... are artificial; others like chewing and speaking are more natural. The protocol had been freely accepted by the two volunteers, students who duly informed had expressed their interest in the results. They had undergone no orthodontic treatment likely to modify their muscular physiology or morphological characters in their growth phase. They were mesodivergent and presented a complete teething without any articular pathology or dental prosthesis.

Motion acquisition was assumed by two digital video cameras; 3D video reconstruction allowed computing the 3D coordinates of a point from the shots of this point by two (or more than two) cameras. Points of location were associated with the maxilla and the mandible to study the TMJ kinematics. To support these points intra-cortical needles near the mandible condyle would have been particularly painful and the necessary precision proscribed any location on cutaneous areas. Tissues are too mobile compared to the close osseous zones. Therefore the two dental arches served as permanent skeletal basis (Figure 1). Two gutters, clutched onto the arches, were moulded in methyl methacrylate. They were custom-made for each volunteer using orthodontics material. A rigid metal snap ring was inserted into each gutter and a plate was glued on each ring. As a coordinate system can be defined with a set of three points, three targets were then stuck on the plates.

At this stage, the displacements of the mandible with respect to the cranium might be determined, but these displacements would be expressed with respect to the upper plate. One appreciates to express the displacements in a functional system. This system must be attached to the human anatomy of the maxilo-facial system. The Camper's plane retained particularly our attention. This

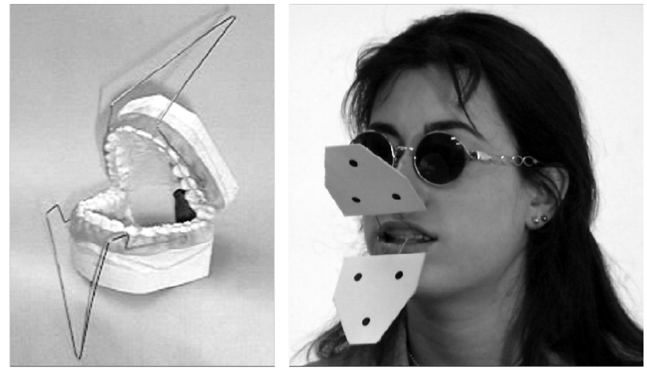


Figure 1: Gutters, Rings, Plates and Targets

plane passes through the condyles centres and the sub-nasal point, osseous point at the base of the nose. The interesting facts are that this plane is fixed with respect to cranium, it passes through the condyles and it is approximately parallel to the cuspide plane for a mesodivergent subject. The distinction between hypodivergent, mesodivergent and hyperdivergent is based on the inclination of the Camper's plane with the cuspide plane.

The inter-incisor point was also useful to analyse how the teeth engage into one another for small displacements.

To identify these morphological and inter-incisor points, a special plate allowing the capture was developed (Figure 2). This plate was equipped with a tip which came in contact with the measured point. Once more, the set of three points on the plate permits to construct a coordinate system and then to deduce the location of the tip, i.e. the location of the morphologic point in contact with the extremity.

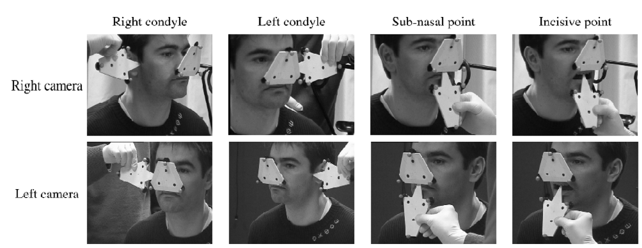


Figure 2: Tip Plate in Different Positions

The morphological coordinate system, Camper's coordinate system, was computed using the condyle centres and the sub-nasal point.

3D reconstruction requires space calibration; a calibration box (Figure 3) completed the necessary equipment. The calibration box presented 18 targets of known coordinates. Video shots of this box allowed to calibrate the 3D space and to define a measurement system.

Recording and data processing were computer-assisted. After shooting with the two cameras, the captures of the images were realized with a video

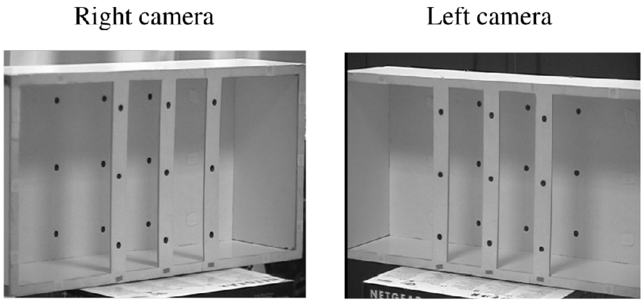


Figure 3: Calibration Box

assembly software (Adobe Premiere). Then the detection of the targets was carried out with an image processing software (Snap32, Biometrics). After that, laboratory software ensured the 3D reconstruction and motion analysis. The software allowed to calculate and to represent the point evolutions presented in the results section. Results and graphics were expressed in the morphological Camper's system.

NUMERICAL METHOD

As the protocol was defined; it became possible to develop the numerical method used to obtain the global motion of the mandible in 3D space. The process was decomposed in the following steps.

3D Reconstruction

The method for 3D reconstruction is based on Direct Linear Transformation method, DLT, developed by Abdel-Aziz and Karara [18]. There is a linear relation between the coordinates of a point of the digitised image and its co-ordinates in a measurement system. The constants of the linear relations are calculated from the calibration. Six points on a calibration box are sufficient to determine the constants; in this experiment, the constants were calculated from eighteen points to reduce the uncertainties. The oversize systems of equations were solved by the "least squares" method to determine first, the experiment constants characterising the camera adjustments and then the space co-ordinates of a filmed point. For more details, this method has been completely developed in [18] [5].

Coordinate System Associated to the Targets of a Plate

Each plate (upper plate fixed on the cranium, lower plate fixed on the mandible and tip plate used for the morphologic points) was equipped with three targets to associate a Cartesian coordinate system. The definition of the coordinate systems S_p of a plate was the following:

– the origin was the barycentre of the three targets,
– the z_p-axis was perpendicular to the plane of the three targets (the external side of the plate points to the positive z_p-axis),
– the x_p-axis was parallel to a straight line defined by two targets,
– the y_p-axis completed the system to satisfy the right-hand rule.

As shown in annex, the transformation from S_m to S_p, denoted $t_{m,p}$, may be computed from the measured coordinates of the targets in S_m.

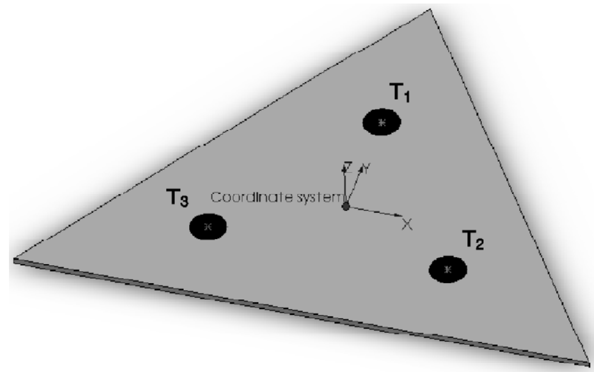


Figure 4: Plate and Coordinate System

Upper Plate as a Reference

In this study, the upper plate represented the reference plate. It allowed defining the location of the other plates during all the experiment. In a first step, every coordinate system was defined with respect to the coordinate system associated to the upper plate.

Let S_{up} be the upper plate coordinate system.

Let S_p be another plate coordinate system.

Let $t_{m,up}$ and $t_{m,p}$ be the transformation from S_m to S_{up} and S_p computed from the measured coordinates of the targets.

Let $t_{p,up}$ be the transformation from S_p to S_{up} (Figure 5). According to equations (a3) and (a2),

$$t_{p,up} = t_{m,up} \circ t_{p,m} = t_{m,up} \circ t_{m,p}^{-1} \quad (1)$$

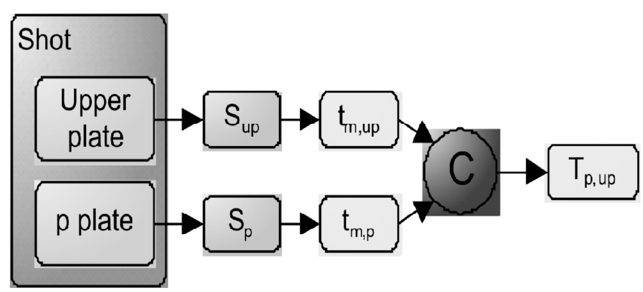


Figure 5: Upper Plate as a Reference

Morphologic Points

At this stage, location of the morphologic points had to be achieved.

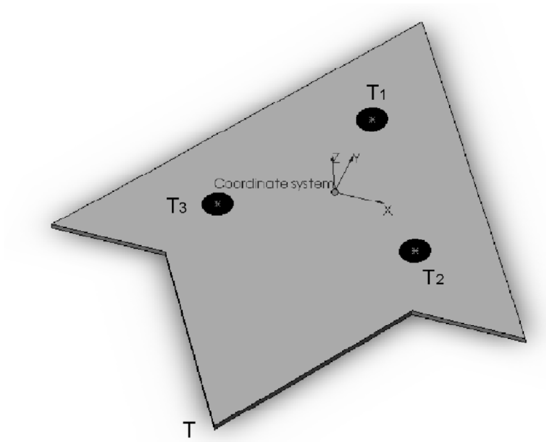


Figure 6: Tip Plate

Let S_t be the tip plate coordinate system (Figure 6).

Let T be the contact point. The coordinates of point T were known in S_t (a pattern with the three targets and the point T, was obtained by printing). According to equation (a1),

$$[\mathbf{O_{up}T}]^{Bup} = t_{t,up}\,([\mathbf{O_tT}]^{Bt}) \qquad (2)$$

Equation (2) made it possible to compute each $[\mathbf{O_{up}T}]^{Bup}$ (Figure 7).

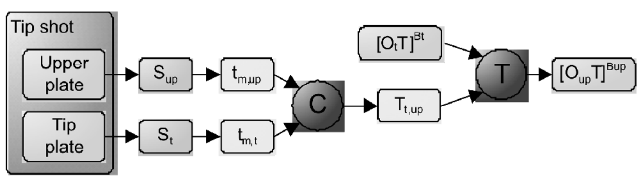


Figure 7 : Morphological Point Coordinates

Similarly to the coordinate system associated to three targets, a coordinate system was associated to the three morphologic points defining the camper plane. The definition of the coordinate systems S_c was the following:

– the origin was the middle of the segment defined by the two condyle centres,

– the y-axis was perpendicular to the Camper's plane, positive y-axis upward,

– the x-axis was parallel to the straight line passing through the two TMJ, positive x-axis toward the volunteer's left,

– the z-axis completed the system to satisfy the right-hand rule.

As shown in annex, the transformation from S_{up} to S_c, denoted $t_{up,c}$, may be computed from the calculated coordinates of the three morphologic points (Figure 8).

Lower Plate Displacement

Let S_{lo} be the lower plate coordinate system.

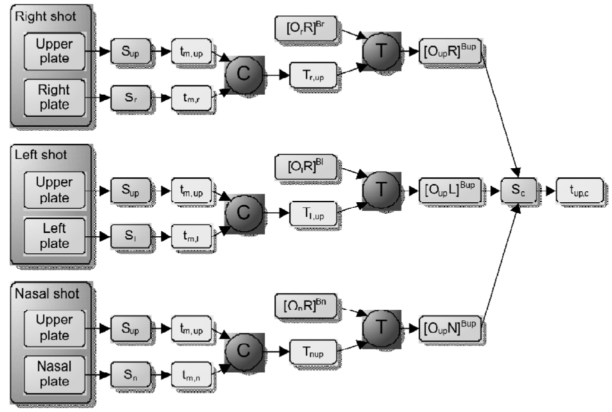


Figure 8: Camper's Coordinate System

Let $t_{m,lo}$ be the transformation from S_m to S_{lo} computed from the measured co-ordinates of the targets.

Let $t_{lo,up}$ be the transformation from S_{lo} to S_{up}. According to equations (a3) and (a2),

$$t_{lo,up} = t_{m,up} \circ t_{lo,m} = t_{m,up} \circ t_{m,lo}^{-1} \qquad (3)$$

Let $t_{lo,c}$ be the transformation from S_{lo} to S_c. According to equations (a3) and (3),

$$t_{lo,c} = t_{up,c} \circ t_{lo,up} = t_{up,c} \circ t_{m,up} \circ t_{m,lo}^{-1} \qquad (4)$$

Movement

Initialization

Motion analysis involved computing of several transformations which correspond to different pictures. One needed a reference position of the lower maxillary for this analyse; this reference was taken in the intercuspidation position. A shot of the volunteer in this position was then selected and considered as the reference shot. All the parameters associated to this reference were indexed by $_0$.

Let $S_{f,0}$ be the coordinate system fixed to the lower maxillary, identical to the Camper's system in the reference position.

$$\begin{aligned} t_{f,c,0} &= I \text{ (identity function)} \\ &= t_{lo,c,0} \circ t_{f,lo,0} \qquad (5) \\ t_{f,lo,0} &= t_{lo,c,0}^{-1} = (t_{up,c,0} \circ t_{lo,up})^{-1} = t_{lo,up,0}^{-1} \circ t_{up,c,0}^{-1} \\ &= (t_{m,up,0} \circ t_{m,lo,0}^{-1})^{-1} \circ t_{up,c,0}^{-1} = t_{m,lo,0} \circ t_{m,up,0}^{-1} \circ t_{up,c,0}^{-1} \qquad (6) \end{aligned}$$

where $t_{up,c,0}$ was equal to $t_{up,c}$ as S_c is fixed to S_{up} during the movement.

$t_{f,lo,0}$ was computed from $t_{m,lo,0}$, $t_{m,up,0}$ and $t_{up,c}$ which themselves could be computed according to the previous equations.

Current Transformation

As S_f is fixed to S_{lo} during the movement, $t_{f,lo}$ was equal to $t_{f,lo,0}$ during the movement. The transformation $t_{f,c}$ was decomposed,

$$t_{f,c} = t_{up,c} \circ t_{lo,up} \circ t_{f,lo} = t_{up,c} \circ t_{lo,up} \circ t_{f,lo,0} \quad (7)$$

$t_{up,c}$ and $t_{f,lo,0}$ were constant and already known.
$t_{lo,up}$ was computed for each shot t (Figure 9).

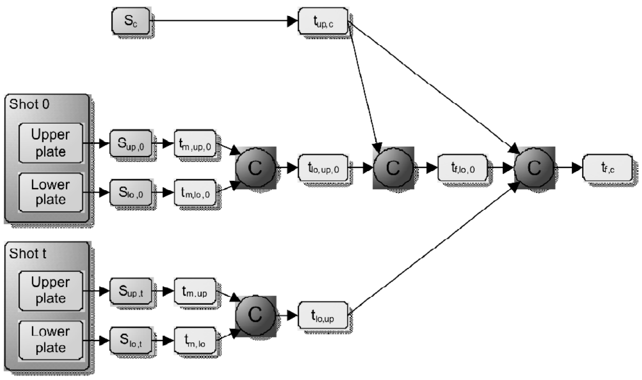


Figure 9: Current Transformation

The position of a point M in S_c was,

$$[O_cM]^{Bc} = t_{f,c}\,([O_fM]^{Bf}) \quad (8)$$

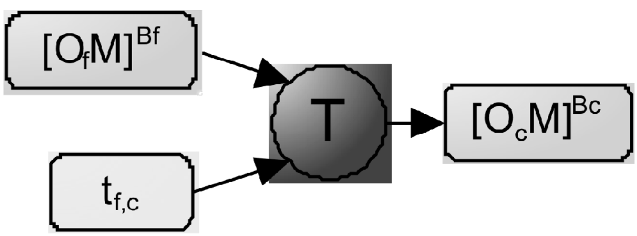


Figure 10: Position of a Point M in S_c

One can be interested in the trajectory of particular points: points R, L, middle of RL and I. These points are particularly interesting for motion analysis. Their co-ordinates were known in S_{up} (Figure 11). They were transformed by $t_{up,f,0}$ (which is equal to $t_{up,c}$) and $t_{f,c}$, so the co-ordinates could be computed in S_c (Figure 12).

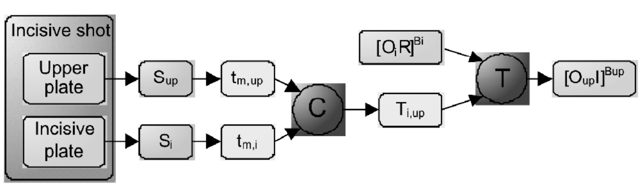


Figure 11: Coordinates of Point I in S_{up}

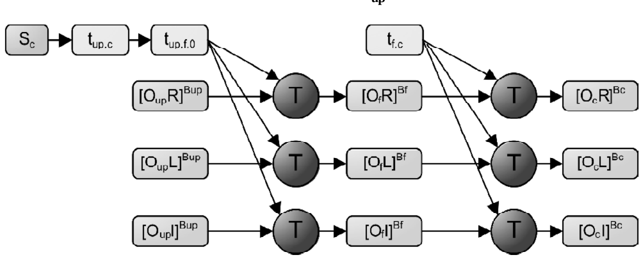
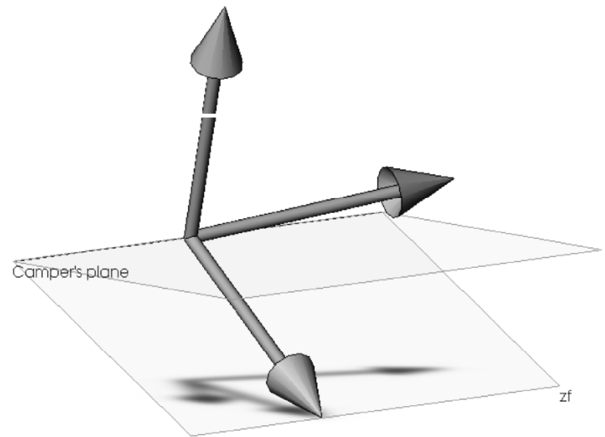


Figure 12: Position points R, L and I in S_c

Others interesting parameters are the projected angles of the mandible in the morphologic planes: a, b and g. These angle cosines were coefficients of the matrix rotation $R_{f,c}$ of $t_{f,c}$.



$$R_{f,c} = \begin{bmatrix} x_2.x_1 & x_2.y_1 & x_2.z_1 \\ y_2.x_1 & y_2.y_1 & y_2.z_1 \\ z_2.x_1 & z_2.y_1 & z_2.z_1 \end{bmatrix} \quad (9)$$

RESULTS

Jaw opening

Figure13 shows the displacements in the sagittal plane of the right condylar centre (A) and of the inter-incisor point (B). The displacement of the condylar centre reproduces the articular eminence profile.

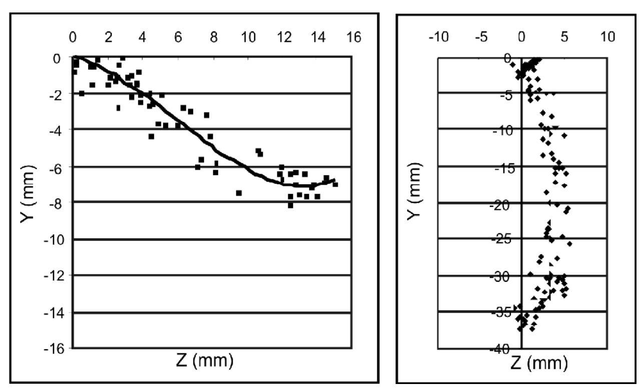


Figure 13: Displacements in the Sagittal Plane of the Right Condylar Centre (A) and of the Inter-incisor Point (B)

On the figure 14, the curve represents the mandible rotation around the transverse axis as a function of the translation of the condyle centre along z-axis. At the onset of the jaw opening, it shows that no pure rotation occurs and that rotation and translation are concomitant.

Lateral Movement

During the lateral movement of the mandible, the two condyle centres have contradictory displacements. Figure

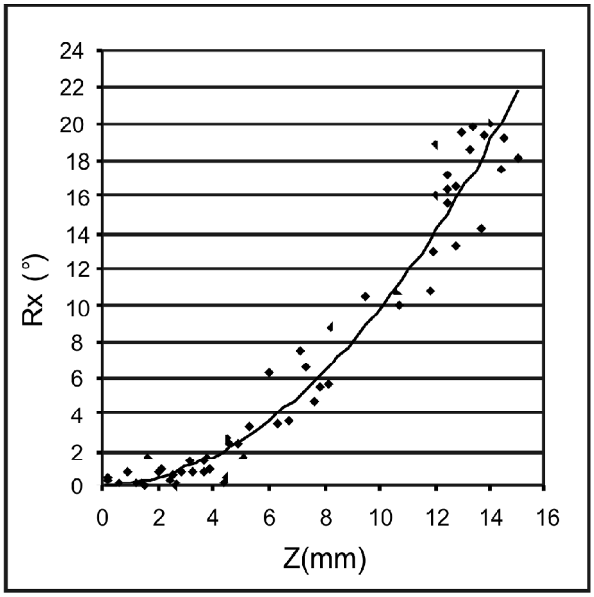
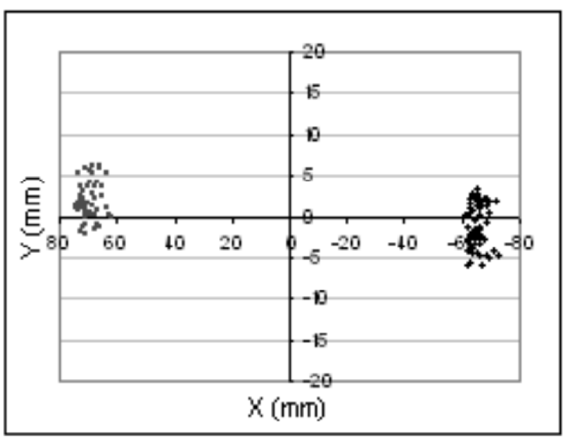


Figure 14: Rotation Around the Transverse Axis Versus Translation along z-axis

15 shows the centre displacements in the frontal plane (A) and in the horizontal plane (B) when the mandible moves to the left side. The left centre moves up and down while the right one goes down and advances, describing the temporal profile.



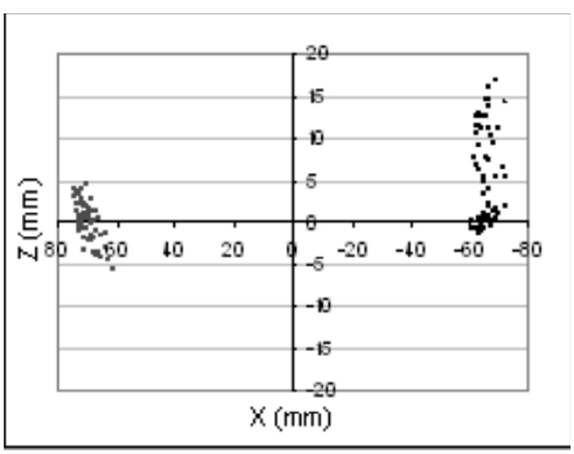


Figure 15: Displacements in the Frontal Plane (A) and in the Horizontal Plane (B)

DISCUSSION AND CONCLUSION

The paper presents the experimental protocol used to measure the 3D displacements of the mandible. The main particularity of the work is to identify the movements in an anatomical reference. The choice of reference was oriented to the camper's coordinate system which ensures repeatability and robustness for comparing results between subjects. We propose that Camper's coordinate system become a standard coordinate system for jaw motion analysis.

The first results obtained with an asymptomatic volunteer are presented for an open-close movement and a lateral movement. Describing the displacements of the condyle centre and of the inter-incisor point in the sagittal, frontal and horizontal planes, these results validate the protocol.

One more contribution concerns the presentation of the relation between rotation and translation during opening movement in a graph; the result confirms the fact that the rotation and translation are concomitant. A statistical study becomes now necessary.

The uncertainty remains under 8% which means that the maximal error value is 1 mm when the displacement reaches 22 mm [19].

One can sometimes observe that interference between the two gutters may occur. Because the skin does not move with respect to the cranium, this difficulty can be bypassed using a kind of helmet fixed onto the cranium instead of the upper gutter.

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This study would not have been possible and validated without the first volunteer's contributions.

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Annex: Transformations

Transformation between two coordinate systems

Let S_1 and S_2 be two coordinate systems.

Let M be a point :

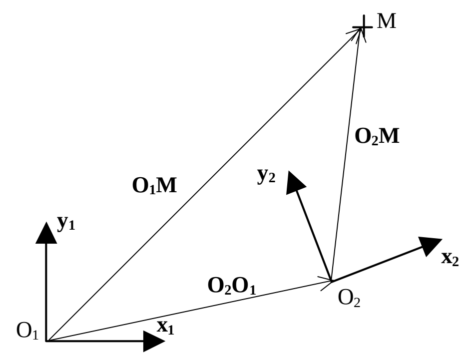

Figure: Transformation between Two Coordinate Systems

The transformation function was denoted $t_{1,2}$.

$$[\mathbf{O_2M}]^{B2} = t_{1,2}\,([\mathbf{O_1M}]^{B1}) \quad \text{(a1)}$$

Inverse Transformation

The inverse transformation function was denoted $t_{1,2}^{-1}$.

$$[\mathbf{O_1M}]^{B1} = t_{1,2}^{-1}\,([\mathbf{O_2M}]^{B2}) = t_{2,1}\,([\mathbf{O_2M}]^{B2}) \quad \text{(a2)}$$

Composition of Transformations

Let S_3 be a third coordinate system.

The composition of transformation functions was denoted:

$$t_{1,2} = t_{2,3} \circ t_{1,2}. \quad \text{(a3)}$$

Coordinate System Associated to the Targets of a Plate

Let S_m be the measurement coordinate system fixed to the calibration box.

Let points T_i be the target centres. The coordinates of each target centre was known in S_m.

Let S_p be the coordinate system associated to the plate (Figure 4). Then, according to the definitions,

$$\mathbf{O_mO_p} = (\mathbf{O_mT_1} + \mathbf{O_mT_2} + \mathbf{O_m})/3 \quad \text{(a4)}$$

$$\mathbf{z_p} = (\mathbf{T_1T_2} \times \mathbf{T_1T_3})/|\mathbf{T_1T_2} \times \mathbf{T_1T_3}| \quad \text{(a5)}$$

$$\mathbf{x_p} = \mathbf{T_3T_2}/|\mathbf{T_3T_2}| \quad \text{(a6)}$$

$$\mathbf{y_p} = \mathbf{z_p} \times \mathbf{x_p} \quad \text{(a7)}$$

As the coordinates of the points T_1, T_2 and T_3 were known in S_m, the coordinates of the vectors $\mathbf{O_mO_p}$, $\mathbf{x_p}$, $\mathbf{y_p}$ and $\mathbf{z_p}$ were also determined in S_m.

Let M be a point.

Then, the transformation described in the annex according to equation (a1) established,

$$[\mathbf{O_pM}]^{Bp} = t_{m,p}\,([\mathbf{O_mM}]^{Bm}) \quad \text{(a8)}$$

where $t_{m,p}$ was computed from $[\mathbf{O_mO_p}]^{Bm}$ (a4), $[\mathbf{x_p}]^{Bm}$ (a6), $[\mathbf{y_p}]^{Bm}$ (a7) and $[\mathbf{z_p}]^{Bm}$ (a5).

Camper's Coordinate System Associated to the Morphologic Points

Let S_c be the Camper's coordinate system.

Let N, L and R be the morphologic points,

- N nasal point,
- L left condyle centre,
- R right condyle centre.

According to the definitions,

$$\mathbf{O_{up}O_c} = (\mathbf{O_{up}L} + \mathbf{O_{up}R})/2 \quad \text{(a9)}$$

$$\mathbf{y_c} = (\mathbf{NL} \times \mathbf{NR})/\|\mathbf{NL} \times \mathbf{NR}\| \quad \text{(a10)}$$

$$\mathbf{x_c} = \mathbf{RL}/\|\mathbf{RL}\| \quad \text{(a11)}$$

$$\mathbf{z_c} = \mathbf{xc} \times \mathbf{yc} \quad \text{(a12)}$$

According to equation (a1),

$$[\mathbf{O_cM}]^{Bc} = t_{up,c}\,([\mathbf{O_{up}M}]^{Bup}) \quad \text{(a13)}$$

where $t_{up,c}$ was computed from $[\mathbf{O_{up}O_c}]^{Bup}$ (a9), $[\mathbf{x_c}]^{Bup}$ (a11), $[\mathbf{y_c}]^{Bup}$ (a10) and $[\mathbf{z_c}]^{Bup}$ (a12) (Figure 8).