International Journal of Computational Vision and Biomechanics
Vol.1 No. 2 (July-December, 2015)

MUK PUBLICATIONS
Open Access Publisher

Road Extraction from High-resolution Satellite Images Using Level Set Methods

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This is an extended version of our paper presented in the CompIMAGE2006. A road extraction algorithm will be discussed through proper mathematical modeling and the applications of the level set methods. We define four speed models to first extract an initial contour, then remove the leakages, eliminate noises and improve the smoothness until it finally reaches the right position of road edges. The proposed method utilizes the characteristics of roads in the high-resolution satellite images and can offer a better and more complete road network. Through using the level set methods, topological changes during the evolution can be handled in a natural way, and its initialization procedure is simple with no need of priori topology information. Edge detection and region detection can be accomplished simultaneously. Several numerical experiments are presented to further demonstrate the correctness of our method.

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1. INTRODUCTION

Road information contained in the satellite images is very important for the update of GIS (Geographic Information System). Many efficient and effective methods like the ones introduced in [1] and [2] have been proposed to extract roads from images with low resolution. While as the developments of remote sensing technologies, people nowadays are confronted with more and more high-resolution satellite images. Processing these images has become the new research focus. Meanwhile, due to some assumptions made by the former extraction methods, such as the roads should have linear structures and negligible widths, most of these methods are not applicable to the high-resolution images. Researchers from different fields with distinct applications begin to concern the information retrievals from high-resolution satellite images.

Compared with the low-resolution images, roads in the high-resolution images have certain width and form a much more complex network. Therefore, extraction process is a little similar to the procedure of shape detection. Among the methods introduced in recent years, models using the snake method and morphological methods are quite popular. For example, Péteri and Ranchin[3] combined snake method with a multi-resolution analysis to do the extraction. Fua and Leclerc[4] proposed a double snake method to handle the extraction; Gruen and Li[5] proposed the LSB-Snake method; Zhang C.S, *et al.*[6] and Zhu C.Q, *et al.*[7] used the morphological methods to perform the extraction. However, some characteristics of the two methods decrease the feasibilities of their applications in this field. The main drawback of the morphologic method is that we can hardly exclude objects with similar intensities as roads', even when they are apparently not connected with roads. Besides, using structuring elements in the morphological operations may break the smoothness of the contour; the snake method has the difficulty of obtaining the prior topology information of the roads, and its requirement that initial road network must be put in a position near the roads is a great constrain to its flexibility. Moreover, when the road network becomes complex, it is difficult to define a proper energy function. While these mentioned problems could be easily handled through using the level set methods.

The level set method was first introduced by Osher and Sethian[8] to track the interface of a closed curve moving in a direction normal to itself. This method has many excellent characteristics like the ability of naturally handling topological changes through implicit expression of curves and high computing efficiency through using the narrow band method or the fast marching method. Therefore, the level set methods have been widely applied in many areas, especially in the image processing during the last ten years. Our paper makes the attempt to apply this method to extract roads from the high-resolution satellite images. We also notice that other effective studies in [9, 10] also successfully applying the level set methods to extract roads.

2. EXTRACTION PRINCIPLE

2.1 *Curve Movement and Contour Extraction*

The contour of an object could be delineated by curves. Suppose these curves are the final status after moving away from their initial positions, then we can extract the desired contours through moving the initial curves in a proper way. Therefore, defining right speed models and finding a proper method for tracking become important for the extraction. Unfortunately, classical methods used for tracking propagation are not convenient when dealing with the movement during which the topologies of curves might change. As the discussions in the pervious section, the level set methods can handle this challenge in a natural way. To facilitate the later discussion, we first give a brief introduction to three most used level set methods in the following sections.

2.2 *Standard Level Set Method*

Level set method, which was first proposed by Osher and Sethian[8], is a numerical method used to solve the propagation problems like crystal growth and combustion. The central idea of the level set method is to represent the evolving curve as the point set $\{x(t) \mid \phi(x(t),t)=0\}$, where ϕ is called the level set function, which means to embed the curve into a higher dimension space. Thus when we differentiate the two sides of the level set, we get the level set equation:

$$\phi_t + F \mid \nabla \phi \mid = 0 , \tag{1}$$

$$\phi(x(t),t) = 0 . \tag{2}$$

where F is the speed of curve. To propagate the level surfaces, we initialize the level set function using the signed distance function, and the positive (minus) sign is chosen if the point is outside (inside) the initial curve. The intrinsic geometric properties of the front can be calculated through the level set function. For example, the normal vector and the curvature can be expressed as:

$$\vec{n} = \nabla \phi / \mid \nabla \phi \mid , \ \kappa = \nabla \cdot \vec{n} = \nabla \cdot (\nabla \phi / \mid \nabla \phi \mid) .$$

Osher and Sethian[8] have discussed that numerical schemes, which are used to solve the hyperbolic partial differential equations and satisfy the hyperbolic conservation law, can be borrowed to solve the level set equation (1), and the implicit calculations of the intrinsic geometric properties of curve could greatly enhance the stabilities of numerical methods. Therefore, level set methods can handle the topological changes easily and give the correct solutions that satisfy the entropy condition[7,11]. We give an example of the evolution process of two curves using the level set method in Figure 1.

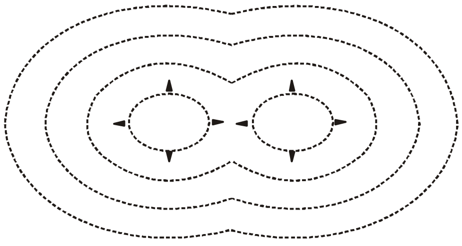


Figure 1: Evolving process of two curves

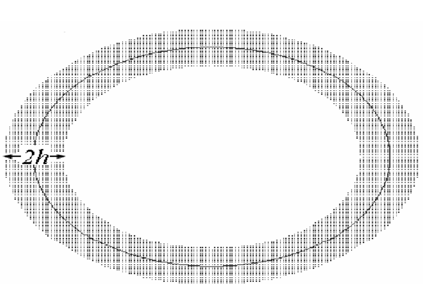


Figure 2: A narrow band region with width *2h*

2.3 *Narrow Band Method*

The standard level set method suffers a draw back of low computing efficiency, so Chopp[12] proposed the narrow band method to handle the problem. The main idea of the narrow band method is to construct a narrow band near the current curve and constrain the computations inside it. Once the front moves near the edge of the band, we rebuild a new band and continue the computation. Figure 2 gives us a simple example of a *2h* width narrow band of an elliptic curve, in which we could constrain the computations during a certain time period. Although the re-initialization step of the narrow band method is still time-consuming, this method does save the computing labor especially when the curve moves in a large region.

2.4 *Fast Marching Method*

When the speed function F only depends on position and does not change its sign, the positions of curve can be gained by solving the Eikonal equation:

$$F \mid \nabla T \mid = 1 . \tag{3}$$

$$\Gamma(t) = \{(x,y) \mid T(x,y) = t\} , F > 0. \tag{4}$$

where $T(x, y)$ is the arrival time as the curve crosses the point (x, y). One numerical scheme used to solve the equation (3) is:

$$[\max(D_{ij}^{-x}T,0)^2 + \min(D_{ij}^{+x}T,0)^2 + \max(D_{ij}^{-y}T,0)^2 + \min(D_{ij}^{+y}T,0)^2] = \frac{1}{F_{ij}^{\ 2}}. \tag{5}$$

where D^- is the backward difference operator; D^+ is the forward difference operator. Having observed the upwind difference structure of (5), Adalsteinsson and Sethian[13] introduced the fast marching method to improve the computing efficiency. The fast marching method can reduce the total operation count to $O(N \log N)$ and avoid iterative calculations. Therefore, this method has much higher computation efficiency than the fore mentioned numerical schemes.

3. EXTRACTION ALGORITHM

In most times, due to the complexity of the background of high-resolution satellite images, the smoothness of road contour will inevitably be disturbed by the noises and discontinues caused by cars and shadows. However, the continuity, smoothness and connectivity of roads are critical for further analysis and applications after the extraction. Therefore, extraction algorithm must guarantee the elimination of those unexpected disturbance and recover the original smooth contour as much as possible. Besides, to better process the road information, we need a method that can extract both the road contour and the road region. The level set methods offer us a good way to do so. The extracted contour curve represents the road edge with its inside representing the road region. Given the connectivity among roads, we establish four speed models to handle the retrieval problem. Our method includes three phases: we first extract an initial contour of the roads through the fast marching method, then remove the leakages caused by the blurred parts of images, and finally eliminate the noises and discontinuities.

3.1 *Extract an Initial Contour of the Roads*

3.1.1 Limitations of the formal speed model

This phase is to extract an initial contour of the roads. Since the roads have certain widths, the extraction process is similar to the shape detection. Malladi, *et al.*[14-16] first used the level set method to perform the edge detection in the medical images. They put an initial curve inside the object and let it move with speed $F_0(x,y) = 1/(1+|\nabla(G_\sigma * I(x,y))|)$ until the computation time satisfies the stop criterion. However, using the above speed model F_0, the curve will reach the edge of images if the computation time is long enough, which means the edges of objects are dependent on computation time. Consequently, due to the inhomogeneous distribution of image intensity, improper placement of the initial curves might lead to the result that while one side of the curve hasn't reached the edge of roads, other sides might have massively leaked into other objects' inside. We give an example of the aforementioned problem in Figure 3. In Figure 3(b), if the initial curve was placed near the left side where the intensity gradient is large enough to block the contour, the edges can be gained through defining proper stop criteria. But if the initial curve is placed like in Figure 3(c), the right bottom side had leaked while the other sides hadn't reached the right positions. Continued computation will lead to massive leakages. This case must be avoided especially for the high-resolution satellite images which usually contain huge amount of pixels and cover large range areas.

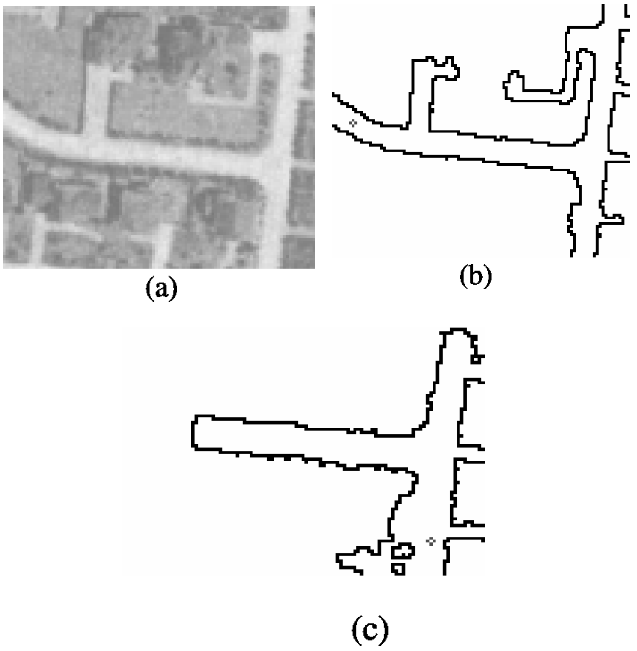


Figure 3: Different results caused by different positions of initial curve: (a) original image; (b) initial curve near the left edge; (c) initial curve near the right-bottom

3.1.2 Procedure

We define the speed function as follows:

$$F_1(x,y) = \begin{cases} 1 & \text{if} \quad |I(x,y) - Ave(x,y,k)| \le Var(x,y,k) \cdot S_{threshold} \\ \varepsilon & \text{if} \quad |I(x,y) - Ave(x,y,k)| > Var(x,y,k) \cdot S_{threshold} \end{cases}. \tag{6}$$

where ε is a small positive real number, Ave(x, y, k) and Var(x, y, k) are respectively the average and the variance of intensity in a disk with radius $R = k$ pixels centered around the point (x, y), $S_{threshold}$ is a positive value used for a threshold. Use the image pixels as the mesh points and put the initial curves to an area in roads not highly affected by the noises. Let the initial curve move with speed F_1. Stop the calculation when the values of speed function on the curve points are all equal to ε.

3.1.3 Explanation

Since we only concern the final position of the evolving curves, we could combine the speed model with a threshold. While like the discussions in [17], the value of speed function on a single point should use its local information to better decrease the possibility of leakage. This requirement is especially important for the satellite images in which different parts have appreciable local characteristics. Therefore, we can use $S(x,y,k) = \dfrac{I(x,y) - Ave(x,y,k)}{Var(x,y,k)}$ to normalize the image intensity on a single point (we define the value of S to be zero when the nominator is equal to zero). The parameter k plays a scale effect. For k = 1, the criterion will have a similar influence of intensity gradient; the increasing of k will combine more pixels to determine the speed value. Given the possibility of homogeneity of certain areas, the *Var*(x, y, k) might be equal to zero, so we revise the judging criterion as the presented in (6). We can also use the attraction force field introduced by Tabb and Ahuja[18] or statistics methods that can utilize regional information to make better decisions.

The number of initial curves used for the fast marching method can be one or more. Theoretically, due to the definition of speed function and the ability of handling topological changes of the fast marching method, our model assures that as long as the initial curves are placed in the road regions not too much affected by the noises, the final results will be the same. The requirement that the initial curve should not be put to a noisy position is to assure that the initial extraction will be correct. We calculate the average intensity of the points on the initial curve and denote the value as I_0, which will be used in the next stage. In the ideal condition, one initial curve will be enough no matter what shape it is (for simplicity, a single pixel point is also a preferred choice). While in practice, we usually use several initial curves and distribute them in different roads, because some roads might be so seriously disturbed by noises that the evolving curves might not be able to move into these regions under the speed F_1. More initial curves mean higher possibility of getting a more complete road network. Even if some roads extracted in this phase are not connected with others due to the noises, we can reconnect them through the later processing. Therefore, the dependency to the initial positions of the curves is greatly reduced. As the speed function F_1 always remain non-negative, we can use the fast marching method to get the arrival time, which allows us to get a high computing efficiency.

3.2 *Remove the Leakages of the Contour*

3.2.1 *Procedure*

In order to get a complete road network, the $S_{threshold}$ defined in 3.1 should not be too small. Consequently, those blurred parts of road edges, which were caused by the influence of weather or objects on the roads, cannot block the contour. In most cases, due to the high expenses and real time requirement, we cannot expect the lost information be transmitted again. This is a problem that almost every extraction algorithm will encounter when dealing with different kinds of images. For the high-resolution satellite images, it is the gradual changes of intensity on the blurred edge points that causes the leakage. However, we can easily notice that regions enclosed by the leakage parts have considerable different image intensity to the road regions'. In other words, although the gradual changes can decrease the intensity gradient among nearby pixels, leading to the result that the value of function S is not large enough to perform the function of blocking the contour, it cannot change the fact that intensity differences among not neighbored pixels are considerable. Therefore, we can use the intensity difference to draw the leakage parts back to the reasonable positions.

3.2.2 *Procedure*

Define the speed function as:

$$F_2(x,y)=\begin{cases} -1 & \text{if } Ave(x,y,k)\notin[I_0-\delta,I_0+\delta] \\ F_0(x,y) & \text{if } Ave(x,y,k)\in[I_0-\delta,I_0+\delta] \end{cases}\cdot \qquad (7)$$

where $F_0(x,y)=1/(1+|\nabla(G_\sigma * I(x,y))|)$ and Ave(x, y, k) has the same definition in (6). Let the curve move with speed F_2. Stop calculating when the curve achieves a steady status.

3.2.3 *Explanation*

Given the intensities distribution of road region, together with the requirements about the initial position, we can use I_0 as the average intensity of road region and $[I_0-\delta, I_0+\delta]$ as the range of road intensity. The leakage parts with intensities not belonging to the given range should contract to the right positions; while the rest parts should move with the speed F_0 to reach the accurate positions of the road edge. Speed model F_0 is used there because using it could decrease certain influence of weak noises[6, 11]. Under the speed model F_2, the inward and outward movements of different parts will let the curve achieve a steady state after the leakages contract to their reasonable positions. We could gain the right contour even the position of initial curve is placed like in Figure 3(c). When the curve achieves the steady state, further calculation will not change the status of the contour. Leakage parts will be diminished and most parts of the contour will approach to the right edge of roads.

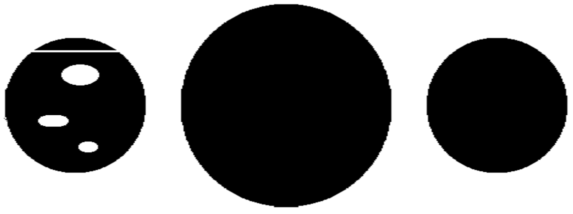
We can initialize the level set function using the contour curve gained in 3.1 and use the narrow band method to get the values of the level set function at the time t. The narrow band method guarantees the right handling of topological changes and improves the computing efficiency in this phase. The state of stability can be achieved quickly because the curves have arrived at the neighborhood of the edges after the movement of phase 3.1.

3.3 *Eliminate the Noises*

3.3.1 *Curve Movements used in the Extraction*

After the movements of 3.1 and 3.2, most parts of the curve have reached the right edge of roads. But the noises, such as the holes and discontinues, still exist in the contour. Keaton and Brokish[9] once proposed to eliminate noises using the speed model $F = \min(\kappa, 0)$. While holes and discontinuities inside the contour do not always have negative curvatures, for example, the curvature of the breakages caused by the crosswalk is zero. So under this speed, noises sometimes cannot be completely removed and the smoothness of the parts with positive curvatures cannot be improved. Besides, some parts with large curvatures might move drastically. The status of the curves might become unstable and hard to control.

What we use to handle this problem is based on the fact that although the noises are in the outside of curves, they are enclosed by the contour. So we can let the curves first move outward and then move inward to eliminate these noises. The entropy condition ensures that once those noises enter the inside of the curves during the outward movement, they will not appear when the curves move inward. We explain this principle in Figure 4. We color the insides of curves with black. The initial curves in Figure 4(a) contain holes and are two distinctive curves with discontinuities between them. When the curves move outward, those holes



(a) (b) (c)

Figure 4: Evolving process of the multi-connective curves: (a) original curves; (b) curves after expansion; (c) curves after contraction

inside the below curve are covered by the curve, and the two curves merged into one. The information of holes and discontinuities between the two curves will be totally lost after certain movements like we presented in Figure 4(b). When the merged curve moves inward, a single curve will be gained with no discontinues and no holds inside it. For the level set methods, this application can be naturally realized through the re-initialization procedure of narrow band method. As level set methods satisfy entropy condition, once the curve are merged into one curve, the information before mergence will be lost, and when the curve moves inward, the right solutions will be gained.

On the other hand, to improve the smoothness of road contour, we need to recover the original smooth contour as much as possible. We first observe the evolving processes of two kinds of movements: the curve moving outward with speed $F_4 = 1 - \varepsilon\kappa$ and moving inward with speed $F_5 = -1 - \varepsilon\kappa$ in a direction normal to itself, where κ is the curvature; ε is a small positive parameter. We divide the current curve into three categories: the straight parts with curvature $\kappa = 0$, the convex parts with $\kappa > 0$ and the concave parts with $\kappa < 0$. From the definition of F_4, the concave part will have a larger speed than the other parts and the convex part will have a smaller speed. This speed definition means the oscillations of the curve will be decreased. A case in point can be seen in Figure 5. Therefore, the moving tendency is to let the curve become more flat with less oscillation. Similar conclusions can be drawn when the curve move inward under the speed model F_5. In fact, Sethian[19] once proved that once $F_\kappa |_{\kappa=0} \leq 0$, the totally oscillation of curve will decrease as the curve moves on. For the above two speed models, both the irregular parts become smoother after the corresponding movements. The irregularity of these parts can be greatly decreased and some parts with large curvatures will be removed.

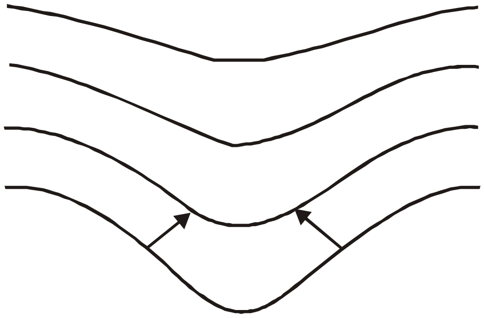


Figure 5: Evolving process of the concave part under speed F_4.

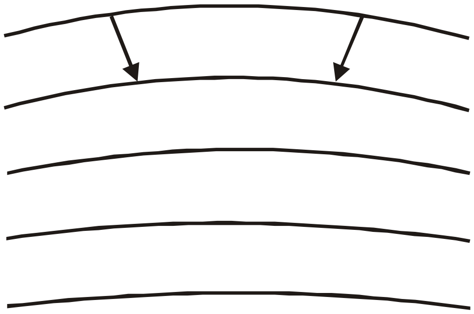


Figure 6. Evolving process of the convex part under speed F_5

3.3.2 Procedure

Move the contour outward with speed F_4 1 – $\varepsilon\kappa$ until the time t_0, and then move the expanded curve inward with speed $F_5 = 1 - \varepsilon\kappa$ until the time $2t_0$.

3.3.3 Explanation

No structuring element is needed there. Noises can be eliminated through the outward movement, and the smoothness of the edges will be improved rather than be disturbed. The value of ε can be seen as a scale parameter or a penalty parameter. In order to achieve the goal of eliminating the noises, improving the smoothness and avoiding the problem of Keaton's speed model, we only need to guarantee that the selection of ε wouldn't change the sign of speed function so that the moving tendency keeps uniform. The value of t_0 mainly depends on the size of objects in the satellite images and the disturbance of the contour. Small value of t_0 may lead to the movements unable to completely eliminate the noises; while large value of t_0 might lead to a wrong result that unconnected roads are connected through the merges of curves and the entropy condition. According to our practices, the range from 5 to 15 is a preferable choice. After this phase, curves with dynamic changes will be smoothed, and the breaks among roads can be reconnected. We can get a more complete and smoother curve in this phase.

3.4 *Full Procedure*

We give a brief description of our algorithm as follows:

Step 1: Put the initial curves inside the road. Let the curves move outward under the speed model F_1. When all the speed values on the curve point equal to ε, go to Step 2.

Step 2: Let the curve move with speed F_2. When the curve achieves a steady status, go to Step 3.

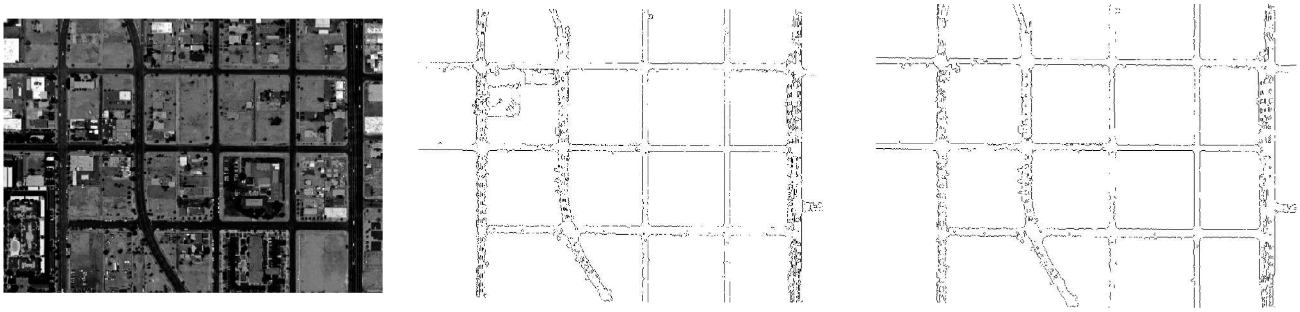
Step 3: Move the curve outward with speed F_4 until the time t_0, then move the curve inward with speed F_5 until the time $2t_0$.

All the variables and their selection rules in the above procedure can be found in their corresponding sections. The Step 1 is to get an initial contour; The Step 2 is to eliminate the noises; The Step 3 is to remove the noises and improve the smoothness. The final curves after the Step 3 are the road contours with their insides representing the road region.

4. NUMERICAL EXPERIMENTS

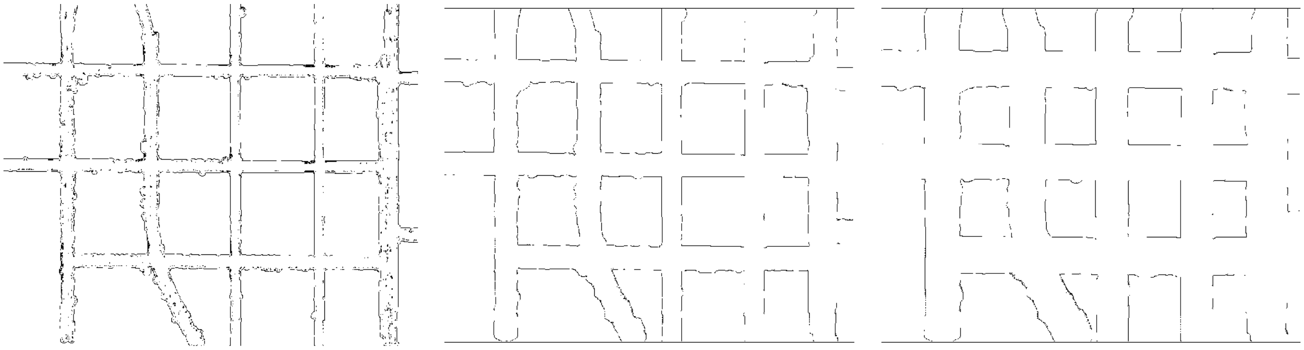
We test our model using several Landsat 7 satellite images. In this section, we give a detailed illustration of the extraction procedure using the two images in [20], and make a contrast with the final results. We also give another example in Figure 13.

For Figure 7(a), Figure 8 and Figure 9 give detailed evolving processes of the contour curve moving outward and inward. For Figure 11(a) and Figure 13(a), Figure 11(b)-(d) and Figure 13(b)-(d) give the brief processes of evolving. Through Figure 8 and Figure 9, we can easily see that as the movement of the curve, noises and oscillations are gradually eliminated. In Figure 11 and Figure 13, we can see that discontinues caused by noises among roads are re-connected through outward and inward movements (For example, the roads located in the left part of Figure 11). To better exhibit the results, we color the road regions white in Figure 10(a), Figure 12(a) and Figure 13(d). Final results using the proposed model are appreciably more regular and reasonable than the initial contours. In Figure 10 and Figure 12, we make a contrast of our final results with the ones we presented in [20]. We can see that better results can be gained using the new speed models. After getting the road region, we can turn the extracted road regions into binary-valued images and extract the linear structure network of roads. We present the linear structure network with original images in Figure 14 using the skeleton method introduced in [21].



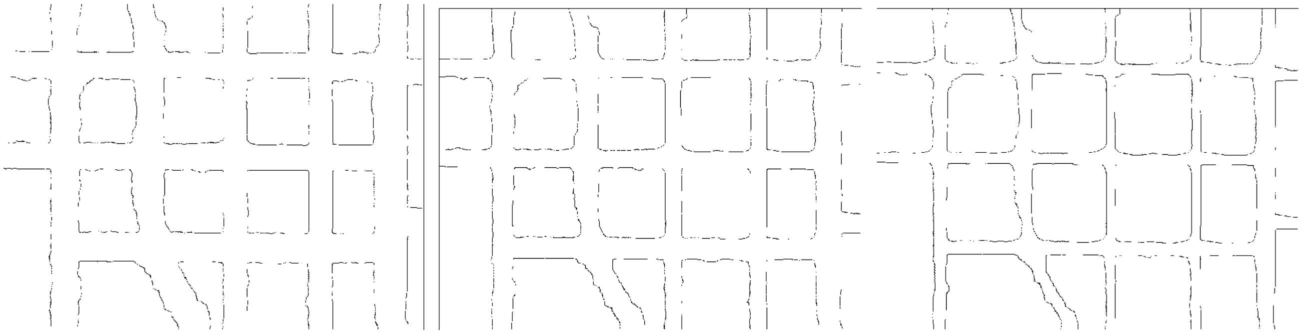
(a) (b) (c)

Figure 7: Extraction process: (a) original image (1024 × 743); (b) initial contour; (c) contour without leakages



(a) (b) (c)

Figure 8: Extraction process of expanding



(a) (b) (c)

Figure 9: Extraction process of contracting

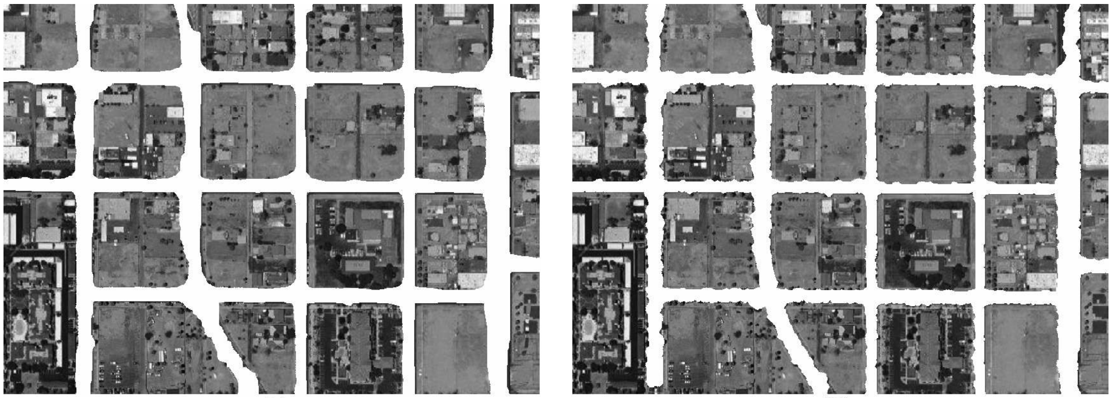
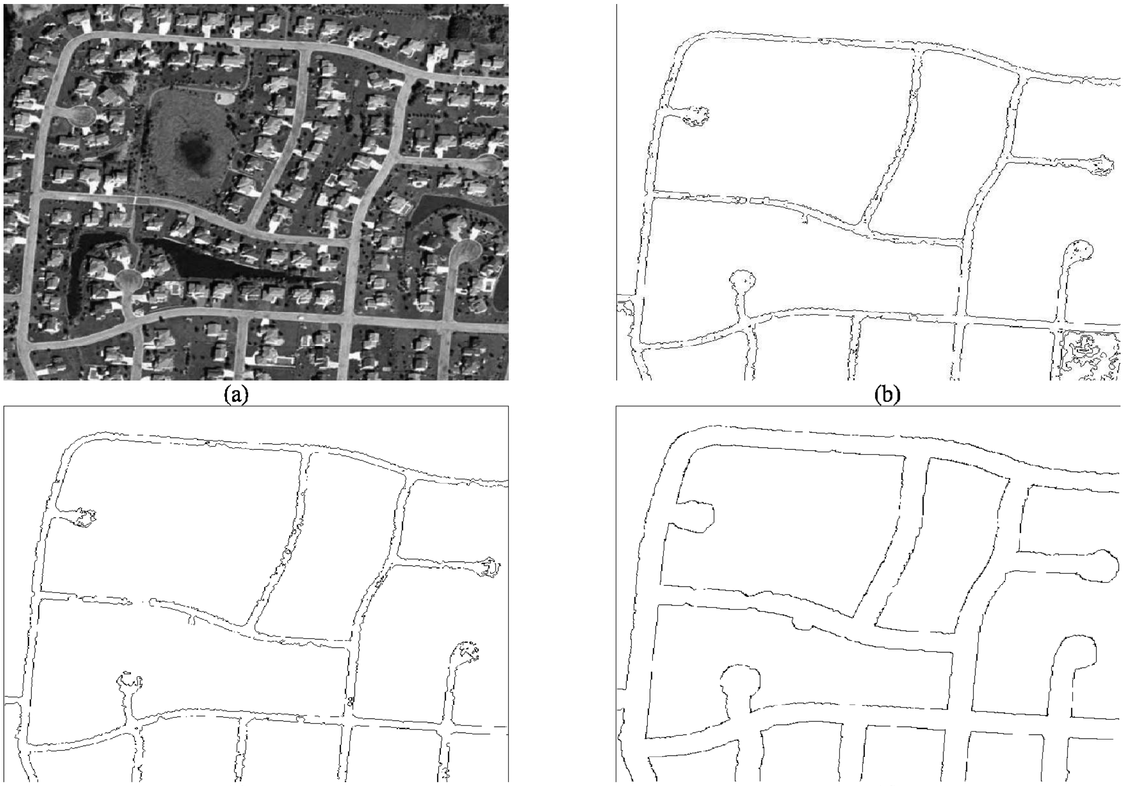

(a) (b)

Figure 10: Contrast between final results: (a) current algorithm; (b) former algorithm.

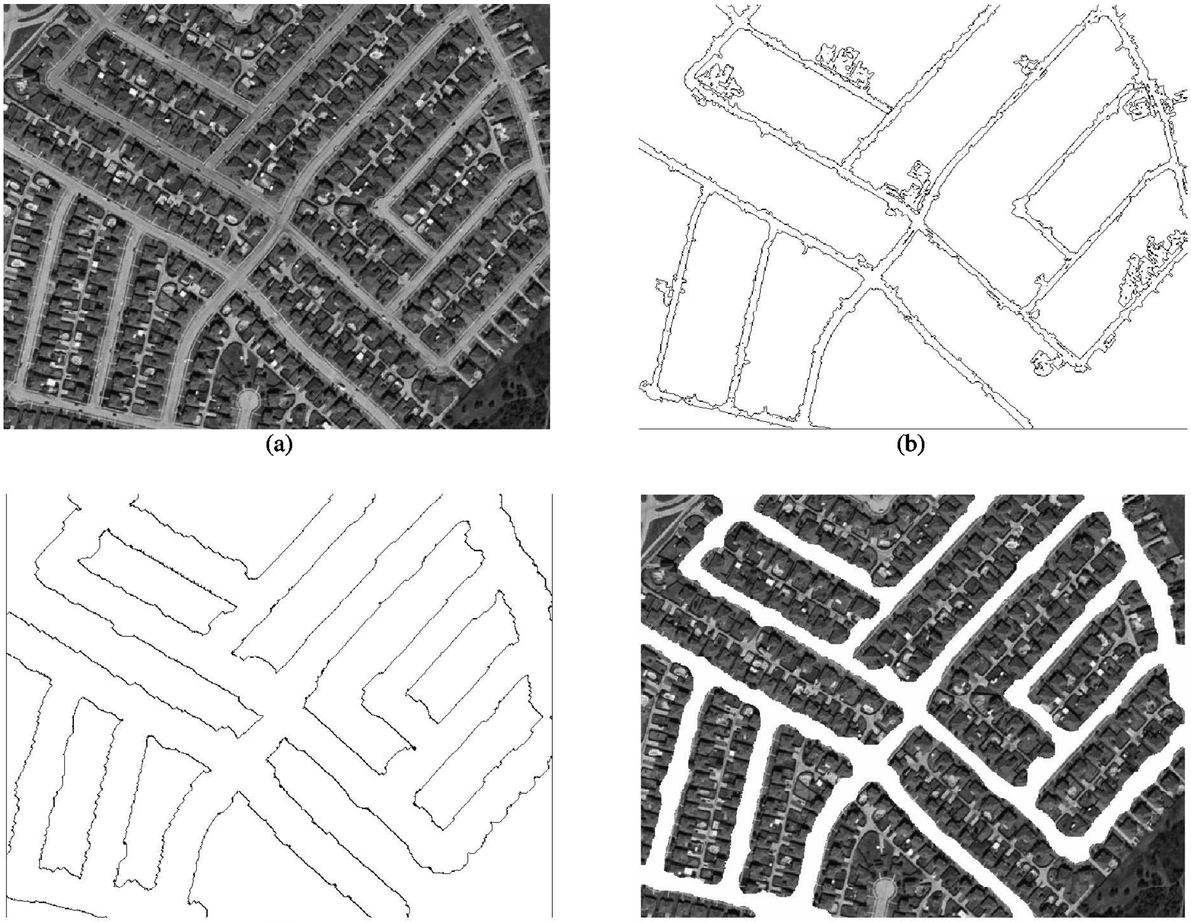

(a) (b)

(c) (d)

Figure 11: Extraction process: (a) original image (667 × 631); (b) initial contour; (c) contour after eliminating the noises; (d) contour after the outward movement


(a) (b)

Figure 12: Contrast between final results: (a) current algorithm; (b) former algorithm



(a) (b)

(c) (d)

Figure 13: Extraction process: (a) original image (686 × 536); (b) initial contour; (c) contour after the outward movement; (d) extracted road region.

(a) (b) (c)

Figure 14. Linear structure of road

5. CONCLUSION AND FUTURE WORK

Through using the level set methods, when the road has branches, the curves split; when the roads are crossed, the curves merge. The topological changes are easily handled. Leakage parts are removed and the noises inside the curves are eliminated. The smoothness of the contour is greatly improved and the parts with large curvatures diminished after the movements.

Roads in the high resolution satellite images usually form a complex network. Through using level set methods, we can avoid the definition of energy functions and do not need an initial guess of road positions. Therefore, our initialization procedure is simple and has much looser constrictions about the initial contour. Several speed models are proposed to handle the extraction, remove the leakages, eliminate the noises and recover the smoothness of the contour, during which we utilize the entropy conditions and high efficiencies of the level set methods. In the final results, both the road contour and the road regions can be gained after the three phases of movements. The numerical experiments have further demonstrated the correctness of our proposed methods.

The drawbacks of our methods can be concluded into two points: First, under certain circumstances, our method may misjudge the objects that are connected with roads and have similar intensities with roads'. Once the contours enter such regions, the second phases and the third phases cannot assure that all the leakages will be removed. The other drawback is that our method is a semi-automated one, which means selecting proper parameters sometimes will be a time-consuming thing. Our future work aims to improve the automation of the method and enhance its general applicability to different conditions.

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