Sampled-Data Control for Fuzzy Time-Delay Systems under Time-Varying Sampling

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Abstract: *Sampled-data control for fuzzy time-delay systems is discussed. When we apply a zero-order control input, the closed-loop system with such a zero-order state feedback control becomes a system with time-varying delay. Here, we take a delay system approach to the sampled-data control problem. First, stability conditions for the closed-loop system are given in terms of linear matrix inequalities (LMIs). Such conditions are derived by using Leibniz-Newton formula and free weighting matrix method for fuzzy time-delay systems under the assumption of time-varying sampling. Then, a design method of sampled-data stabilizing controller for fuzzy time-delay systems is proposed. Numerical examples are given to illustrate our sampled-data control design.*

Keywords: *Takagi-Sugeno fuzzy systems, Time-delay systems, Sampled-data control, Stabilization.*

1. INTRODUCTION

In the past two decades, a fuzzy system approach for nonlinear control systems has been considerably active because Takagi-Sugeno fuzzy system is one of the most systematic representations to describe nonlinear systems. In fact, nonlinear control design and analysis based on Takagi-Sugeno fuzzy systems have provided successful results. Since the work by Tanaka and Sugeno in [4] where stability analysis and stabilizing controller design of Takagi-Sugeno continuous-time system were first made, various relaxed stability conditions have been given, and stabilization methods based on those conditions have been proposed (for example, [5], [6] and references therein.). Recently, theory has been extended to a class of fuzzy systems with time-varying delays. Sufficient stability conditions for systems with time-varying delays were obtained in terms of LMIs (for example, [7], [8]).

These results are mainly on continuous-time fuzzy systems. It is, however, difficult to implement continuous-time controllers directly to practical nonlinear systems by using digital devices. To overcome such difficulty, we look for sampled-data control. A class of sampled-data fuzzy systems has been considered in [3] where an approach based on a jump system, which is the form of continuous- and discrete-time model, has been employed. Sampled-data stabilization of fuzzy systems was considered in [1] and [9] where sufficient stability conditions for a fuzzy system with sampled-data inputs were obtained and a design method of a stabilizing sampled-data state feedback controller was proposed. The approach taken in [1] and [9] was closely related to a time-varying delay system approach because a fuzzy system with zero-order sampled-data control input results in the closed-loop system with time-varying state delays. Furthermore, sampled-data control for fuzzy time-delay systems was considered in [2] where different membership functions for the control were introduced.

In this paper, we consider the sampled-data control for Takagi-Sugeno fuzzy time-delay systems. We take an input delay approach to such a control design problem. When we consider the zero-order control to a fuzzy time-delay system, the closed-loop system becomes a fuzzy system with multiple time-delay. An appropriate Lyapunov function, together with Leibniz-Newton formula and free weighting matrix method, which are known to reduce the conservatism in stability conditions ([7]), are employed to obtain sufficient conditions that guarantee the asymptotic stability of the closed-loop system. [1] and [9] consider stabilization for fuzzy systems, but their conditions are rather conservative. We develop less conservative delay-dependent conditions and propose a design method of sampled-data state feedback control of a fuzzy time-delay system. A numerical example is given to illustrate a design method of sampled-data state feedback stabilizing controllers for fuzzy time-delay systems.

2. FUZZY TIME-DELAY SYSTEMS

In this section, we introduce Takagi-Sugeno fuzzy time-delay systems. Consider the Takagi-Sugeno fuzzy model, described by the following IF-THEN rules:

IF $\xi_1(t)$ is M_{i1} and ... and $\xi_p(t)$ is M_{ip},

THEN $\dot{x}(t) = A_i x(t) + A_{di} x(t-\tau) + B_i u(t)$

where $x(t) \in \Re^n$ is the state and $u(t) \in \Re^m$ is the control input. τ is an unknown constant that satisfies $0 \le \tau \le \tau_M$ where τ_M is a known constant. The matrices A_i, A_{di} and B_i are constant matrices of appropriate dimensions. r is the number of IF-THEN rules. M_{ij} are fuzzy sets and $\xi_1, \ldots, \xi_p$ are premise variables. We set $\xi = [\xi_1 \ \dots \ \xi_p]^T$ and $\xi(t)$ is assumed to be given or to be a measurable function.

The state equation is described by

$$\dot{x}(t) = \sum_{i=1}^{r} \lambda_i(\xi(t))\{A_i x(t) + A_{di} x(t-\tau) + B_i u(t)\} \quad (1)$$

where $\lambda_i(\xi) = \frac{\beta_i(\xi)}{\sum_{i=1}^{r}\beta_i(\xi)}, \beta_i(\xi) = \prod_{j=1}^{p} M_{ij}(\xi_j)$ and $M_{ij}(\cdot)$ is the grade of the membership function of M_{ij}. We assume $\beta_i(\xi(t)) \ge 0$, $i = 1, \dots, r$, $\sum_{i=1}^{r}\beta_i(\xi(t)) > 0$ for any $\xi(t)$. Hence $\lambda_i(\xi(t))$ satisfy $\lambda_i(\xi(t)) \ge 0$, $i = 1, \dots, r$, $\sum_{i=1}^{r}\lambda_i(\xi(t)) = 1$ for any $\xi(t)$.

We consider the sampled-data control input. It may be represented as delayed control as follows;

$$u(t) = u_d(t_k) = u_d(t-(t-t_k)) = u_d(t-h(t)),\ t_k \le t \le t_{k+1} \quad (2)$$

where u_d is a zero-order control signal and the time-varying delay $0 \le h(t) = t - t_k$ is piecewise linear with the derivative $h(t) = 1$ for $t \ne t_k$. t_k is the time-varying sampling instant satisfying $0 < t_1 < t_2 < \dots < t_k < \dots$. Sampling interval $h_k = t_{k+1} - t_k$ may vary but it is assumed to be bounded. Thus, we assume $h(t) \le t_{k+1} - t_k = h_k \le h_M$ for all t_k where h_M is a known constant.

Our problem is to find a sampled-data state feedback controller that stabilizes the system (1). We consider the following rules for a controller:

IF $\xi_1(t_k)$ is M_{i1} and ... and $\xi_p(t_k)$ is M_{ip},
THEN $u(t) = K_i x(t_k)$, $i = 1, \dots, r$

where K_i are to be determined. Then the natural choice of a controller is given by

$$u(t) = \sum_{i=1}^{r} \lambda_i(\xi(t_k))K_i x(t_k). \quad (3)$$

We represent a piecewise control law as a continuous-time one with a time-varying piecewise continuous (continuous from the right) delay $h(t) = t - t_k$ as given in (2). Thus we look for a state feedback controller of the form

$$u(t) = \sum_{i=1}^{r} \lambda_i(\xi(t_k))K_i x(t-h(t)). \quad (4)$$

The closed-loop system (1) with (4) is given by

$$\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(\xi(t))\lambda_j(\xi(t_k))\{A_i x(t) + A_{di}x(t-\tau) + B_i K_j x(t-h(t))\}. \quad (5)$$

3. SAMPLED-DATA CONTROL

Here we consider the sampled-data control of a fuzzy time-delay system.

3.1. Stability Analysis

First, we make a stability analysis of the closed-loop system (5).

Theorem 3.1. Given control gain matrices K_i, $i = 1, \dots, r$, the closed-loop system (5) is asymptotically stable if there exist matrices $P > 0$, $Q \ge 0$, $R \ge 0$, $Y > 0$, $Z_1 > 0$, $Z_2 > 0$,

$$N_{ij} = \begin{bmatrix} N_{1ij} \\ N_{2ij} \\ N_{3ij} \\ N_{4ij} \\ N_{5ij} \end{bmatrix}, S_{ij} = \begin{bmatrix} S_{1ij} \\ S_{2ij} \\ S_{3ij} \\ S_{4ij} \\ S_{5ij} \end{bmatrix}, M_{ij} = \begin{bmatrix} M_{1ij} \\ M_{2ij} \\ M_{3ij} \\ M_{4ij} \\ M_{5ij} \end{bmatrix}, W_{ij} = \begin{bmatrix} W_{1ij} \\ W_{2ij} \\ W_{3ij} \\ W_{4ij} \\ W_{5ij} \end{bmatrix} i, j = 1, \cdots, r \text{ and } T = \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{bmatrix}$$

such that

$$\Phi_{ij} = \begin{bmatrix} \Phi_{11ij} & h_M N_{ij} & h_M S_{ij} & h_M M_{ij} & \tau_M W_{ij} \\ h_M N_{ij}^T & -h_M Z_1 & 0 & 0 & 0 \\ h_M S_{ij}^T & 0 & -h_M Z_1 & 0 & 0 \\ h_M M_{ij}^T & 0 & 0 & -h_M Z_2 & 0 \\ \tau_M W_{ij}^T & 0 & 0 & 0 & -\tau_M Y \end{bmatrix} < 0, \quad i, j = 1, \cdots, r \quad (6)$$

where $\Phi_{11ij} = \Phi_{1ij} + \Phi_{2ij} + \Phi_{2ij}^T + \Phi_{3ij} + \Phi_{3ij}^T$,

$$\Phi_{1ij} = \begin{bmatrix} Q+R & 0 & 0 & 0 & P \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -R & 0 & 0 \\ 0 & 0 & 0 & -Q & 0 \\ P & 0 & 0 & 0 & \tau_M Y + h_M(Z_1+Z_2) \end{bmatrix},$$

$$\Phi_{2ij} = \begin{bmatrix} N_{ij} + M_{ij} + W_{ij} & -N_{ij} + S_{ij} & -M_{ij} - S_{ij} & -W_{ij} & 0 \end{bmatrix},$$

$$\Phi_{3ij} = \begin{bmatrix} -TA_i & -TB_{2i}K_j & 0 & -TA_{di} & T \end{bmatrix}.$$

Proof: First, it follows from the Leibniz-Newton formula that the following equations hold for any matrices N_{ij}, S_{ij}, M_{ij}, and W_{ij}, the forms of which are given in Theorem 3.1.

$$2\sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(\xi(t))\lambda_j(\xi(t_k))\zeta^T(t)N_{ij}\left[x(t) - x(t-h(t)) - \int_{t-h(t)}^{t}\dot{x}(s)ds\right] = 0, \quad (7)$$

$$2\sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(\xi(t))\lambda_j(\xi(t_k))\zeta^T(t)S_{ij}\left[x(t-h(t)) - x(t-h_M) - \int_{t-h_M}^{t-h(t)}\dot{x}(s)ds\right] = 0, \quad (8)$$

$$2\sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(\xi(t))\lambda_j(\xi(t_k))\zeta^T(t)M_{ij}\left[x(t) - x(t-h_M) - \int_{t-h_M}^{t}\dot{x}(s)ds\right] = 0, \quad (9)$$

$$2\sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(\xi(t))\lambda_j(\xi(t_k))\zeta^T(t)W_{ij}\left[x(t) - x(t-\tau) - \int_{t-\tau}^{t}\dot{x}(s)ds\right] = 0 \quad (10)$$

where $\zeta(t) = [x^T(t)\ x^T(t-h(t))\ x^T(t-h_M)\ x^T(t-\tau)\ \dot{x}^T(t)]^T$. It is also clear from the closed-loop system (5) that the following is true for any matrix T.

$$2\sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(\xi(t))\lambda_j(\xi(t_k))\zeta^T(t)T\left[\dot{x}(t) - A_i x(t) - \right.$$

$$A_{di}x(t-\tau)-B_iK_jx(t-h(t))\big]=0. \quad (11)$$

Now, we consider the following Lyapunov functional:

$$V(x_t)=V_1(x)+V_2(x_t)+V_3(x_t)$$

where $x_t = x(t+\theta)$, $-h \le \theta \le 0$,

$$V_1(x)=x^T(t)Px(t),$$

$$V_2(x_t)=\int_{t-\tau}^{t}x^T(s)Qx(s)ds+\int_{t-h_M}^{t}x^T(s)Rx(s)ds,$$

$$V_3(x_t)=\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)Y\dot{x}(s)dsd\theta+\int_{-h_M}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)(Z_1+Z_2)\dot{x}(s)dsd\theta,$$

and $P > 0$, $Q \ge 0$, $R \ge 0$, $Y > 0$, $Z_1 > 0$, $Z_2 > 0$ are to be determined. Then, we take the derivative of $V(x_t)$ with respect to t along the solution of the system (5) and add (7)-(11):

$$\begin{aligned}
\frac{d}{dt}V(x_t)&=2\dot{x}^T(t)Px(t)+x^T(t)(Q+R)x(t)\\
&-x^T(t-\tau)Qx(t-\tau)-x^T(t-h_M)Rx(t-h_M)\\
&+\dot{x}^T(t)(\tau Y+h_M(Z_1+Z_2))\dot{x}(t)-\int_{t-h_M}^{t}\dot{x}^T(s)(Z_1+Z_2)\dot{x}(s)ds-\\
&\int_{t-\tau}^{t}\dot{x}^T(s)Y\dot{x}(s)ds\\
&\le 2\dot{x}^T(t)Px(t)+x^T(t)(Q+R)x(t)-x^T(t-\tau)Qx(t-\tau)-\\
&x^T(t-h_M)Rx(t-h_M)+\dot{x}^T(t)(\tau_MY+h_M(Z_1+Z_2))\dot{x}(t)\\
&-\int_{t-h(t)}^{t}\dot{x}^T(s)Z_1\dot{x}(s)ds-\int_{t-h_M}^{t-h(t)}\dot{x}^T(s)Z_1\dot{x}(s)ds\\
&\int_{t-h_M}^{t}\dot{x}^T(s)Z_2\dot{x}(s)ds-\int_{t-\tau}^{t}\dot{x}^T(s)Y\dot{x}(s)ds\\
&+2\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(\xi(t))\lambda_j(\xi(t_k))\zeta^T(t)N_{ij}\left[x(t)-x(t-h(t))-\int_{t-h(t)}^{t}\dot{x}(s)ds\right]\\
&+2\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(\xi(t))\lambda_j(\xi(t_k))\zeta^T(t)S_{ij}\left[x(t-h(t))-x(t-h)-\int_{t-h_M}^{t-h(t)}\dot{x}(s)ds\right]\\
&+2\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(\xi(t))\lambda_j(\xi(t_k))\zeta^T(t)M_{ij}\left[x(t)-x(t-h_M)-\int_{t-h_M}^{t}\dot{x}(s)ds\right]\\
&+2\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(\xi(t))\lambda_j(\xi(t_k))\zeta^T(t)W_{ij}\left[x(t)-x(t-\tau)-\int_{t-\tau}^{t}\dot{x}(s)ds\right]\\
&+2\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(\xi(t))\lambda_j(\xi(t_k))\zeta^T(t)T\big[\dot{x}(t)-A_ix(t)-A_{di}x(t-\tau)-\\
&B_{2i}K_jx(t-h(t))\big]\\
&\le\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(\xi(t))\lambda_j(\xi(t_k))\{\zeta^T(t)\Psi_{ij}\zeta(t)-\int_{t-h(t)}^{t}[\zeta^T(t)N_{ij}\\
&+\dot{x}^T(s)Z_1]Z_1^{-1}[N_{ij}^T\zeta(t)+Z_1\dot{x}(s)]ds\\
&-\int_{t-h_M}^{t-h(t)}[\zeta^T(t)S_{ij}+\dot{x}^T(s)Z_1]Z_1^{-1}[S_{ij}^T\zeta(t)+Z_1\dot{x}(s)]ds\\
&-\int_{t-h_M}^{t}[\zeta^T(t)M_{ij}+\dot{x}^T(s)Z_2]Z_2^{-1}[M_{ij}^T\zeta(t)+Z_2\dot{x}(s)]ds\\
&-\int_{t-\tau}^{t}[\zeta^T(t)W_{ij}+\dot{x}^T(s)Y]Y^{-1}[W_{ij}^T\zeta(t)+Y\dot{x}(s)]ds\} \quad (12)
\end{aligned}$$

where $\Psi_{ij}=\Phi_{11ij}+h_MN_{ij}Z_1^{-1}N_{ij}^T+h_MS_{ij}Z_1^{-1}S_{ij}^T+h_MM_{ij}Z_2^{-1}M_{ij}^T+\tau_MW_{ij}Y^{-1}W_{ij}^T$. Now if (6) is satisfied, then by Schur complement formula we have

$$\Psi_{ij}<0,\ i,\ j=1,\cdots,r. \quad (13)$$

If (13) holds, we have $\sum_i^r\sum_j^r\lambda_i(\xi(t))\lambda_j(\xi(t_k))\zeta^T(t)\Psi_{ij}\zeta(t)<0$, which implies that $\dot{V}(x_t)<0$ because $Y > 0$, $Z_1 > 0$ and $Z_2 > 0$ and the last four terms in (12) are all less than 0. By Schur complement formula, this proves that conditions (6) suffice to show the asymptotic stability of the system (5).

Remark 3.1. The free weighting matrices T_i, $i = 1, \dots,$ 5 are employed to express the relationships between the terms $x(t)$, $x(t)$, $x(t-h(t))$ and $x(t-\tau)$. These matrices relax the conditions (6). Similarly, the free weighting matrices N_{kij}, S_{kij}, M_{kij}, W_{kij}, $k = 1, \dots, 5$, $i, j = 1, r$ are used to describe the relationships between the terms $x(t)$, $x(t-h(t))$, $x(t-h_M)$, $x(t-\tau)$ and their integral terms, and make the conditions (6) delay-dependent. This implies that the Leibniz-Newton formula is taken into account in the conditions, which reduces their conservatism in stability conditions. In addition, unlike [2], we do not use an inequality like $-2a^Tb \le a^TRa + b^TR^{-1}b$ to avoid the conservatism.

3.2. State Feedback Control

Now, we seek a design method of the sampled-data control for fuzzy time-delay systems based on Theorem 3.1.

Theorem 3.2. Given scalars t_i, $i = 1, \dots, 5$, the sampled-data controller (3) stabilizes the system (1) if there exist matrices $\bar{P}>0,\bar{Q}\ge 0,\ \bar{R}\ge 0,\bar{Y}>0,\bar{Z}_1>0,\bar{Z}_2>0, L, G_j$

$$\bar{N}_{ij}=\begin{bmatrix}\bar{N}_{1ij}\\ \bar{N}_{2ij}\\ \bar{N}_{3ij}\\ \bar{N}_{4ij}\\ \bar{N}_{5ij}\end{bmatrix},\bar{S}_{ij}=\begin{bmatrix}\bar{S}_{1ij}\\ \bar{S}_{2ij}\\ \bar{S}_{3ij}\\ \bar{S}_{4ij}\\ \bar{S}_{5ij}\end{bmatrix},\bar{M}_{ij}=\begin{bmatrix}\bar{M}_{1ij}\\ \bar{M}_{2ij}\\ \bar{M}_{3ij}\\ \bar{M}_{4ij}\\ \bar{M}_{5ij}\end{bmatrix}\text{ and }\bar{W}_{ij}=\begin{bmatrix}\bar{W}_{1ij}\\ \bar{W}_{2ij}\\ \bar{W}_{3ij}\\ \bar{W}_{4ij}\\ \bar{W}_{5ij}\end{bmatrix}i,\ j=1,\cdots,r$$

such that

$$\Theta_{ij}=\begin{bmatrix}\Theta_{11ij} & h_M\bar{N}_{ij} & h_M\bar{S}_{ij} & h_M\bar{M}_{ij} & \tau_MW_{ij}\\ h_M\bar{N}_{ij}^T & -h_M\bar{Z}_1 & 0 & 0 & 0\\ h_M\bar{S}_{ij}^T & 0 & -h_M\bar{Z}_1 & 0 & 0\\ h_M\bar{M}_{ij}^T & 0 & 0 & -h_M\bar{Z}_2 & 0\\ \tau_MW_{ij}^T & 0 & 0 & 0 & -\tau_M\bar{Y}\end{bmatrix}<0,\quad i,\ j=1,\cdots,r \quad (14)$$

where $\Theta_{11ij}=\Theta_1+\Theta_{2ij}+\Theta_{2ij}^T+\Theta_{3ij}+\Theta_{3ij}^T,$

$$\Theta_1 = \begin{bmatrix} \bar{R} & 0 & 0 & 0 & \bar{P} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\bar{R} & 0 & 0 \\ 0 & 0 & 0 & -\bar{Q} & 0 \\ \bar{P} & 0 & 0 & 0 & \tau_M \bar{Y} + h_M(\bar{Z}_1 + \bar{Z}_2) \end{bmatrix}$$

$$\Theta_{2ij} = \begin{bmatrix} \bar{N}_{ij} + \bar{M}_{ij} + \bar{W}_{ij} & -\bar{N}_{ij} + \bar{S}_{ij} & -\bar{M}_{ij} - \bar{S}_{ij} & -W_{ij} & 0 \end{bmatrix},$$

$$\Theta_{3ij} = \begin{bmatrix} -t_1 A_i L^T & -t_1 B_i G_j & 0 & -t_1 A_{di} L^T & t_1 L^T \\ -t_2 A_i L^T & -t_2 B_i G_j & 0 & -t_2 A_{di} L^T & t_2 L^T \\ -t_3 A_i L^T & -t_3 B_i G_j & 0 & -t_3 A_{di} L^T & t_3 L^T \\ -t_4 A_i L^T & -t_4 B_i G_j & 0 & -t_4 A_{di} L^T & t_4 L^T \\ -t_5 A_i L^T & -t_5 B_i G_j & 0 & -t_5 A_{di} L^T & t_5 L^T \end{bmatrix}$$

In this case, state feedback control gains in (3) are given by

$$K_i = G_i L^{-T}. \tag{15}$$

Proof: We let $T_i = t_i \bar{L}, i = 1, \cdots, 5$ where t_i are given scalars, and substitute them into the conditions (6). If conditions (6) hold, it follows that (5, 5)-block of Φ_{11ij} must be negative definite. It follows that $T_5 + T_5^T = t_5 (\bar{L} + \bar{L}^T) < 0$, which implies that $\bar{L}$ is nonsingular if $t_5 \neq 0$. Then we calculate $\Theta_{ij} = \Sigma\Phi_{ij}\Sigma^T$ with

$$\Sigma = \mathrm{diag}\begin{bmatrix} L & L & L & L & L & L & L & L \end{bmatrix}$$

where $L = \bar{L}^{-1}$. Defining $\bar{P} = LPL^T$, $\bar{Q} = LQL^T$, $\bar{R} = LRL^T$, $\bar{Y} = LYL^T$, $\bar{Z}_i = LZ_iL^T$, $i = 1, 2$, $\bar{N}_{kij} = LN_{kij}L^T$, $\bar{S}_{kij} = LS_{kij}L^T$, and applying Schur complement formula, we obtain Θ_{ij} in (14) where we define $G_j = K_j L^T$, and Θ_{11ij}, Θ_1, Θ_{2ij} and Θ_{3ij} are given as in Theorem 3.2. If the conditions (14) hold, state feedback control gain matrices K_i are obviously given by (15).

Remark 3.2. Conditions in Theorem 3.2 are not strict LMIs unless scalars t_i, $i = 1, ..., 5$ are specified. One way is to find optimal t_i, $i = 1, ..., 5$ is to use a numerical software like Matlab with optimization toolbox fminsearch.

4. EXAMPLE

Consider a fuzzy time-delay system (1) where $r = 2$, $\lambda_1(x_1) = 1 - \sin^2 x_1$, $\lambda_2(x_1) = 1 - \lambda_1(x_1)$ and

$$A_1 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1.5 \end{bmatrix}, A_{d1} = \begin{bmatrix} -2 & -0.5 \\ 0 & -1 \end{bmatrix},$$

$$A_{d2} = \begin{bmatrix} -2 & -0.5 \\ 0 & -1.5 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 1.5 \end{bmatrix}.$$

When the upper bound of time-delay is given by $\tau_M = 1$, Theorem 3.2 gives the maximum upper bound of the sampling interval $h_M = 0.100$ for which a stabilizing sampled-data controller (3) exists. For $h_M = 0.100$, with $t_1 = 1.64$, $t = 0.0$, $t_3 = 0.24$, $t_4 = 0.47$, and $t_5 = 1.0$, (14) is satisfied by

$$\bar{P} = \begin{bmatrix} 15.0286 & -34.2628 \\ -34.2628 & 195.7312 \end{bmatrix}, \bar{Q} = \begin{bmatrix} 10.8635 & -28.2987 \\ -28.2987 & 352.3587 \end{bmatrix},$$

$$\bar{R} = \begin{bmatrix} 1.9690 & -4.6885 \\ -4.6885 & 11.1827 \end{bmatrix},$$

$$\bar{Y} = \begin{bmatrix} 15.5269 & -27.2787 \\ -27.2787 & 47.9271 \end{bmatrix}, \bar{Z}_1 = \begin{bmatrix} 5.9234 & -44.5072 \\ -44.5072 & 335.9034 \end{bmatrix},$$

$$\bar{Z}_2 = \begin{bmatrix} 0.7586 & -1.5512 \\ -1.5512 & 3.9711 \end{bmatrix},$$

$$\bar{L} = \begin{bmatrix} -10.0767 & 19.0482 \\ 18.7626 & -62.3930 \end{bmatrix}, G_1 = \begin{bmatrix} -20.2724 & -153.2056 \end{bmatrix},$$

$$G_2 = \begin{bmatrix} -20.2731 & -153.2033 \end{bmatrix}.$$

Then, Theorem 3.2 gives the control gains K_1 = [– 6.0940 – 4.288], K_2 = [– 6.0937 – 4.2879] in (3).

Next, assume the maximum sampling interval $h_M = 0.05$. Theorem 3.2 guarantees the maximum upper bound of time-delay $\tau_M = 1.347$. With $t_1 = 1$, $t = 0.79$, $t_3 = 0.33$, $t_4 = -0.48$, and $t_5 = 1.33$, (14) is satisfied by

$$\bar{P} = 10^3 * \begin{bmatrix} 0.0199 & -0.0589 \\ -0.0589 & 1.0740 \end{bmatrix}, \bar{Q} = 10^3 * \begin{bmatrix} 0.0073 & -0.0181 \\ -0.0181 & 1.9457 \end{bmatrix},$$

$$\bar{R} = \begin{bmatrix} 4.7944 & -14.3053 \\ -14.3053 & 43.1445 \end{bmatrix},$$

$$\bar{Y} = \begin{bmatrix} 19.3788 & -56.1462 \\ -56.1462 & 162.6849 \end{bmatrix}, \bar{Z}_1 = 10^3 * \begin{bmatrix} 0.0076 & -0.0620 \\ -0.0620 & 1.4988 \end{bmatrix},$$

$$\bar{Z}_2 = \begin{bmatrix} 1.2315 & -3.8414 \\ -3.8414 & 23.8568 \end{bmatrix},$$

$$\bar{L} = \begin{bmatrix} -12.0378 & 30.8176 \\ 38.2882 & -192.3205 \end{bmatrix}, G_1 = \begin{bmatrix} -21.0988 & 686.7995 \end{bmatrix},$$

G_2 = [–21.1067 686.6412].

In this case, Theorem 3.2 gives the control gains K_1 = [– 15.0708 – 6.5715], K_2 = [– 15.0652 – 6.5696] in (3).

5. CONCLUSION

We considered the sampled-data control problem for fuzzy time-delay systems. Our method employed an input delay system approach. We first showed the closed-loop system with the sampled-data state feedback control became the state delayed system. Then we gave sufficient conditions for the closed-loop system to be asymptotically stable. Based on such conditions we proposed a design method of the sampled-data controller for fuzzy time-delay systems.

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