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# DYNAMICS OF PARALLEL DEVELOPMENT OF THE BOND MARKET INDICES IN THE US MARKET AND ITS MULTIDIMENSIONAL COPULA MODELS

TOMÁŠ BACIGÁL, JOZEF KOMORNÍK, MAGDA KOMORNÍKOVÁ, AND CUONG NGUYEN

Abstract. In this paper (which is a substantially extended version of a conference paper from SMTDA 2016 [10]) we focus our attention to 3-dimensional copula models of returns of indices of US financial markets (various bond indices have been investigated in the literature much less than stock indices). We have gained interesting experience in constructing Vine copula models. Although, for our particular data (comprising two triples of bond indices: US Investment Bond indices and US Corporate Bond indices), the global dominance of more traditional classes of elliptic (especially Student type) 3-dimensional copulas was demonstrated (and some conclusions concerning optimizations of investment portfolios can be based on fairly simple arguments), the optimal local Vine copulas helps to obtain more insight in the detailed development of the investigated triples of investments.

## 1. Introduction

In this paper we apply 3-dimensional copula models to two triples of time series of returns of indices of US financial markets (using daily data from Bloomberg). The first triple US Investment Bond indices (US IBI) includes the Investment grade bond index (Igbi), High yield corporate bond index (Hybi), and the Investment grade bond index SP500 (SP) in the period from January 2010 to April 2015. Here we follow the approach of Hong et al. [8] that decomposed corporate bonds into investment grade and high-yield bonds claiming that the returns of the second group can be predicted by past stock market returns. The second triple of US Corporate Bond indices (US CBI) contains the Bank of America Merrill Lynch US Corporate Bond Index (ML), the Barclays US Corporate & Investment Grade Index (Bar), Dow Jones Corporate Bond Index (DJ) in the (longer) period from January 1997 to May 2014. Our results show high values of Kendall's correlation coefficients as well as tail dependencies between all couples of this triple of indices. Very interestingly (from the investor's point of view), the correlations between the returns of Hybi and Igbi (as well as their tail dependencies) are remarkably weak.

The paper is a substantially extended version of the contribution to SMTDA 2016 conference [10]. It is organized as follows. First we recall some theory about copulas, their classes and construction used in higher dimensions, then review

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estimation and goodness of fit methods used in our analysis. Finally we discuss results and conclude.

## 2. Theory

Copulas are fundamental tools for modelling dependence between/among random variables leaving alone their marginal distributions. Due to Sklar [15]

$$F(x_1, ..., x_n) = C\left[F_1(x_1), ..., F_n(x_n)\right],$$

where $F$ is joint cumulative distribution function of random vector $(X_1, ..., X_n)$, $F_i$ is marginal cumulative distribution function of $X_i$, and $C : [0,1]^n \to [0,1]$ is a copula which is a n-increasing function with 1 as neutral element and 0 as annihilator, see e.g. monograph Nelsen (2006) [11]. Besides three fundamental copulas

$$M(x_1, ..., x_n) = \min(x_1, ..., x_n), \quad W(x_1, x_2) = \max(x_1 + x_2 - 1, 0),$$

$$\Pi(x_1, ..., x_n) = \prod_{i=1}^{n} x_i,$$

which model perfect positive dependence, perfect negative dependence (not applicable for $n > 2$) and independence, respectively, there exist numerous parametric classes, such as Archimedean, Extreme-Value and elliptical copulas. Within the last one there belong such important parametric families as *Gaussian* copulas

$$C_G(x_1, ..., x_n) = \Phi\left[\Phi_1^{-1}(x_1), ..., \Phi_n^{-1}(x_n)\right]$$

and *Student* $t$-copulas

$$C_t(x_1, ..., x_n) = t\left[t_1^{-1}(x_1), ..., t_n^{-1}(x_n)\right],$$

(where $\Phi$ and $t$ are joint distribution functions of multivariate normal and Student $t$ distributions, similarly $\Phi_i^{-1}$ and $t_i^{-1}$, $i = 1, ..., n$ are univariate quantile functions related to $X_i$), able to flexibly describe dependence in multidimensional random vector. On the contrary, the *Archimedean* class

$$C_A(x_1, ..., x_n) = \varphi^{(-1)}\left[\varphi(x_1), ..., \varphi(x_n)\right]$$

(with generator $\varphi : [0,1] \to [0,\infty]$ and its pseudo-inverse $\varphi^{(-1)}$) is much easier to handle, yet it is reasonably useful only in two-dimensional case. However it can be a building block in a so-called pair-copula construction originally proposed in Joe (1996) [9] in terms of distributions functions, later reformulated in terms of densities by Bedford & Cooke (2001) [2] and organized by Bedford & Cooke (2002) [3] in a graphical way involving a sequence of nested trees (vines), see Figure 1 for illustration. More about Vine copulas can be found in [1, 14] and [4], here we outline the construction of three-dimensional probability density function $f$ (needed in our analysis)

$$\begin{aligned} f(x_1, x_2, x_3) &= f_1(x_1) \cdot f_{2|1}(x_1, x_2) \cdot f_{3|12}(x_1, x_2, x_3) = \\ &= f_1(x_1) \cdot c_{12}\left[F_1(x_1), F_2(x_2)\right] \cdot f_2(x_2) \cdot \\ &\quad \cdot c_{31|2}\left[F_{3|2}(x_2, x_3), F_{1|2}(x_1, x_2)\right] \cdot c_{23}\left[F_2(x_2), F_3(x_3)\right] \cdot f_3(x_3) \end{aligned} \tag{2.1}$$

where $f_i$ is a (marginal) probability density function of $X_i,\ i = 1, 2, 3,$

$$f_{i|j}(x_i, x_j) = \frac{f(x_i, x_j)}{f_j(x_j)}$$

is conditional density function of $X_i$ given $X_j$. A copula density $c_{ij}$ couples $X_i$ and $X_j$ while $c_{ij|k}$ couples bivariate marginal distributions of $X_i, X_k$ and $X_j, X_k, i, j, k \in \{1, 2, 3\}, i \neq j \neq k \neq i$. Finally,

$$F_{i|j} = \frac{\partial C_{ij}\left[F_i(x_i), F_j(x_j)\right]}{\partial F_j(x_j)}$$

is a conditional cumulative distribution function of $X_i$ given $X_j$. The construction (2.1) represented by Figure 1 is one of the three possible pair–copula decompositions, which, graphically, are both canonical (C-) and drawable (D-) vine trees. In more than three dimensions, C–vines and D–vines are just small subsets of a more general class - regular vines.

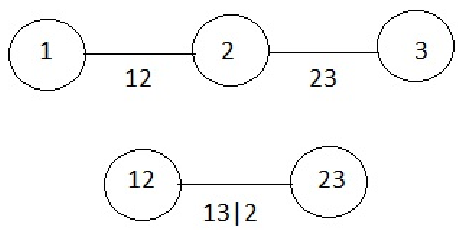


FIGURE 1. Vine tree corresponding to construction (2.1) with the $2^{nd}$ variable as a root node

## 3. Methods

Given $m$ observations $\{X_{j,i}\}_{i=1,...,m}$ of $j$-th random variable $X_j,\ j = 1, 2, 3,$ the parameters $\theta$ of all copulas under consideration were estimated by maximizing the likelihood function

$$L(\theta) = \sum_{i=1}^{m} \log\left[c_\theta(U_{1,i}, U_{2,i}, U_{3,i})\right], \tag{3.1}$$

where $c_\theta$ denotes density of a parametric copula family $C_\theta$, and

$$U_{j,i} = \frac{1}{m+1} \sum_{k=1}^{m} \mathbf{1}(X_{j,k} \leq X_{j,i}), \quad i = 1, ..., m,$$

are so-called pseudo-observations. Goodness-of-fit was performed by a test proposed by Genest et al. [6] and based on empirical copula process using Cramer-von Misses test statistic

$$S_{CM} = \sum_{i=1}^{m} \left[C_\theta(U_{1,i}, U_{2,i}, U_{3,i}) - C_m(U_{1,i}, U_{2,i}, U_{3,i})\right]^2 \tag{3.2}$$

with empirical copula $C_m(\mathbf{x}) = \frac{1}{m}\sum_{i=1}^{m}\prod_{j=1}^{3}\mathbf{1}(X_{j,i} \leq x_j)$ and indicator function $\mathbf{1}(A) = 1$ whenever $A$ is true, otherwise $\mathbf{1}(A) = 0$.

The random variable with the strongest association to the others was selected as a root node 2 in the first vine tree (upper level on Figure 1) and bivariate copulas as nodes in the vine forest (set of vine trees) were chosen by minimizing Akaike information criterion.

All calculations were done in R [12] with the help of packages copula [7] and VineCopula [13]. Besides the usual parametric families of Archimedean class such as Gumbel, Clayton, Frank and Joe copulas (see e.g. [9, 11]) in bivariate case we used also their rotations $C_\alpha$ by angle $\alpha$ defined

$$C_{90}(x_1, x_2) = x_2 - C(1 - x_1, x_2),$$

$$C_{180}(x_1, x_2) = x_1 + x_2 - 1 + C(1 - x_1, 1 - x_2) \quad \text{survival copula,}$$

$$C_{270}(x_1, x_2) = x_1 - C(x_1, 1 - x_2),$$

that are implemented in R package VineCopula [13].

## 4. Results

We can see graphs of the considered two triples of time series in the Figure 2 and Figure 3. In Figure 2, we can observe that (expectedly for after-crisis period) the dynamics of growth of the High yield corporate bond index (Hybi) was stronger than that of the Investment grade bond index (Igbi), but both of them mostly trace the SP500 Index (SP). In Figure 3 we see that (expectedly) the Merrill Lynch US Corporate Bond Index (ML) mostly leads the remaining two in the considered triple (with deeper losses in the crisis period).

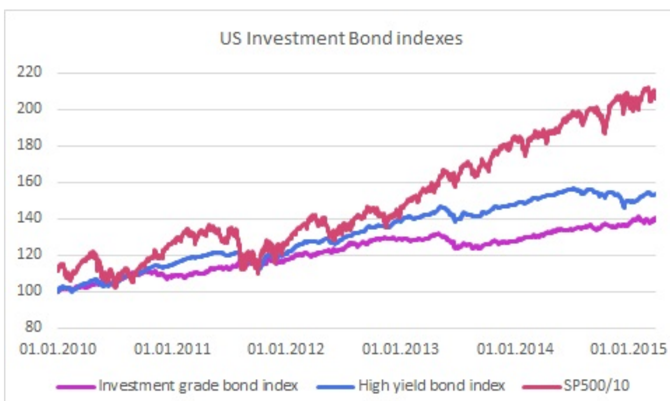


Figure 2. US Investment Bond indexes

All indices are computed in terms of returns (see Figures 4 and 5)

$$return_i = \log\frac{index_i}{index_{i-1}}, \quad i = 2, 3, ..., n.$$

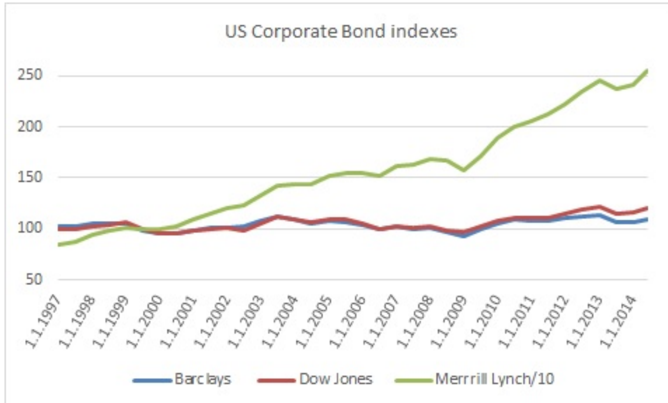


FIGURE 3. US Corporate Bond indexes

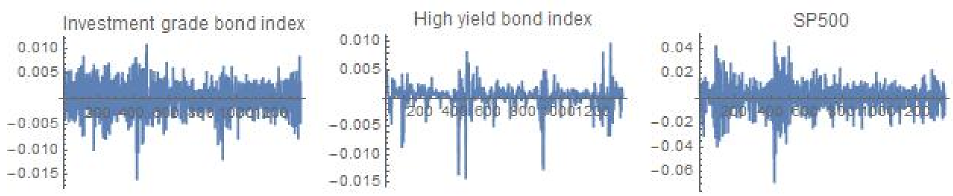


FIGURE 4. Returns of High yield corporate bond index (left), Investment grade bond index (middle), Investment grade bond index SP500 (right)

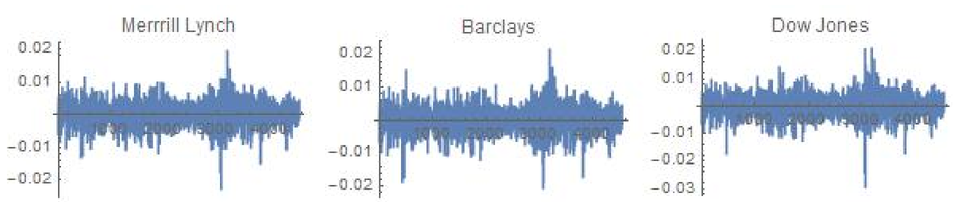


FIGURE 5. Returns of Bank of America Merrill Lynch US Corporate Bond Index (left), Barclays US Corporate & Investment Grade Index (middle), Dow Jones Corporate Bond Index (right)

Before further analyses, we filtered all considered time series of returns by ARMA-GARCH filters

$$X_t - c_0 = \sum_{i-1}^{P} a_{0,i}(X_{t-i} - c_0) + e_t + \sum_{j-1}^{Q} b_{0,j} e_{t-j},$$

$$e_t = h_t \eta_t,$$

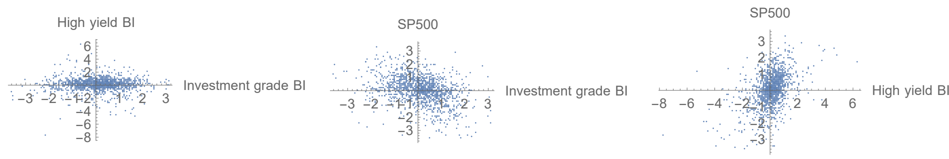


FIGURE 6. Scatter plots for all couples of the (filtered) returns of US IBI

$$h_t^2 = \omega_0 + \sum_{i=1}^{q} \alpha_{0,i} e_{t-i}^2 + \sum_{j=1}^{q} \beta_{0,j} h_{t-j}^2,$$

where $X_1, ..., X_n$ are the observations, $c_0 = E[X_t],\ t = 1, ..., n,\ a_{0,i},\ i = 1, ..., P$ are the AR coefficients, $b_{0,j},\ j = 1, ..., Q$ are the MA coefficients, $(\eta_t)$ is a sequence of independent and identically distributed (i.i.d.) random variables such that $E[\eta_t] = 0, E[\eta_t^2] = 1$, $\omega_0 > 0$, $\alpha_{0,i} \geq 0,\ i = 1, ..., q$ and $\beta_{0,j} \geq 0,\ j = 1, ..., p$ (for more details see e.g. [5]).

Results of the introductory standard analysis of the residuals for both triples are presented in Table 1, Figure 6, Table 2 and Figure 7.

In Table 1 we observe that the data high yield (Hybi) and investment grade (Igbi) bonds are practically uncorrelated, while their Kendall's correlation coefficient wit SP500 (SP) data exhibit slight correlations of opposite signs. Those numerical results are graphically illustrated in the Figure 6.

TABLE 1. Values of the Kendall's correlation coefficient for all couples of the (filtered) returns of US IBI

| | Hybi | Igbi | SP |
|---|---|---|---|
| Hybi | 1 | 0.0005 | 0.2070 |
| Igbi | 0.0005 | 1 | -0.2620 |
| SP | 0.2070 | -0.2620 | 1 |

Very interestingly, Table 2 presents high values of Kendall's correlation coefficients for all three couples of the second triple of the residuals of the returns of the considered indices. They are illustrated in the Figure 7.

TABLE 2. Values of the Kendall's correlation coefficient for all couples of the (filtered) returns of US CBI

| | Bar | DJ | ML |
|---|---|---|---|
| Bar | 1 | 0.825 | 0.826 |
| DJ | 0.825 | 1 | 0.836 |
| ML | 0.826 | 0.836 | 1 |

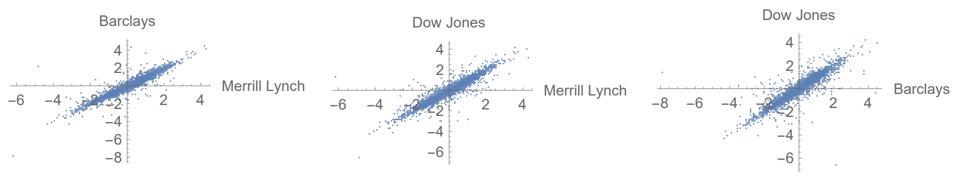


FIGURE 7. Scatter plots for all couples of the (filtered) returns of US CBI

We extended our analyses by examining developments of the Kendall's correlations. In Figure 8, we see the development of Kendall's correlation coefficients calculated in semi-annual frequency from data in neighboring annual interval that overlap by six months with corresponding intervals that provide data for left and right neighboring values of Kendall's correlation coefficients. Altogether, we have calculated a sequence of 9 values of Kendall's correlation coefficients. The last of them was calculated from the interval of 16 months. We see that for the couple Hybi & Igbi the values of these coefficients are largely located within the significance limits $\pm 0.083$ for test of zero value (with two slight exceptions). However the corresponding values for the couple Hybi & SP are completely out of such interval and for the couple Igbi & SP only slightly enter that interval.

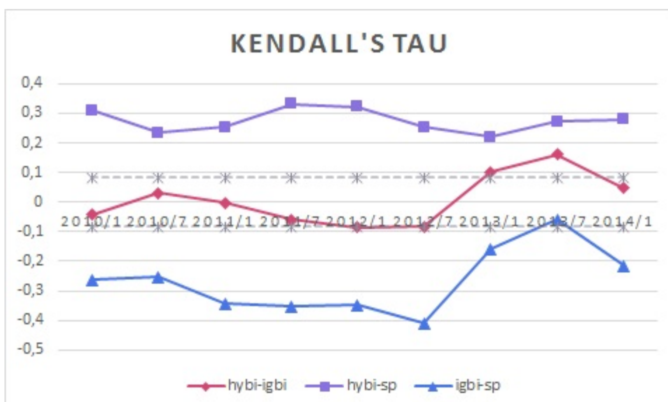


FIGURE 8. Evolution of Kendall's $\tau$ for all couples of the (filtered) returns of US IBI with 95% insignificance band

For the second triple with longer time period, we have chosen annual frequency of calculations of Kendall's correlation coefficients over the intervals of 24 months overlapping by 12 months with the intervals for calculation of the neighboring values of Kendall's correlation coefficient. Altogether, we have calculated a sequence of 17 such values. The last of them was calculated from the interval of 17 months. We can see (Figure 9) that all three correlation coefficients exhibit extremely parallel development and their values do not approach the significance limits for tests of their zero value.

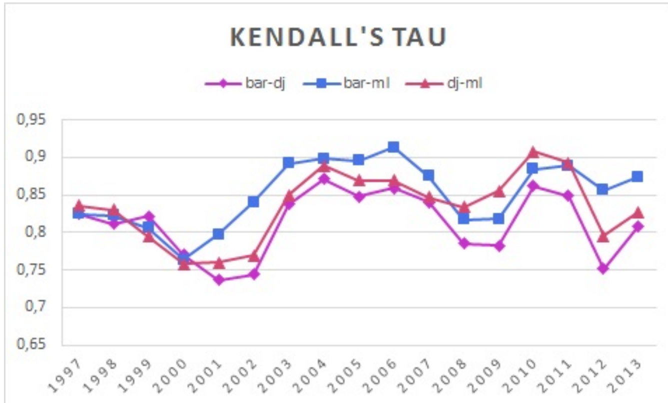


FIGURE 9. Evolution of Kendall's $\tau$ for all couples of the (filtered) returns of US CBI

We have calculated global copula models for the first triple (US IBI) of modelled variable. The best copula with respect to CvM test statistic is the trivariate t-copula with test statistic $S_{CM} = 0.041$ and parameters $\rho_{hybi,igbi} = 0.007$, $\rho_{hybi,sp} = 0.400$, $\rho_{igbi,sp} = -0.369$ and degrees of freedom $df = 8$, very closely followed by normal copula with $S_{CM} = 0.045$ and the best Vine copula ($S_{CM} = 0.095$) consisting of bivariate t-copulas in tree 1 (SP500 as its root node) and normal copula in tree 2.

Then we continued in searching models for the 9 time intervals described above (for which sequence of Kendall's correlation coefficient was calculated). A Vine copula was identified (tree structure) for each interval but estimated also for all the other intervals, thus we got the selection of 8 best fitting Vine copula structures and their corresponding sequences of estimated Vine copulas. Similarly we estimated a sequence of 9 Gaussian and a sequence of 9 Student t-copulas.

Their corresponding Cramer-von Misses GOF test statistic (mean squared distance from empirical copula) is displayed in Figure 10 and it shows slightly superior performance of elliptical copulas over Vine copulas throughout the whole analyzed period. We see that in most individual time intervals, the difference between the best Vine class copulas and the best optimal Student class copulas are almost negligible. Moreover for most remaining time intervals (except for one centered at 2014), the above differences are far from being remarkably big. This phenomenon could provide opportunities for finding more optimal local models in the class of convex combinations of the best local Student class copulas and the best Vine class copulas. The diversity of Vine copulas reveals us significant changes in the shape of dependence structure since 2013, accompanied by the changes in the dependence strength between Igbi and the others (Figure 8). The eight Vine copula structures chosen to represent dependence shape were identified in each period by applying selection criteria as mentioned in Methods. From Figure10 (see also Table3) we may observe that after 2013 no Gaussian pair-copula in vine trees was chosen while copulas with lower tail dependence in tree 2 and t-copula for Igbi &

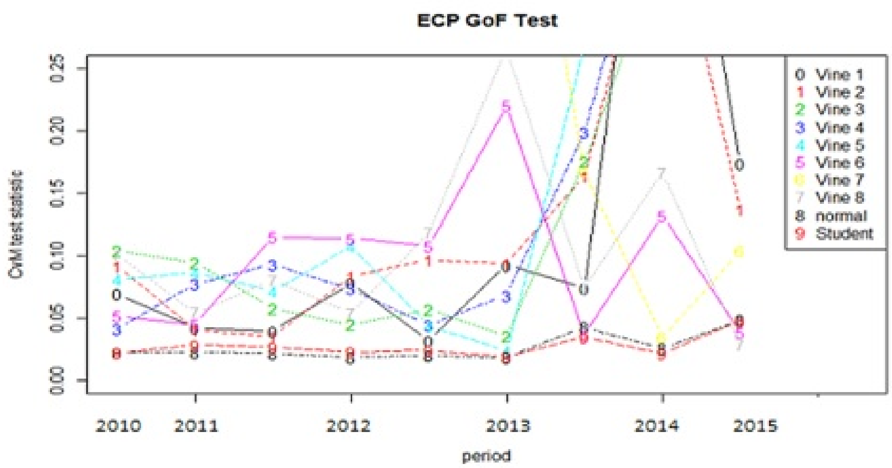


FIGURE 10. Evolution of Cramer-von Mises test statistic for (filtered) returns of US IBI

TABLE 3. Models for (filtered) returns of US IBI from Figure 10

| Notation | Tree 1 | | Tree 2 |
|---|---|---|---|
| | Couple | Copula | Copula |
| Vine 1 | Hybi & SP<br>Igbi & SP | Gaussian<br>$\text{Gumbel}_{90}$ | Frank |
| Vine 2 | Hybi & SP<br>Igbi & SP | Student t<br>$\text{Gumbel}_{90}$ | Clayton |
| Vine 3 | Hybi & SP<br>Igbi & SP | Survival Gumbel<br>Gaussian | Gaussian |
| Vine 4 | Hybi & SP<br>Igbi & SP | Gaussian<br>$\text{Gumbel}_{270}$ | Frank |
| Vine 5 | Hybi & SP<br>Igbi & SP | Gaussian<br>Gaussian | Gaussian |
| Vine 6 | Hybi & SP<br>Igbi & SP | Survival Gumbel<br>Student t | Survival Gumbel |
| Vine 7 | Hybi & SP<br>Igbi & SP | Survival Gumbel<br>Student t | Student t |
| Vine 8 | Hybi & SP<br>Igbi & SP | Student t<br>$\text{Gumbel}_{90}$ | Survival Gumbel |

SP couple were preferred. Interestingly, Vine 6 and 7 look similar in shape (though Vine 6 add lower tail dependence in tree 2) yet they quite strongly alternate in ability to describe dependence. Especially interesting is the fact that the couple Hybi & Igbi appear in the Tree 1 of the Vines 1 — 7 that is signalizing their uniformly very low dependence (that is also supported by the fact that in almost all local time intervals the Gaussian 3-dimensional copula is provided almost equally strong competitor as the Student copula).

TABLE 4. Models for (filtered) returns of US CBI from Figure 11

| Notation | Tree 1 | | Tree 2 | |
|---|---|---|---|---|
| | Couple | Copula | Couple | Copula |
| Vine 1 | Bar & DJ<br>DJ & ML | Student t<br>Student t | Bar & ML — DJ | Student t |
| Vine 2 | Bar & ML<br>DJ & ML | Student t<br>Student t | Bar & DJ — ML | Student t |
| Vine 3 | DJ & Bar<br>DJ & ML | Student t<br>Student t | DJ & ML — Bar | Student t |
| Vine 4 | Bar & ML<br>DJ & ML | Survival Gumbel<br>Student t | Bar & DJ — ML | Student t |

Next we calculated global copula models for the second triple (US CBI) of the considered series. The best copula with respect to CvM test statistic is the trivariate t-copula with test statistic $S_{CM} = 0.082$ and parameters $\rho_{bar,dj} = 0.967$, $\rho_{bar,ml} = 0.977$, $\rho_{dj,ml} = 0.973$ and degrees of freedom df = 2 (normal copula scored $S_{CM} = 0.766$) while the most successful Vine copula with $S_{CM} = 0.353$ uses bivariate t-copulas in each pair and Merril Lynch as its root node variable.

Similarly, we continued in searching copula models for 17 time intervals of US CBI (for which sequence of Kendall's correlation coefficient had been calculated previously) in the same way as described for US IBI.

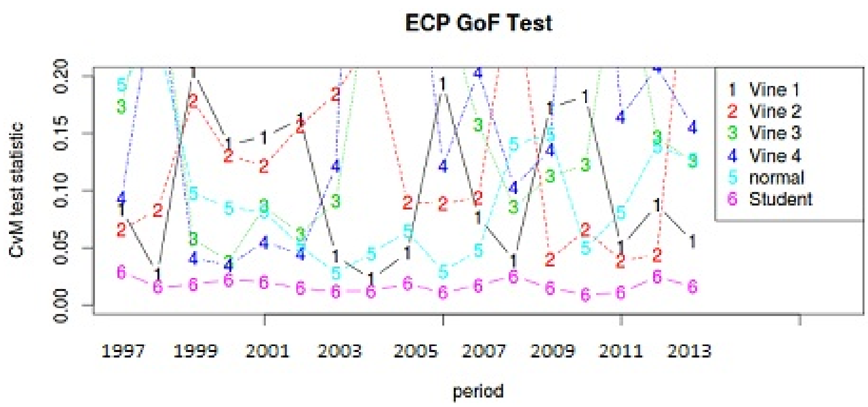


FIGURE 11. Evolution of Cramer-von Mises test statistic for (filtered) returns of US CBI

Figure 11 (with Table 4) shows superior position of t-copula in modelling dependence among US CBI returns and is vastly contained also in vines. This copula captures both tail dependences in the contrary to normal copula which does not appear among other copulas. Again, we can observe that for a large part of individual time intervals, the differences between the best local Vine copulas and the best local Student copulas are almost negligible and for a vast majority of the remaining time intervals, the above differences are far from being remarkably big. Therefore, we can again hope for finding optimal local models in the class of convex combinations of the best local Student class and Vine class copulas. What varies throughout the periods is the selection of a root node variable which is most of the time Merrill Lynch index replaced by Dow Jones in 2003 and 2005.

Finally, Figure 12 and Figure 13 contain comparable graphs of the parameters development and tail dependence coefficients of the optimal three–parametric Student 3D models for the triples US CBI and US IBI.

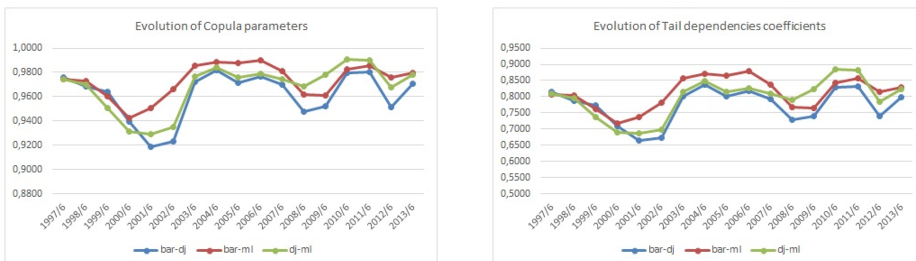


FIGURE 12. Evolution of parameters (left) and tail dependence coefficients (right) of the optimal 3-parametric Student class 3D copula for US CBI


FIGURE 13. Evolution of parameters (left) and tail dependence coefficients (right) of the optimal 3-parametric Student class 3D copula for US IBI

## 5. Conclusion and future work

Analyzing mutual development of bond indices is interesting and important for investors, risk managers and policy makers. Application of more dimensional copulas is bringing a new insight and experience for modelling activities. The most important conclusion for investors is the observation that the dependence between returns of high-yield and investment grade indices has been very low. With respect to optimal portfolio building (using the class of the US IBI indices), we can start from the dominant profitability of the SP 500 index. However the other two components of the considered triple can enhance hedging effect of possible combined portfolio since they are (even locally) almost uncorrelated and exhibit small opposite sign correlations with SP 500. We, however, should keep in mind that the Student class models imply heavy tails (which is a good news for speculators but not for conservative hedgers). The situation seems to be more simple in case of the second triple of US CBI indices. Since filtered returns of the components of this triple exhibit permanent significantly high values of the Kendall's correlation, any portfolio created by them could be more effectively represented by their most profitable component ML. We, however, should again keep in mind that the Student models indicate heavy tails (with the consequences mentioned above).

Although in our concrete case the elliptic Student type copulas globally dominated the description of the considered returns of investment, the applications of Vine copulas provide useful contributions concerning many interesting detailed information related to their local development.

More thorough investigations of convex combinations elliptic and Vine copulas can open new challenges for further research.

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Tomáš Bacigál: Faculty of Civil Engineering, Slovak University of Technology, Radlinského 11, 810 05 Bratislava, Slovakia
*E-mail address*: Tomas.Bacigal@stuba.sk
*URL*: http://www.math.sk

Jozef Komorník: Faculty of Management, Comenius University, Odbojárov 10, P.O.BOX 95, 820 05 Bratislava, Slovakia
*E-mail address*: Jozef.Komornik@fm.uniba.sk

Magda Komorníková: Faculty of Civil Engineering, Slovak University of Technology, Radlinského 11, 810 05 Bratislava, Slovakia
*E-mail address*: Magdalena.Komornikova@stuba.sk
*URL*: http://www.math.sk

Cuong Nguyen: Faculty of Commerce, Lincoln University NZ, Christchurch, 7647 Canterbury, New Zealand
*E-mail address*: Cuong.Nguyen@lincoln.ac.nz