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Switched Gain Control of Continuous LTI Systems with Asymmetrical Input Constraints

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Abstract: Based on the positive invariance concept, a new and easy methodology of performing the convergence rate to the origin is proposed for invariant linear continuous-time systems with asymmetrical input constraints. It enables to benefit from a large set of admissible initial states to the cost of a slow transient's system convergence rate performance. When the state tends to the origin, the control law is switched to larger and larger gains and the convergence rate to the origin is made better and better. This control law is computed off-line and it guarantees that input bounds are never exceeded without causing input saturations. The importance of the proposed approach with respect to existing ones is shown through an example.

1. INTRODUCTION

In many applications, linear systems subject to state and/or input constraints are frequently encountered because such constraints are generally associated to physical limitations of the actuators. The respect of this constraints can be accomplished by designing suitable feedback control laws. In many cases, this can be done by constructing positively invariant domains inside the set of the constraints, ([1], [2], [3], [4], [5], [6], [7], [8], [9], [10], [11])in which case, saturation does not occur. A second way to deal with the constraints is given by saturation allowance, in this case, system is non linear and significant efforts are made to guarantee the stability of the closed-loop system ([12], [13], [14], [15], [16] and reference therein). In this work, we treat the saturation avoidance. The main purpose of this approach is to design a controller which can stabilize the system while maintaining its state vector inside a positively invariant set. The use of the invariance positive concept in the regulation of linear systems under constraints is motivated by its simplicity with respect to other approaches. Other applications have being derived from this concept. Particularly, one which consists in using large set of initial states while the constraints on the control vector are respected, ([17], [18]). In fact, a dilemma appears between the size of this set and the convergence rate of the closed-loop system. To overcome this dilemma, the proposed method consists in tolerating bad initial convergence rate performance, which leads to the use of a large set of initial states and when the state tends to the origin, the control law is switched to larger and larger gains. In [17], an optimal control law based on the quadratic Lyapunov functions is used leading to symmetrical nested ellipsoidal domains, while in [18], the non-quadratic asymmetrical Lyapunov functions are used, leading to asymmetrical nested polyhedral domains which are more convenient to the real constraint type. In spite of the simplicity of the second method, some questions have not been dealt with. In particular, as will be shown, the system matrix $A(\dot{x} = Ax + Bu,\ x \in \mathbb{R}^n,\ u \in \mathbb{R}^m)$ must possess at least $(n - m)$ stable eigenvalues. In the opposite case, the method developed in [18] cannot be applied, i.e., one can not introduce fictitious entries in a given system as done in [3] and improve its convergence rate to the origin.

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In this paper, the asymmetrical constraints on the control vector are considered. The proposed approach consists in starting the closed-loop system with modest convergence rate performance and finishing it with better one using an, a *priori* chosen, number N of predetermined regulator gains F_i, $i = 1, \ldots, N$. It consists in switching the feedback control law gain from F_{i-1} to F_i, $i = 1, \ldots, N$ when the state crosses the polyhedral domain belonging to F_i. Each polyhedral domain is positively invariant with respect to the corresponding closed-loop system. These domains are asymmetric and not necessarily nested, which constitutes a great advantage in the sense that, applying the proposed control law, the union of the obtained domains is positively invariant and it can be taken as a large set of initial states. The pole assignment method studied in [9] is used to compute state feedback gains which assign faster and faster dynamics while the constraints on the control vector are respected.

The particular case of linear systems with single input is also studied. In this case, it is shown that all the gain feedback matrices are proportional to the initial gain.

The paper is organized as follows: Preliminary results are stated in section II. Section III is reserved to the main results together with some comments and two illustrative examples.

1.1 Notations

- For two vectors x, y of $\mathbb{R}^n$, $x \leq y$ (respectively, $x < y$) if and only if $x_i \leq y_i$ (respectively $x_i < y_i$), $i = 1, \ldots, n$.
- $\mathbb{I}_n$ is the identity matrix of $\mathbb{R}^n$; A^T, $\lambda_i(A)$ and $\sigma(A)$ denote the transpose, the i^{the} eigenvalue and the spectrum of matrix A, respectively.
- For $\eta \in \mathbb{R}$, $\eta^+ = \sup(\eta, 0)$, $\eta^- = \sup(-\eta, 0)$.
- For $\mu \in \mathbb{C}$, $Re(\mu)$ is the real part of μ.
- $\mathbb{R}^m_+ = \{x \in \mathbb{R}^m / x_i \geq 0,\ i = 1, \ldots, m\}$
- For two subsets S_1 and S_2 of $\mathbb{R}^m$, $S_1 \backslash S_2 = \{x \in \mathbb{R}^m / x \in S_1 \text{ and } x \notin S_2\}$
- $\mathrm{int}\mathbb{R}^m_+$ is the interior of $\mathbb{R}^m_+$.
- If $H = (h_{ij})_{1 \leq i, j \leq m} \in \mathbb{R}^{m \times m}$ then,

$$\widetilde{H} = \begin{bmatrix} H_1 & H_2 \\ H_2 & H_1 \end{bmatrix} \in R^{2m \times 2m}$$

$$H_1(i,j) = \begin{cases} h_{ij} & if \quad i = j \\ h_{ij}^{+} & if \quad i \neq j \end{cases} \qquad H_2(i,j) = \begin{cases} 0 & if \quad i = j \\ h_{ij}^{-} & if \quad i \neq j \end{cases}$$

2. PROBLEM STATEMENT AND PRELIMINARY RESULTS

This note is devoted to the study of linear systems described by:

$$\dot{x} = Ax + Bu \tag{1}$$

$x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ is the control vector, A and B are constant matrices with appropriate dimensions and (A, B) is stabilizable. The control vector u is constrained to evolve in the set Ω defined by:

$$\Omega = \{u \in \mathbb{R}^m / -u_{\min} \leq u \leq u_{\max}\} \tag{2}$$

$u_{\min}$, $u_{\max} \in \text{int}\mathbb{R}^m_+$ are fixed vectors. Consider the unconstrained state feedback control law,

$$u = F_0 x,\ F_0 \in \mathbb{R}^{m\times n} \tag{3}$$

The control is admissible only if the state is constrained to evolve in the domain given by,

$$D(F_0, u_{\min}, u_{\max}) = \{x \in \mathbb{R}^n / - u_{\min} \leq F_0 x \leq u_{\max}\} \tag{4}$$

Taking into account (3), system (1) becomes,

$$\dot{x} = (A + BF_0)\, x \tag{5}$$

which represents an autonomous system equation. Consider the projection $z = F_0 x$ then $\dot{z} = F_0(A + BF_0)x$. If there exists a matrix H_0 such that $F_0(A + BF_0) = H_0 F_0$ then system (5) and domain (4) become respectively,

$$\dot{z} = H_0 z \tag{6}$$

$$D(\mathbb{I}_m, u_{\min}, u_{\max}) = \{z \in \mathbb{R}^m / - u_{\min} \leq z \leq u_{\max}\} \tag{7}$$

Definition 2.1: A subset D of $\mathbb{R}^m$ is said to be positively invariant with respect to system (6) if for every $z(t_o) \in D$, $z(t) \in D$, $\forall\ t \geq t_o$.

Definition 2.2: Let S_1 and S_2 be two subsets of $\mathbb{R}^m$ satisfying $S_1 \subset S_2$, we say that S_1 is S_2-attractive with respect to system (6) if for every $z(t_o) \in S_2$, there exist $t^* \geq$ to such that $z(t) \in S_1$ for every $t \geq t^*$.

The following result recalls the non-quadratic asymmetrical Lyapunov function.

Theorem 2.1: ([6]): Function

$$v(z) = \max_i \max\left(\frac{z_i^+}{u_{\max}^i}, \frac{z_i^-}{u_{\min}^i}\right) \tag{8}$$

with $u_{\min} > 0$, $u_{\max} > 0$, which is continuous positive definite, is a Lyapunov function of system (6), and domain $\mathrm{D}(\mathbb{I}_m, u_{\min}, u_{\max}) = \{z \in \mathbb{R}_m = v(z) \leq 1\}$ is a stability domain of system (6) if and only if:

$$\widetilde{H_0} U \leq 0, \tag{9}$$

where

$$U = \begin{bmatrix} u_{\max} \\ u_{\min} \end{bmatrix} \in \mathbb{R}^{2m}$$

Recall that ([6], [16]) $\dot{v}\,(z)$ is the directional derivative of function v at z in the direction of H_{0z} with $\dot{z}\,(t) = H_0 z(t)$. That is,

$$\dot{v}\,(z) = \lim_{\varepsilon\to 0^+} \frac{v(z + \varepsilon H_0 z) - v(z)}{\varepsilon}$$

We also have, if $\widetilde{H_0} U \leqslant -\beta U$, then $\dot{v}(z) \leqslant -\beta v(z)$ for any $\beta \geqslant 0$ ([6]).

In order to apply this result to system (1) with (3), we have to look for a couple of matrices (H_0, F_0) such that:

$$\begin{cases} F_0 A + F_0 B F_0 = H_0 F_0 \\ \widetilde{H_0} U \leq 0 \end{cases} \tag{10}$$

For this, consider the following approach, named inverse procedure, where H_0 is chosen to be a stable diagonalizable matrix such that:

$$\widetilde{H_0} U < -\varepsilon U \,, \tag{11}$$

where ε is a positive real number ($0 < \varepsilon$). The gain matrix F_0 is obtained by solving the following equation:

$$XA + XBX = H_0 X. \tag{12}$$

For the resolution of this equation, one can refer to ([9]).

3. MAIN RESULTS

The method presented here is built up from the inverse procedure: The stable diagonalizable matrix H_0 is chosen such that its eigenvalues are as close as possible to the imaginary axis, so the initial convergence rate performance of the corresponding system (6) is bad. Furthermore, there must exist a real $\varepsilon > 0$ such that inequality (11) is satisfied while equation (12) is solved to obtain the feedback gain F_0. Our objective is to start the closed loop with the dynamics of H_0 and finish it with those of H_N through N successive feedback control gain switches. In order to compute the N remaining gains, we begin by the computation of matrix H_i as follows,

$$H_i = H_0 - \beta_i \mathbb{I}_m, \ i = 1, \ldots, N. \tag{13}$$

Where

$$\beta_i = \varepsilon(\alpha^i - 1)$$

$\alpha > 1$ is the rate with which the convergence improvement is made as it will be shown in the second item of Remark 3.1. Feedback gain matrices F_i, $i = 1, \ldots, N$ are obtained from equation (12) off-line and stored because the feedback control is going to be switched N times.

Consider the following control law,

$$u(t) = F_i x(t) \tag{14}$$

Using the gain F_i, the closed-loop system is described by:

$$\dot{x}(t) = (A + BF_i)x(t) \tag{15}$$

To compute matrices H_i, $i = 1, \ldots, N$, one needs to know α. For this, we choose β_N which characterizes the eigenvalues of matrix H_N, since $\lambda_i(H_N) = \lambda_i(H_0) - \beta_N$, $i = 1, \ldots, m$, and compute α according to the following lemma:

Lemma 3.1: Consider system (1) with matrix H_0 satisfying (11) and matrices H_i, $i = 1, \ldots, N$ obtained from (13), if the integer N and the scalar β_N are fixed then the scalar α is given by,

$$\alpha = \exp\left(\frac{1}{N}\log\left(1 + \frac{\beta_N}{\varepsilon}\right)\right) > 1 \tag{16}$$

Proof: The real ε is known by virtue of equation (11) which is satisfied. From (13), if β_N is fixed then H_N is also fixed. Since $\beta_N = \varepsilon(\alpha^N - 1)$, equation (16) is then obtained.

It is worth noting that the spectrum of matrix $(A + BF_i)$ is the union of the spectrum of matrix H_i and the stable and desired eigenvalues of matrix A [9]. If the dominant eigenvalue of $(A + BF_i)$, i.e., the eigenvalue $\lambda_d \in \sigma(A + BF_i)$ satisfying $\mathrm{Re}(\lambda_d) = \max_j (\mathrm{Re}(\lambda_j(A + BF_i)))$, $j = 1, \ldots, n$, is an eigenvalue of H_i, then the convergence rate of system (15) is improved in the state space. Else, the convergence rate is improved only for system $\dot{u} = H_i u$ in the control space. Without loss of generality, we suppose that the dominant eigenvalue of $(A + BF_i)$ is an eigenvalue of H_i, $i = 0, \ldots, N$. Now we present a result concerning the positive invariance property of each domain $D(F_i, u_{\min}, u_{\max})$ with respect to the corresponding system.

Theorem 3.1: Consider system (1) with (14), where H_0 satisfies (11) and matrices H_i, $i = 1, \ldots, N$, computed from (13) then, every domain $D(F_i, u_{\min}, u_{\max})$ is positively invariant with respect to the corresponding system (15), $i = 1, \ldots, N$.

Proof: The first equation of (10) is satisfied because F_i is obtained from the resolution of equation (12). From (13), one obtains $\widetilde{H_i} = \widetilde{H_0} - \beta_i \mathbb{I}_{2m}$. Since H_0 satisfies (11), we can write $\widetilde{H_i}U = \widetilde{H_0}U - \beta_i U < -\varepsilon U - \varepsilon(\alpha^i - 1)U = -\varepsilon\alpha^i U$, that is $\widetilde{H_i}U < -\varepsilon\alpha^i U < 0$, the second equation of (10) is then satisfied.

Remarque 3.1

- Clearly, the origin 0 is a common interior point of all the predetermined domains $D_i = D(F_i, u_{\min}, u_{\max})$, $i = 1, \ldots, N$, which are neighborhoods of 0. Consequently, their intersection $\Delta_{\cap} = \bigcap_{i=0}^{N} D_i$ is a neighborhood of 0.
- If H_0 satisfies (11) then $Re(\lambda_i(H_0)) < -\varepsilon$, $i = 1, \ldots, m$, (see [6]). From the proof of Theorem 3.1, $\widetilde{H_i}U < -\varepsilon a^i U$ which implies that $Re(\lambda_i(H_j)) < -\varepsilon\alpha^j$, $i = 1, \ldots, m$, $j = 1, \ldots, N$. Consequently, α is the rate of convergence improvement when F_{i-1} is changed to F_i in the control law (14).

Comments

1. The obtained domains are not nested. This property is an advantage of this method; If the initial state $x(0)$ is chosen in $\Delta_{\cup} = \bigcup_{i=0}^{N} D_i$, the control law must be applied adequately: The gain F_k which will be used corresponds to the greatest integer k such that $x(0) \in D_k$. The controller gain F_k is then kept for all times $t \geqslant 0$ until the first time $t_1 > 0$ when $x(t_1) \in D_j$ for some $j > k$, then switches to F_j (j is the greatest integer such that $x(t_1) \in D_j$). Then, t_1 and $x(t_1)$ are taken as the initial time and initial state of an ordinary linear system. Once t^*, the first time such that $x(t^*) \in D_N$, is reached, the gain switches to the static gain F_N for all future times, leading to the asymptotic stability of the system inside D_N. At the switching instant, a discontinuity on the control trajectory appears but the imposed constraints remain respected.
2. In the study of the constrained regulation problems, it is important to obtain a large set of initial states such that if the state is initialized therein, then input bounds are never exceeded without causing any saturation. It is known that the size of this domain is limited by the constraints. This approach enables us to enlarge the domain of initial states to the cost of a slow transient's system convergence rate performance.
3. The other advantage of the proposed technic is the mastery of the spectrum of the closed-loop system. Indeed, if the eigenvalues of H_0 are known, those of H_i are obtained by simple translation using the coeffcient $(-\beta_i)$. This constitutes an improvement of the dynamics because $\beta_i > 0$ for $i = 1, \ldots, N$.
4. The previous results can also be justified by a Lyapunov functions argument. In fact, the function $v(u)$ given by (8), satisfies inequality $\dot{v}(u) < -\varepsilon\alpha^i v(u)$ ([6]). Since $\alpha > 1$, the rate with which $v(u)$ decreases inside D_i is better than the one inside D_{i-1}, $i = 1, \ldots, N$.

The following result is about the positive invariance property of $\Delta_\cup$.

Corollary 3.1: According to the proposed control law, $\Delta_\cup$ is positively invariant with respect to system (1) with (14).

Proof: The proof results immediately from the first item of the comments.

Theorem 3.2: $\Delta_\cap$ *is* $\Delta_\cup$-attractive with respect to system (1) and (14).

Proof: Let $x(t_o)$ be in $\Delta_\cup \backslash \Delta_\cap$ and $k_o \in [0, \ldots, N]$ be the greatest integer such that $x(t_o)$ be in D_{k_o}. Since system $\dot{x} = (A + BF_{k_o})x$ is asymptotically stable and $0 \in D_{k_o}$, then, if $k_0 < N$, there exists $t_1 \geqslant to$ and $k_1 > k_o$ such that $x(t_1) \in D_{k_1}$. The same idea is applied on t_1 and k_1 to find t_2 and k_2, etc.... Consequently, there exists t_N such that $x(t_N) \in D_N$. From the asymptotic stability property of the system inside D_N i.e., $\lim_{t\to\infty} x(t) = 0$, and the fact that $\Delta_\cap \subset D_N$ is a neighborhood of 0, there exists t^* such that $x(t) \in \Delta_\cap$ for every $t \geqslant t^*$, this ends the proof.

The interest of this result is to confirm that the initial convergence rate performance depends on the initial state but the final one is always imposed by the eigenvalues of H_N.

When our interest is focused on the size of the initialization domain and not on the performance improvement, the following result shows that $\Delta_\cup$ can be used as a large domain of stability for system (1) by applying a feedback control law with a fixed gain depending on the initial state.

Corollary 3.2: For every $x_o = x(0) \in \Delta_\cup$, there exists a gain F_k, $0 \le k \le N$, such that system (15) with $i = k$, is asymptotically stable and the control law $u = F_k x$ is admissible.

Proof: Obvious.

Note that in the case of Corollary 3.2, the dynamics of the closed-loop system depend on the initial state because there is no gain switching.

3.1 Particular Case: Single Input Linear Systems

In this case, all matrices F_i, $i = 1, \ldots, N$, are proportional to F_0. This can be seen from the fact that, considered as linear forms of $\mathbb{R}^n$, the F_i's have the same kernel. Since F_0 is nonzero, there is a scalar ψ_i such that $F_i = \psi_i F_0$. The value of ψ_i is given by the following proposition:

Proposition 3.1: In the particular case of single input linear systems such that $\sigma(A)$ has $(n - 1)$ stable eigenvalues, we have the following equations:

$$\begin{cases} F_i = \psi_i F_0 \\ \psi_i = 1 - \beta_i (H_0 - \lambda^*)^{-1} \end{cases} \quad i = 1, \ldots, N.$$

where the real λ^* is the eigenvalue of A which we want to change.

Proof: Let V be an eigenvector of A associated to the eigenvalue λ^*. Note that V is not in the kernel of F_0 which is spanned by the eigenvectors of A associated to the stable eigenvalues. So $F_0 V$ is a nonzero scalar. Now, we have

$$\lambda^* F_0 V = F_0 A V = (H_0 F_0 - F_0 B F_0) V = (H_0 - F_0 B) F_0 V$$

Then, one has $\lambda^* = H_0 - F_0 B$. In similar way, we get: $\lambda^* = H_i - F_i B$. Hence:

$$\begin{aligned} H_0 - F_0 B &= H_i - F_i B \\ &= H_0 - \beta_i - \psi_i F_0 B \end{aligned}$$

This leads to: $\psi_i(H_0-\lambda^*) = H_0-\lambda^*-\beta_i$. That is, $\psi_i = 1-\beta_i(H_0-\lambda^*)^{-1}$.

To illustrate this property, consider the following example.

Example 3.1: Consider system (1) with,

$$A = \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix},\ B = \begin{bmatrix} 1.5 \\ 0 \end{bmatrix}$$

(A, B) is controllable, $u_{\min} = 4$, $u_{\max} = 5$, that is $U = [\,5\ \ 4\,]^T$, the eigenvalues of A are $\sigma(A) = \{-4.2361\ \ 0.2361\}$. We hope to start with the spectrum $\{-0.2\ \ -4.2361\}$ and finish with the spectrum $\{-4.2\ \ -4.2361\}$ in three steps. That is, $N = 3$, $h_0 = -0.2$ and $\beta_3 = 4$. Using $\varepsilon = 0.19$, inequality (11) is satisfied. From equation (16) we obtain $\alpha = 2.8043$, which leads to the following results: $h_1 = -0.5428$, $h_2 = -1.5041$ and $h_3 = -4.2$. Furthermore,

$$\widetilde{H_i}U < -\varepsilon\alpha^i U,\ i = 0, \ldots, 3.$$

Equation (12) is solved four times to obtain feedback gains F_i, $i = 0, \ldots, 3$. The obtained results are:

$$F_0 = [-0.2907\quad -0.4704\,],\quad F_1 = [\,-0.5193\quad -0.8402],$$

$$F_2 = [-1.1601\quad -1.8772\,],\quad F_3 = [\,-2.9574\quad -4.7851]$$

and,

$$\psi_1 = 1.7861\ \ \psi_2 = 3.9905\ \ \psi_3 = 10.1722$$

It is easy to see that, $F_1 = \psi_1 F_0$ $F_2 = \psi_2 F_0$ $F_3 = \psi_3 F_0$ In this case, the obtained domains are nested because $\psi_3 > \psi_2 > \psi_1 > 1$, so,

$$D(F_3, u_{\min}, u_{\max}) \subset D(F_2, u_{\min}, u_{\max}) \subset D(F_1, u_{min}, u_{\max}) \subset D(F_0, \mathrm{u}_{\min}, u_{\max})$$

Example 3.2: Consider system (1) with,

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix},\ B = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}$$

with $u_{\min} = 2$, $u_{\max} = 1$. The system does not possess a stable eigenvalues ($n-m = 1$). An easy way of calculous to overcome the existence of $(n-m)$ stable eigenvalues problem is the introduction of a fictitious entry v (see [3]), with $v_{\min} = 3$ and $v_{\max} = 2.5$. In this case all the spectrum of the closed loop will be assigned. The counterpart is that the polyhedral domains become bounded. Big are $v_{\min}$ and $v_{\max}$, large are $D(F_i, u_{\min}, u_{\max})$, $i = 0, \ldots, N$, but matrix H_0 must be chosen such that equation (11) is satisfied. The augmented system is described by A and

$$B_a = \begin{bmatrix} 0.5 & 0 \\ 1 & 0 \end{bmatrix}.$$

We want to start the closed loop with the spectrum $\{-1\ -0.7\}$ and finish it with the spectrum $\{-4\ \ -3.7\}$ in three steps. That is $N = 3$ and $\beta_3 = 3$. Let

$$H_0 = \begin{bmatrix} -1 & 0.05 \\ 0 & -0.7 \end{bmatrix}$$

In this case $U = [\ 1\ \ 2.5\ \ 2\ \ 3]^T$. *With* $\varepsilon = 0.4$, inequality (11) is satisfied. From equation (16) we obtain $\alpha = 2.0408$, which leads to the following results:

$$\sigma(H_0) = \{-1 \quad -0.7\}$$
$$\sigma(H_1) = \{-1.4163 \ \ -1.1163]$$
$$\sigma(H_2) = \{-2.2660 \ \ -1.9660]$$
$$\sigma(H_3) = \{-4 \quad -3.7\}$$

Furthermore,

$$\widetilde{H_i}U < -\varepsilon\alpha^i U,\ i = 0, \ldots, 3.$$

Equation (12) is solved four times to obtain feedback gains F_i, $i = 0, \ldots, 3$. The obtained results are:

$$F_0 = \begin{bmatrix} -0.7 & -1.35 \\ 9.8 & 4.9 \end{bmatrix},\ F_1 = \begin{bmatrix} -1.5811 & -1.7421 \\ 35.3005 & 7.2737 \end{bmatrix},$$

$$F_2 = \begin{bmatrix} -4.4549 & -2.0045 \\ 175.1665 & -10.2809 \end{bmatrix},\ F_3 = \begin{bmatrix} -14.8 & -0.3 \\ 1095.2 & -273.8 \end{bmatrix}.$$

Note that the convergence rate performance of the closed-loop system increases through the predetermined domains while the constraints on the control vector are respected. In figure 1, the induced domains $D(F_0, u_{min},$

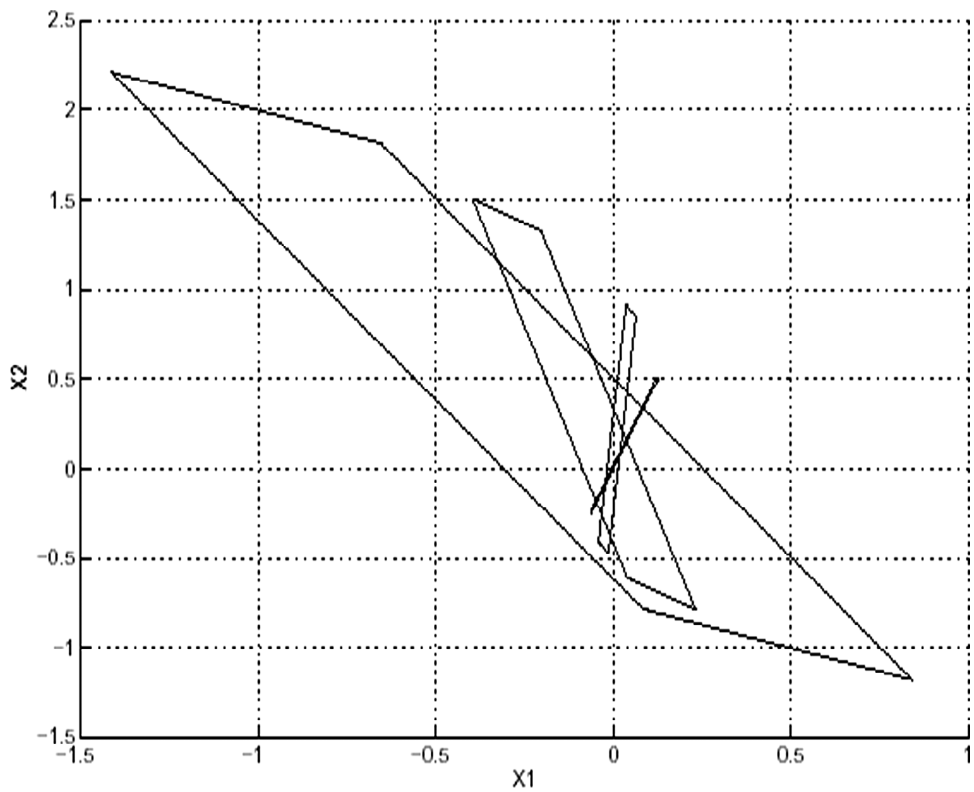

Figure 1: The Induced Domains in the State Space

u_{max}), $D(F_1, u_{\min}, u_{\max})$, $D(F_2, u_{\min}, u_{\max})$ and $D(F_3, u_{\min}, u_{\max})$ are presented in decreasing size sens. In figure 2, the components of the state and the control are presented with initial gain (dotted lines) and with the proposed control law (solid lines).

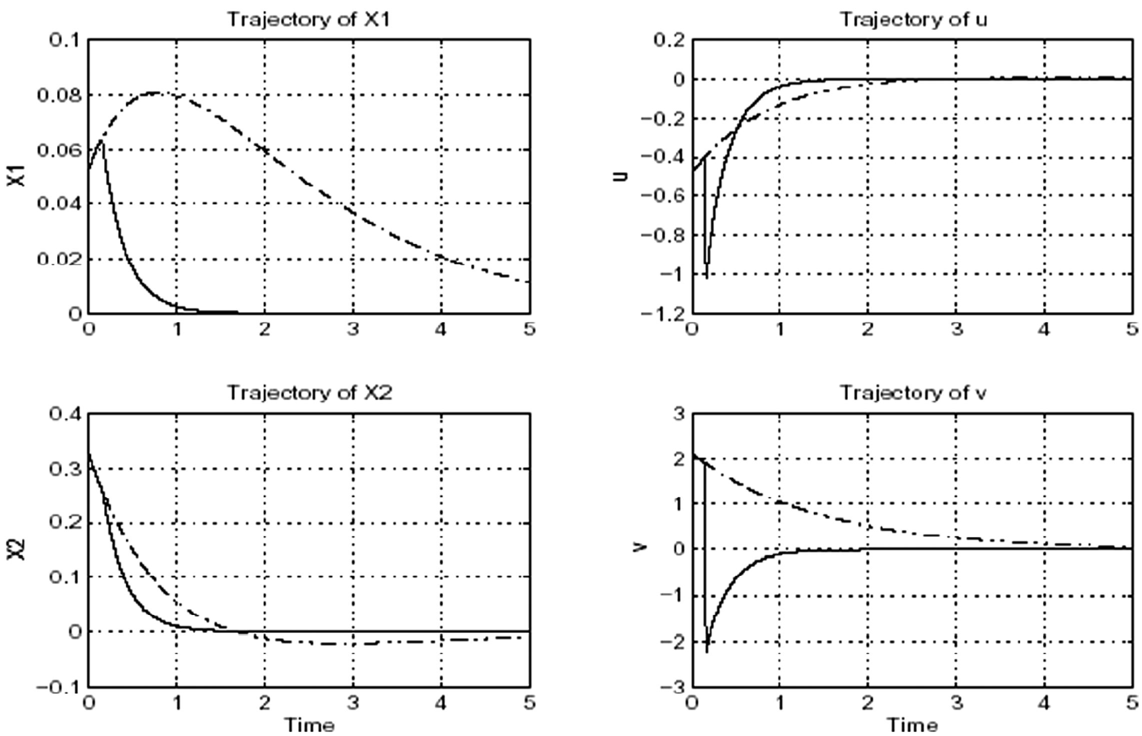

Figure 2: Dotted Lines Represent the Trajectories with Initial Gain. Solid Lines Represent the Trajectories with the Proposed Control Law

Remarque 3.2

- If fictitious entries are used to overcome the existence of (n–m) stable eigenvalues in the spectrum of A then, matrix H_0 cannot be chosen diagonale because in this case, its eigenvectors θ_i, $i = 1, \ldots, n$, are the canonical basis of $\mathbb{R}^n$, so $B_a\theta_i = 0$, $i = m + 1, \ldots, n$, consequently, equation (12) cannot be solved.
- The gains F_i really used to realize the control law (14) are

$$F_0 = [\,-0.7 \quad -1.35\,],\ \ F_1 = [-1.5811 \quad -1.7421\,],$$
$$F_2 = [\,-4.4549 \quad -2.0045\,],\ \ F_3 = [\,-14.8 \quad -0.3].$$

because v is a fictitious entry introduced to overcome the problem of the existence of $(n - m)$ stable eigenvalues in $\sigma(A)$ ([3]). In this case, the control law is applied on the initial system and not the augmented one.

4. CONCLUSION

A new approach is introduced to build a piecewise linear continuous-time systems with asymmetrical constraints on the control vector. The method is based on the positive invariance concept. Its application leads to overcome some limitations encountered in this kind of problems. Furthermore, it enables the mastery of the spectrum of the closed-loop system during all the steps of the gains switching without any control saturation. The increase in the computation time is avoided because all calculous are done off-line. After the last switch, the desired convergence rate is reached. Two examples are given to illustrate the obtained results.

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