Journal of Mathematical Control Science and Applications (JMCSA)
Vol. 3 No. 2 (July-December, 2017), ISSN : 0974-0570

Stream Quality Control via a Constrained Nonlinear Time-delay Model

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Abstract: This paper addresses the problem of water quality control in streams, taking into consideration transportation time delays, system constraints, imposed by water quality standard authorities, and economical aspects associated with waste treatment. To achieve these requirements, pollution control is realized through controlling both pollutant station discharge and pollution levels in effluents to be discharged in the stream. This leads to nonlinear system dynamics, and hence a constrained nonlinear control problem to be solved. A technique is developed to solve this problem and applied to control the concentration of the biochemical oxygen demand and dissolved oxygen in a three reach river system.

Keywords: Water quality control, Optimal control theory, Constrained optimization problem, Distributed systems, Time delay systems.

1. INTRODUCTION

Since the industrial revolution and the ever increasing dependence on heavy industries, major environmental problems have been created. One of these problems is the increase in pollution levels in streams. Pollutants in rivers can be categorized into four main categories: chemical, radio-active, heat, and biological. It is important, when evaluating the impact of industrial discharges on the river's ecosystem; to not only consider their collective characteristics, such as biochemical oxygen demand (BOD) and the amount of suspended solids, but also their content of specific inorganic and organic substances. Biological pollution is mainly a result of the dumping of sewage and fertilizers containing nutrients, such as nitrates and phosphates, into rivers. These nutrients, when present in excess amounts, over stimulate the growth of bacteria, aquatic plants and algae, which consequently: clog the waterways, use up dissolved oxygen (DO) as they decompose, and block light to deeper waters. All these factors contribute negatively and lead to many problems one of which is the respiration ability of fish and other invertebrates that reside in the water. Therefore, it is necessary to keep water quality in streams within a certain threshold in order to sustain aquatic life.

Three options are available in controlling industrial wastewater. In the first one, wastewater is treated to a fixed level, stored in tanks and discharged in a controlled manner into the water body. In the second option, wastes are discharged in the stream with a constant rate and pollution control is carried out through variable wastewater treatment. In the third option, wastewater can be treated completely at the plant and either reused or discharged directly into a receiving water body. However, from an economical point of view, increasing treatment levels beyond certain limits will dramatically increase the cost.

The river system is divided into reaches, where a reach is a stretch of the river, of some convenient length, which has a waste treatment facility at its beginning. Since rivers are characterized by their geographically distributed nature, transportation delay of pollutants between adjacent reaches, along the river basin, cannot be neglected. Thus, time delay models have to be used to represent the dynamics of such a system. From a practical

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point of view, it is often impossible to satisfy water quality standards while maintaining waste water treatment within reasonable levels, to avoid dramatic increases in cost. However, it is possible to achieve these requirements by combining different methods of pollution control in streams. In this paper, the first two methods are combined to satisfy both water quality standards as well as the economical aspects of the problem. In other words, variable wastewater treatment and effluent discharge are both used to control pollution levels in the river system. Accordingly, a bilinear model is produced. Moreover, since effluent will be discharged in a controller manner, wastes have to be, firstly, stored in tanks and then discharged in the river system in a controlled manner. The dynamics of the water volume in these tanks have to be considered while modeling the system, taking into consideration the constraints imposed by the limiting capacities of the tanks. As a result, we end up with a nonlinear optimization problem with time delays and system constraints to be solved.

Time delay control systems, have been solved using different approaches. One class of these approaches is based on using auxiliary variables to approximate the time delay variables, then applying the well developed technique to control the behavior of the extended system model [1,2]. The main drawbacks of this technique are the increased dimensionality of the model and time delay approximation, which may lead to an unstable system. Robust control methodologies constitute a class of these techniques which have been used to design feedback controllers to guarantee system stability. A lot of work is reported in the literature on this subject, among them we quote the works [3-7] and references therein.

On the other side, constrained linear quadratic control problem (LQP) have been solved using many techniques. The most attractive are based on model predictive control (MPC) [8-11], and anti-windup class of approaches [12, 13].

Recently, a new approach has been developed to solve continuous time constrained LQP [14, 15], which has been extended to handle discrete time systems [16, 17], as well as constrained LQP with time delays [18-20].

Clearly most of the work presented in literature deals with either time delay systems or LQP's with constraints on states and/or control. To the knowledge of the authors, time delay nonlinear constrained optimization problems have not been fully addressed. Since many practical control systems, such as the problem at hand, have such characteristics, it is worth trying to develop an algorithm solve thus complex problems.

In this paper, after developing the nonlinear model for water quality control in streams, the approach developed in [11,12] is extended to deal with nonlinear optimization problems with time delays and system constraints. Then, an application to a three reach river system is used to verify our theoretical investigation.

The rest of the paper is organized as follows. Section 2 is devoted to the model description. The approach used to solve this problem and the proposed algorithm, are presented in Section 3 and 4 respectively. In Section 5, simulation results of a BOD-DO control problem of the three reach river system are demonstrated. Finally, the paper is concluded in Section 6.

2. MODEL DESCRIPTION

Consider the following second-order state space model which describes the BOD-DO relationship at some average point in the *ith* reach, in which perfect mixing is assumed to take place, [5, 22]:

$$\text{BOD } (z)\text{: } \dot{z}_i = -(k_1 + k_3 + \frac{Q_1 + Q_E}{V_i})z_i + \frac{Q_{i-1}}{V_i} z_{i-1} + \frac{m_i Q_E}{V_i} \tag{1}$$

$$DO\ (q):\ \dot{q}_i = -(k_2 + \frac{Q_1 + Q_E}{V_i})q_i - k_1 z_i + \frac{Q_{i-1}}{V_i} q_{i-1} + k_2 q_i^s - \frac{k_4}{A_x dx} \quad (2)$$

where: z_i and q_i are, respectively, the concentration of BOD (mg/l) and DO (mg/l), k_1 is the rate of decay of BOD, k_2 is the re-aerations rate, k_3 is the rate of loss of BOD due to settling, $\frac{k_4}{A_x dx}$ is the removal of DO due to bottom sludge requirement and q^s is the concentration of DO at saturation level. Q_{Ei} is the flow rate of effluent, m_i is the concentration of BOD in effluent to be discharged, Q_i and Q_{i-1} are the stream flow rates in reaches i and i–1 respectively, and V_i is the volume of water.

The above model is based on a single reach of the river which has only one effluent input. Moreover, it is assumed that the concentrations of BOD and DO in the $(i-1)^{th}$ reach, i.e. z_{i-1} and q_{i-1} , affect the i^{th} reach instantaneously. For a more realistic assumption, transportation time delay between adjacent reaches has to be taken into consideration. Among the different approaches used to represent transportation delay [5,22], we choose the distributed time delay model. Accordingly, the BOD and DO model of the i^{th} reach takes the form:

$$\dot{z}_i(t) = -(k_1 + k_3)z_i(t) + \frac{Q_1}{V_i}\sum_{j=1}^{\theta} a_j z_{i-1}(t-\tau_j) - \frac{Q_i + Q_{Ei}}{V_i} z_i(t) + \frac{Q_{Ei}}{V_i} m_i \quad (3)$$

$$\dot{q}_i(t) = -k_1 z_i(t) - k_2 q_i(t) + \frac{Q_i}{V_i}\sum_{j=1}^{\theta} a_j q_{i-1}(t-\tau_j) - \frac{Q_i + Q_{Ei}}{V_i} q_i + k_2 q^s - \frac{k_4}{A_x dx} \quad (4)$$

where θ is the number of delays.

By combining the two methods stated above for water quality control, i.e. both variable effluent flow rate and BOD concentration in effluent to be discharged in the water body, this will in turn, contribute to the model as follows:

$$\dot{z}_i(t) = -(k_1 + k_3)z_i(t) + \frac{Q_1}{V_i}\sum_{j=1}^{\theta} a_j z_{i-1}(t-\tau_j) - \frac{Q_i + \bar{Q}_{Ei}}{V_i} z_i(t) + \frac{\bar{Q}_{Ei} + \Delta Q_{Ei}(t)}{V_i}(\bar{m}_i + \Delta m_i(t)) \quad (5)$$

$$\dot{q}_i(t) = -k_1 z_i(t) - k_2 q_i(t) + \frac{Q_i}{V_i}\sum_{j=1}^{\theta} a_j q_{i-1}(t-\tau_j) - \frac{Q_i + \bar{Q}_{Ei} + \Delta Q_{Ei}(t)}{V_i} q_i(t) + k_2 q^s - \frac{k_4}{A_x dx} \quad (6)$$

where: $\bar{Q}_{Ei}$ and $\bar{m}_i$ are the nominal values, while $\Delta Q_{Ei}(t)$ and $\Delta m_i(t)$ are the deviations from these values. Since we have variable effluent discharge, effluents from polluter stations have to be stored in tanks. We assume that the input rates to these tanks are constant, while the output flows are variable and controllable. Therefore, the dynamic equation describing the volume of the effluent stored in the tank is given by:

$$\dot{\eta}_i(t) = F_i \frac{\bar{Q}_{Ei}}{V_i} - \frac{\Delta Q_{Ei}(t)}{V_i} \quad (7)$$

where: $\dot{\eta}_i(t)\ \frac{\dot{V}_{Ti}}{V_i}, \dot{V}_{Ti}$ is the rate of change of the volume of the i^{th} tank, F_i is the inflow rate of effluent into the tank, assumed constant, and $\frac{\bar{Q}_{Ei} + \Delta Q_{E_i}(t)}{V_i}$ is its outflow rate. Assuming that the inflow rate equals the nominal outflow rate of the effluent, i.e. $F_i = \frac{\bar{Q}_{Ei}}{V_i}$, we get:

$$\dot{\eta}_i(t) = \frac{\Delta Q_{Ei}(t)}{V_i} \tag{8}$$

One has to note that $\eta_i(t)$ is bounded by upper and lower limits imposed by the tank capacity.

Let $= \Delta m_i\ (t)\ u_{1i}\ (t)$, $\frac{\Delta Q_{Ei}(t)}{V_i} = u_{2i}(t)$, and defining the control and state vectors as follows:

$u^T(t) = [u_{11}(t)\ u_{21}(t)\ \ldots\ u_{11}\ (t)\ u_{2l}\ (t)]$ and
$x^T(t) = [z_1(t)\ q_1(t)\ \eta_1(t) \ldots\ z_l(t)\ q_l(t)\ \eta_l(t)\ ]$

with l is the number of reaches in the river system. Combining equations (5), (6) and (8), the system can be described by the following state variable model:

$$\dot{x}(t) = A_o x(t) + \sum_{j=1}^{\theta} A_j x(t - \tau_j) + Bu(t) + f(x,u,t) + d \tag{9}$$

with, $x(t_o) = x_o$

where: $x \in R^n$, $u \in R^m$, are, respectively, the state and control vectors, $n=3l$, $m = 2l$, $A_j \in R^{n\times n}$; $j \in \{1,\ldots,\theta\}$, $B \in R^{n\times m}$, are the linear parts of the system matrices, θ is a known positive integer representing the number of delays in the state vector, $f(x, u, t) \in R^n$ is a vector function that includes the nonlinear terms in the model, and $d \in R^n$ is a constant vector.

According to water quality standards, the concentration of BOD must not exceed a maximum threshold, whilst the concentration of DO must not be less than a minimum value, otherwise fish and other aquatic life residing in the water body would decease.

Moreover, from an economical point of view, waste water treatment must not go beyond a maximum value, otherwise operating costs will increase dramatically. Based on these physical and economical considerations, the following constraints need to be added to our model:

$$\underline{x} \leq x\ (t)\ x \leq \bar{x} \tag{10}$$

$$\underline{u} \leq u\ (t) \leq \bar{u} \tag{11}$$

where: $\underline{x}$, $\bar{x}$, $\underline{u}$, $\bar{u}$ are the lower and upper bounds of the state and control vectors (element by element).

It will be assumed in the rest of the paper that:

$$x(t - \tau_j) = x^d\ \forall t - \tau_j < 0; j \in \{1,\ldots., \theta) \tag{12}$$

where, x^d is the desired steady-state or nominal values of the state variables.

Associated with the above system is a performance index J, to be minimized with respect to x and u, of the form:

$$\min_{x,u} J = \frac{1}{2}\int_{t_o}^{t_f} (\left\|x(t) - x^d\right\|_Q^2 + \left\|u(t)\right\|_R^2)dt \tag{13}$$

where: $Q > 0 \in R^{n\times n}$, $R > 0 \in R^{m\times m}$ are block diagonal weighting matrices, with the number of blocks equaling the number of reaches in the river, while t_o and t_f are the initial and final times, respectively, and are assumed to be known.

3. THE DEVELOPED APPROACH

Using a procedure similar to that given in [14, 15], and assuming that three delays represent the effluent's transportation delay from the beginning, middle and end of reach, $i - 1$, the above optimization problem can be reformulated as follows:

$$\min_{x,\, x^o,\, u,\, u^o} J = \frac{1}{2}\int_{t_o}^{t_f} (\left\|x^o(t) - x^d\right\|_{Q_1}^2 + \left\|u(t)\right\|_{R_1}^2 + \left\|u(t) - u^o(t)\right\|_{R_2}^2)\, dt \tag{14}$$

subject to :

$$\dot{x}(t) = A_o x(t) + \sum_{j=1}^{3} A_j x^o(t - \tau_j) + Bu(t) + f(x^o, u^o, t) + d \tag{15}$$

with $x(t_o) = x_o$

$$x(t) = x^o(t) \tag{16}$$

$$u(t) = u^o(t) \tag{17}$$

$$\underline{x} \le x^o(t) \le \overline{x} \tag{18}$$

$$\underline{u} \le u(t) \le \overline{u} \tag{19}$$

where $\frac{1}{2}\left\|u(t) - u^o(t)\right\|_{R_2}^2$ is a convexifying term which may be added to speed up the convergence rate of the algorithm. At the end of convergence, $u^o \to u$, thus the convexifying term will not contribute to the value of the cost function.

The Hamiltonian of the above problem is given by:

$$H(x, x^o, u, u^o, \lambda, \pi, \beta, t) \triangleq \frac{1}{2}\left\|x^o(t) - x^d(t)\right\|_{Q_1}^2 + \frac{1}{2}\left\|u(t)\right\|_{R_1}^2 + \frac{1}{2}\left\|u(t) - u^o(t)\right\|_{R_2}^2$$

$$+\lambda^T(t)[A_o x(t) + \sum_{j=1}^{3} A_j x^o(t - \tau_j) + Bu(t) + f(x^o, u^o, t) + d] + \pi^T(t)[x(t) - x^o(t)] + \beta^T(t)[u(t) - u^o(t)] \tag{20}$$

where: $\lambda(t) \in R^n$ is the co-state vector, while $\pi(t) \in R^n$ and $\beta(t) \in R^m$ are the Lagrange multipliers associated with the equality constraints (16) and (17) respectively.

The necessary conditions of optimality give:

$$\frac{\partial H}{\partial u(t)} = 0 \Rightarrow u(t) = (R_1 + R_2)^{-1} [R_2\, u^o(t) - B^T \lambda(t) - \beta(t)]$$

$$= \Gamma(u^o, \lambda, \beta, t) \tag{21}$$

However, to satisfy the constraints given by (19), the control vector which minimizes the Hamiltonian is given by [21]:

$$u_{qi}(t) = \begin{cases} \underline{u}_{qi} & \text{if} \quad \Gamma_{qi}(u^o,\lambda,\beta,t) < \underline{u}_{qi} \\ \Gamma_{qi}(u^o,\lambda,\beta,t & \text{if} \quad \underline{u}_{qi} \le \Gamma_{qi}(u^o,\lambda,\beta,t) \le \overline{u}_{qi} \\ \overline{u}_{qi} & \text{if} \quad \Gamma_{qi}(u^o,\lambda,\beta,t) > \overline{u}_{qi} \end{cases} \tag{22}$$

where $q \in \{1, 2\}$ for the i^{th} reach.

$$\frac{\partial H}{\partial \lambda(t)} = \dot{x}(t) \Rightarrow \dot{x}(t) = A_o x(t) + \sum_{j=1}^{3} A_j x^o(t-\tau_j) + Bu(t) + f(x^o,u^o,t) + d \tag{23}$$

with $x(t_o) = x_o$

$$\frac{\partial H}{\partial x(t)} = -\dot{\lambda}(t) \Rightarrow \dot{\lambda}(t) = -A_o^t \lambda(t) - \pi(t) \tag{24}$$

with $\lambda(t_f) = 0$

$$\frac{\partial H}{\partial u^o(t)} = 0 \Rightarrow \beta(t) = R_2 u^o(t) - R_2 u(t) + \frac{\partial f^T(x^o,u^o,t)}{\partial u^o}\lambda(t) \tag{25}$$

$$\frac{\partial H}{\partial x^o(t)} = 0 \Rightarrow x^o(t) = Q_1^{-1}[Q_1 x^d - \sum_{j=1}^{3} A_j^T \lambda(t+\tau_j) - \frac{\partial f^T(x^o,u^o,t)}{\partial x^o}\lambda(t) + \pi(t)]$$

$$= \psi(x^o, u^o, \lambda, \pi, t) \tag{26 }$$

Since $x^o(t)$ is obtained for an algebraic equation, it can be treated by the same way as the control variables. Therefore, to satisfy system constraints given by (18), the coordinating vector $x^o(t)$, which minimizes the Hamiltonian, is given by [21]:

$$x_{pi}^o(t) = \begin{cases} \underline{x}_{pi} & \text{if} \quad \Psi_{\text{pi}}(x^o,u^o,\lambda,\pi,t) < \underline{x}_{pi} \\ \psi_{pi}(x^o,u^o,\lambda,\pi,t & \text{if} \quad \underline{x}_{pi} \le \psi_{pi}(x^o,u^o,\lambda,\pi,t) \le \overline{x}_{pi} \\ \overline{x}_{pi} & \text{if} \quad \Psi_{pi}(x^o,u^o,\lambda,\pi,t) > \overline{x}_{pi} \end{cases} \tag{27}$$

where $p \in \{1, 2, 3\}$ and i is the i^{th} reach.

$$\frac{\partial H}{\partial B(t)} = 0 \Rightarrow u^o(t) = u(t) \tag{28}$$

Finally we have:

$$\frac{\partial H}{\partial \pi(t)} = x(t) - x^o(t) \tag{29}$$

this leads to the updating algorithm for π (t) given by:

$$\pi^{k+1}(t) = p^k(t) + \alpha^k l^k(t) \tag{30}$$

where: k is the iteration number, $l^k(t)$ can be specified according to the selected algorithm (steepest descent, conjugate gradient, etc.), while α^k has to be positive to insure the maximization of the dual function w.r.t. the dual variable $\pi^k(t)$. In our procedure, and throughout this paper, we applied the gradient technique to update $\pi^k(t)$, with $l^k(t)$, $= x^k(t) - x^{ok}(t)$

4. THE ALGORITHM

Based on the above necessary conditions of optimality, the following two level algorithm is proposed:

Initialization: Give an initial guess to the vectors $\pi^k(t)$, $\beta^k(t)$, $u^{ok}(t)$, $x^{ok}(t)$ and put the iteration number $k = 1$.

Level # 1

Step (1): If $k = 1$, then

Calculate the co-state vector $\lambda^k(t)$ by backward integration of equation (24), with the final condition $\lambda(t_f) = 0$.

Go to level # 2.

Else

Calculate the error criterion:

$$error = \sqrt{\sum_{t=t_0}^{t_f} \left\| x^k(t) - x^{ok}(t) \right\|^2}$$

If error < ε, where ε is a pre-specified small constant, record the trajectories and stop.

Else

Step (2): Update the trajectories of the Lagrange multiplier vectors $\pi^k(t)$, $\beta^k(t)$ using (30) and (25) respectively. Then, calculate $u^{ok}(t)$ from (28), and $\lambda^k(t)$ using (24). Finally calculate x^{ok} from (27) while using $x^{o(k-1)}(t)$, $\lambda^k(t)$, $\pi^k(t)$, $u^{ok}(t)$.

End if

End if

Level # 2

Step (3): Calculate $u^k(t)$ using (22) and $x^k(t)$ by forward integration of (23), with initial condition $x(t_o) = x$.

Put $k = k + 1$. **Go to level # 1.**

End

Based on the above algorithm, it is worth to state the following remarks:

1. No additional multiplier (Kuhn-Tacker parameters) are needed to satisfy inequality constraints since the satisfaction of $\dfrac{\partial H}{\partial \pi} = 0$ insures the satisfaction of the inequality constraints.

2. In addition to $\dfrac{1}{2}\left\| u(t) - u^o(t) \right\|_{R_2}^2$, another convexifying term, $\dfrac{1}{2}\left\| x(t) - x^o(t) \right\|_{Q_2}^2$, could be added to the cost function to further improve the convergence rate of the algorithm. Again, as $k \to \infty \Rightarrow x^{ok} \to x^k$, and the original cost function will not be modified. However, in this case, the necessary conditions of optimality will be modified. Although the state and co-state equations will be coupled, we do not need to solve the TPBVP at the same level, since $\dot{\lambda}(t)$ is solved at the higher level while $\dot{x}(t)$ is integrated at the lower level. Therefore, the algorithm stays relatively simple since what is required is to solve simple vector differential and algebraic equations.

5. SIMULATION

Consider a three reach river system, the i^{th} of which is represented by (5), (6), (8). Typical values of the system parameters are given by:

$k_1 = 0.32$, $k_2 = 0.2$, $k_3 = 0.0$, $\dfrac{Q_i}{V_i} = 0.9$, $\dfrac{Q_{Ei}}{V_i} = 0.1$, $q^s = 10$ mg/l $\dfrac{k_4}{V_i} = 0.1$, number of delays = 3, $a_1 = 0.15$, $a_2 = 0.7$, $a_3 = 0.15$, $\tau_1 = 0$, $\tau_2 = 0.5$, $\tau_3 = 1.0$

After transforming the model into the form described by equations (9-11), the system has been simulated over a period of 10 days with the following set of data concerning the initial conditions, the steady state values, the constant vector d, and the upper and lower bounds of the state and control trajectories:

$$x_o^T = [10\;\; 7\;\; 0.01\;\; 5.937\;\; 6\;\; 0.01\;\; 5.707\;\; 4.5614\;\; 0.01]$$

$$x^{d^T} = [4.053\;\; 8\;\; 0.01\;\; 5.937\;\; 6\, 0.01\;\; 5.707\;\; 4.5614\;\; 0.01]$$

$$d^T = [5.35\;\; 10.9\, 0\;\; 4.19\;\; 1.9\, 0\;\; 2.19\;\; 1.9\;\; 0]$$

$$\underline{x}^T = [0\;\; 6\;\; 0.001\;\; 0\;\; 5.6\;\; 0.001\;\; 0\;\; 4.22\;\; 0.001]$$

$$x^{-T} = [10\;\; open\;\; 0.055\;\; 6.55\;\; open\;\; 0.034\;\; 6\;\; open\;\; 0.013]$$

$$\underline{u}^T = [-0.24\;\; -0.1\;\; -0.35\;\; -0.1\;\; -0.059\;\; -0.1]$$

$$\overline{u}^T = [0\;\; 0.1\;\; 0\;\; 0.1\;\; 0\;\; 0.1]$$

The concentrations of BOD along the 3 reaches are shown in Figs. (1), (6), (11), whilst those of DO are presented by Figs.(2), (7), (12). The changes in the volume of effluent held by each of the reach's tank are given in Figs. (3), (8), (13). Figs. (4), (9), (14) show the variations of waste water treatments, around their nominal values. Finally, the changes of effluent discharges are demonstrated in Figs. (5), (10), (15). For the purpose of comparison with the unconstrained case, each graph includes both the optimal constrained and unconstrained state or control trajectories.

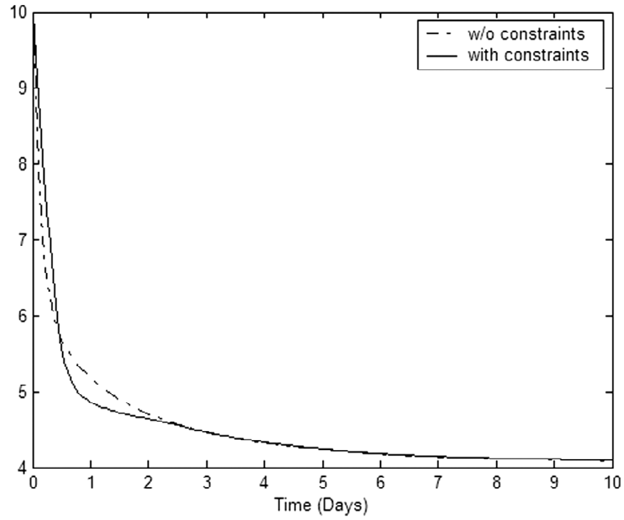

Figure 1: BOD Concentration in the 1st Reach

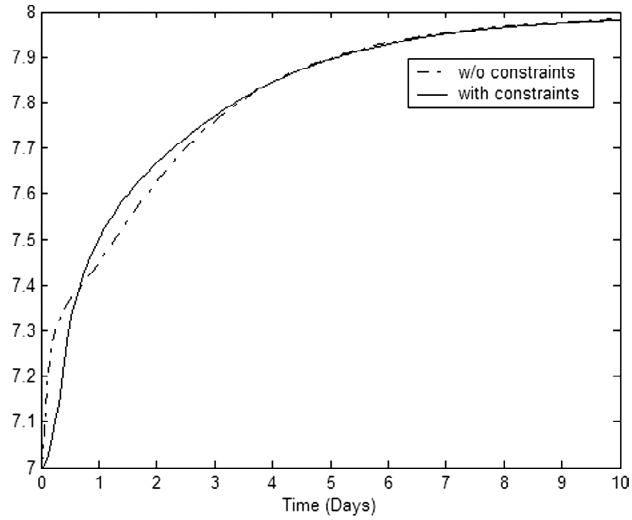

Figure 2: DO Concentration in the 1st Reach

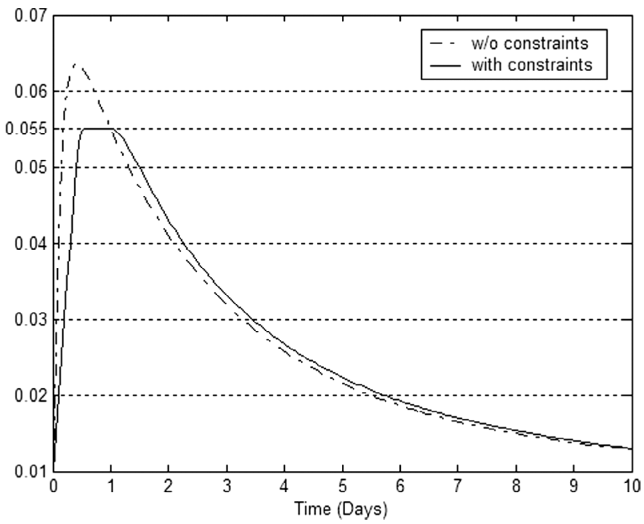

Figure 3: Tank Volume of the 1st Reach

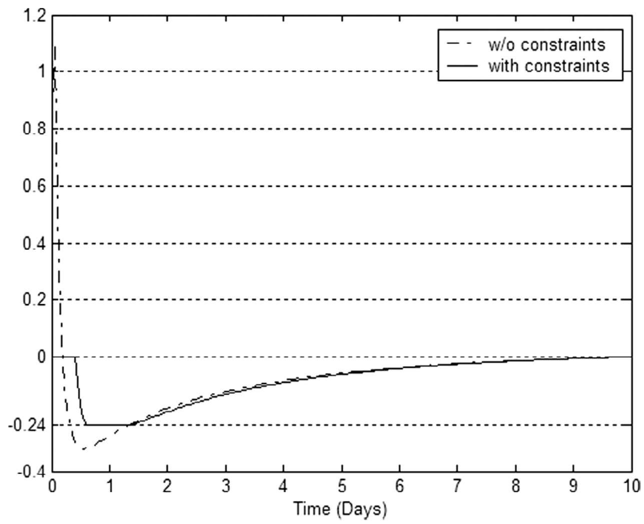

Figure 4: Variable Treatment Control of the 1st Reach

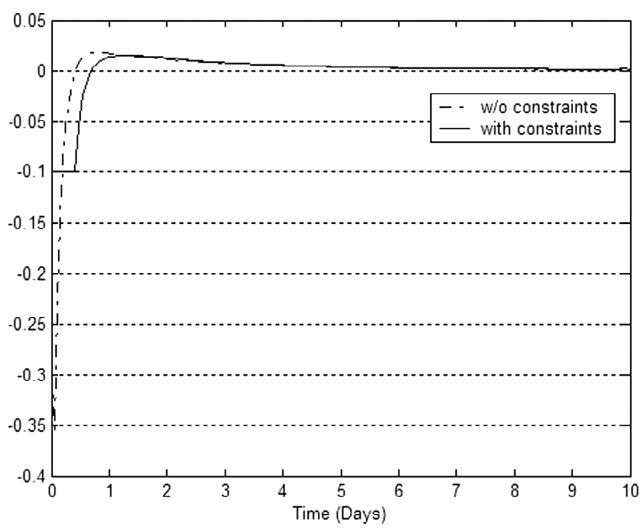

Figure 5: Discharge Control of the 1st Reach

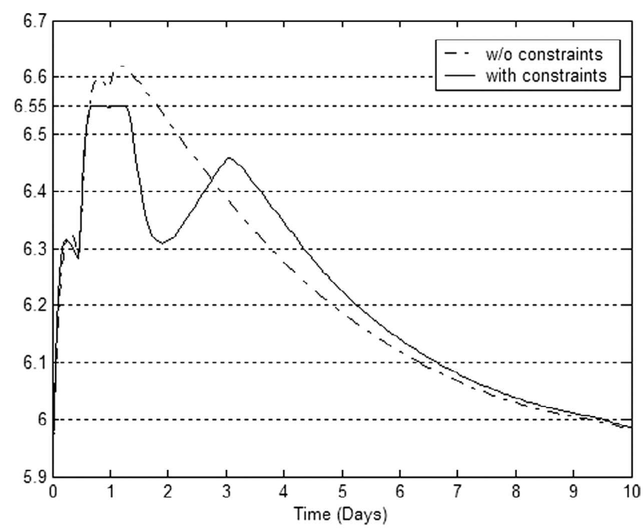

Figure 6: BOD Concentration in the 2nd Reach

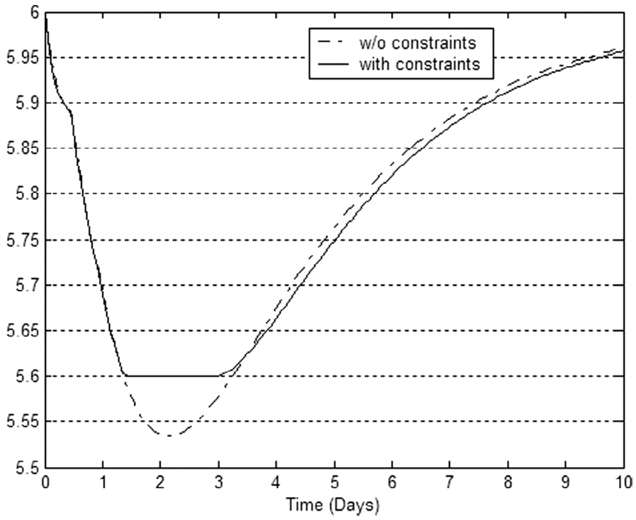

Figure 7: DO Concentration in the 2nd Reach

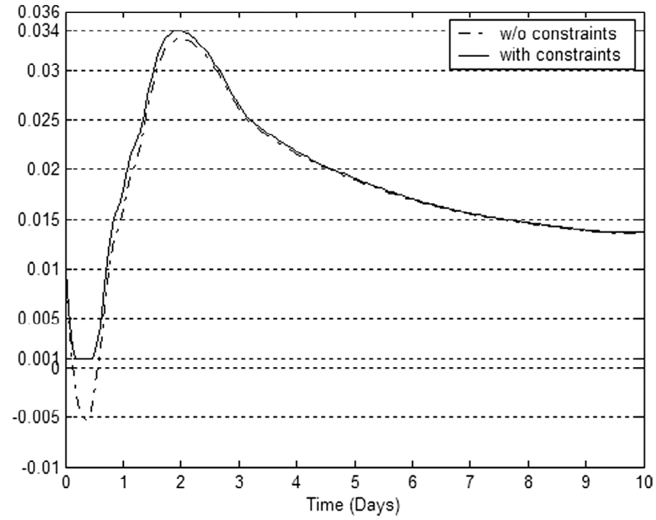

Figure 8: Tank Volume of the 2nd Reach

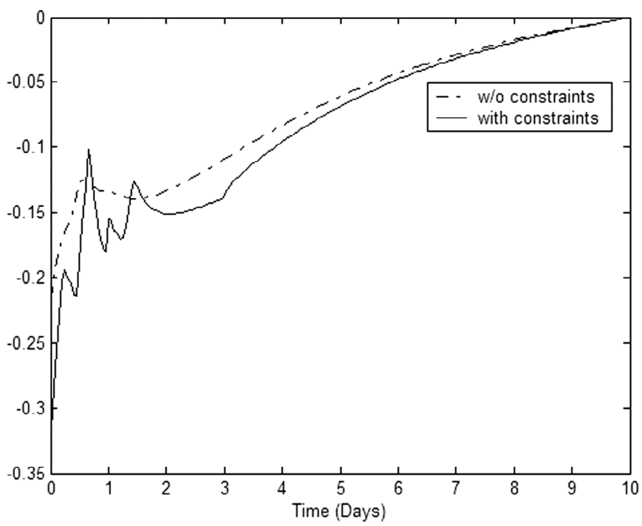

Figure 9: Variable Treatment Control of the 2nd Reach

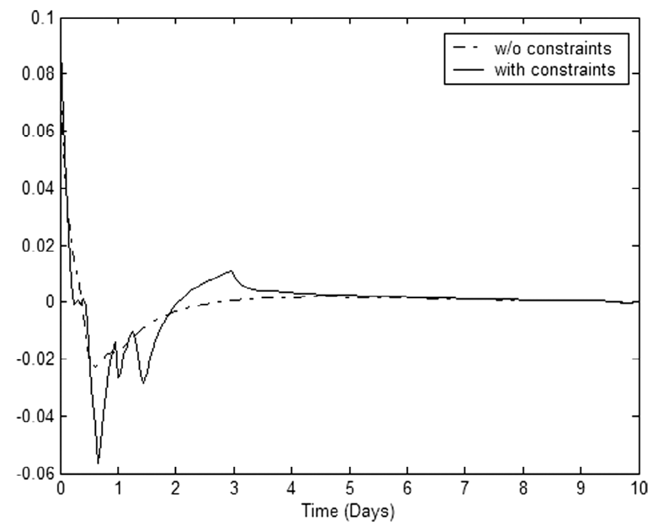

Figure 10: Discharge Control of the 2nd Reach

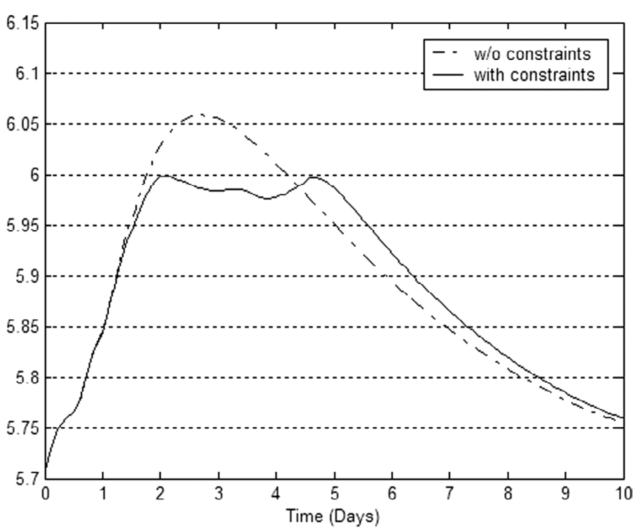

Figure 11: BOD Concentration in the 3rd Reach

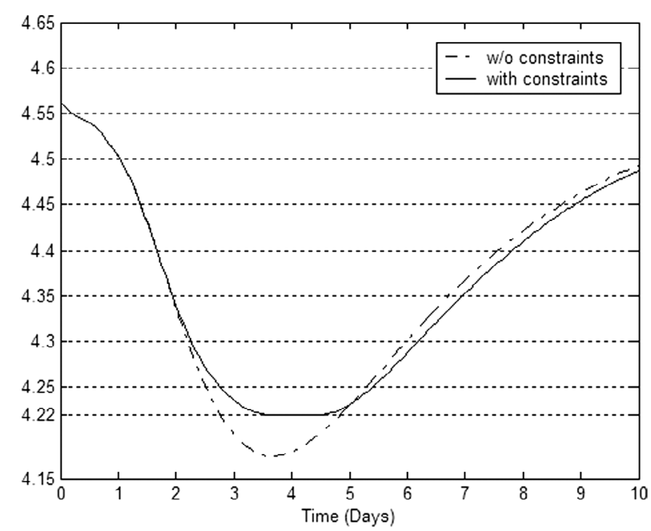

Figure 12: DO Concentration in the 3rd Reach

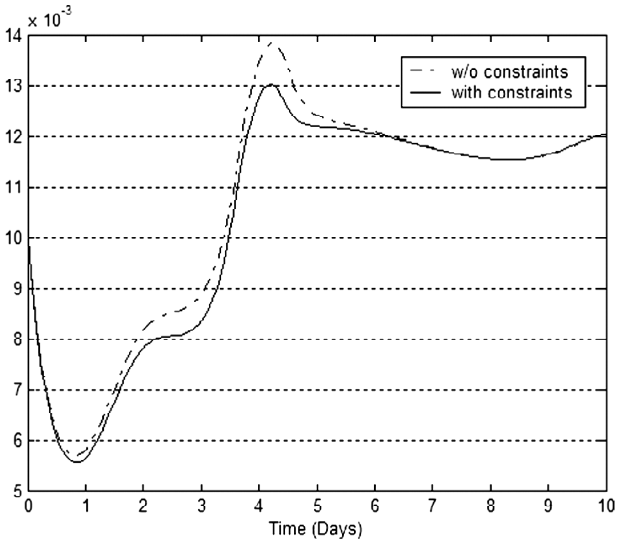

Figure 13: Tank Volume of the 3rd Reach

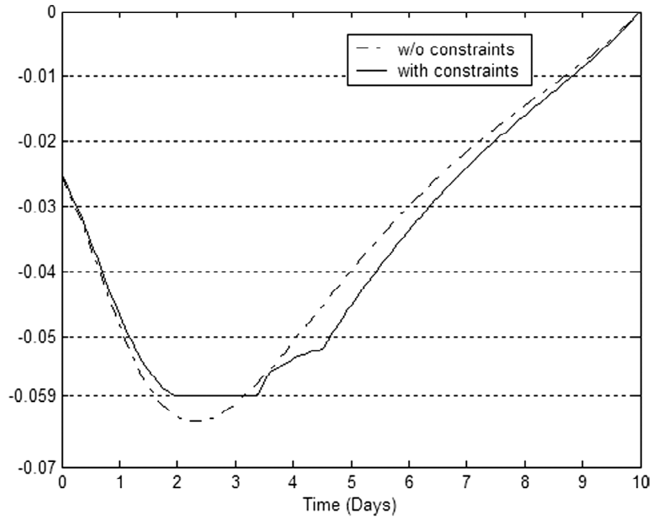

Figure 14: Variable Treatment Control of the 3rd Reach

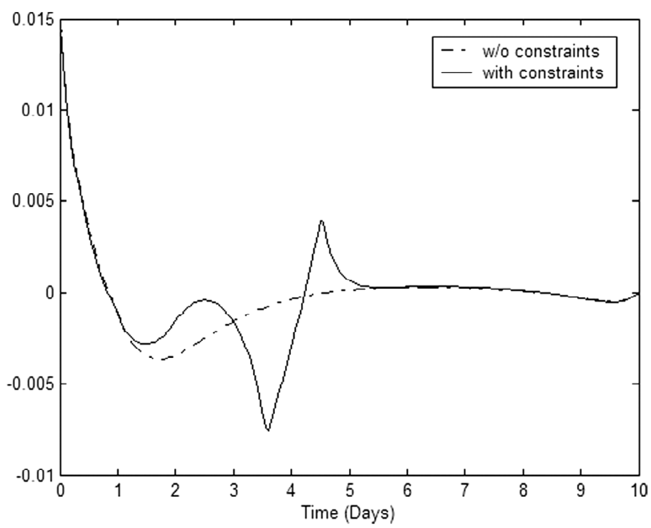

Figure 15: Discharge Control of the 3rd Reach

It should be noted that using a linear control strategy, i.e. without the addition of the second controller [18-20], namely $\frac{\Delta Q_{Ei}(t)}{V_i}$, it was not possible to simultaneously satisfy the constraints imposed on both the maximum level of BOD and the minimum level of DO. In this case, since it was physically unrealizable to achieve these requirements, the algorithm did not converge and reached a limit cycle.

6. CONCLUSION

In this paper, continuous nonlinear optimization problem with time delays and system constraints is investigated. A new multilevel algorithm is developed to solve this problem. Besides satisfying system constraints, the developed algorithm will not increase the dimensionality of the system due to time delayed state variables. A simulation on a three reach river system is carried out to show the applicability and efficiency of the developed technique. It can be seen that the algorithm was successful in reaching the optimal solution while satisfying systems constraints.

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