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# Delay-Dependent Robust Stabilization for Uncertain Discrete Stochastic Fuzzy Time-Delay Systems

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**Abstract:** This paper deals with the problem of robust stabilization for uncertain discrete stochastic fuzzy systems with time-varying delays. The uncertainties are time-varying but norm-bounded. The purpose is to design a state feedback fuzzy controller such that the resulting closed-loop system is robustly stochastically stable. Based on Takagi-Sugeno (T-S) fuzzy model, a delay-dependent sucient condition for the solvability of the problem is obtained by utilizing a proper Lyapunov functional together with the linear matrix inequality (LMI) approach. Finally, a numerical example is provided to demonstrate the applicability of the proposed design method.

**Keywords:** Delay-dependent stabilization; Discrete Takagi-Sugeno (T-S) fuzzy model; Linear matrix inequality; Stochastic systems; Time-varying delays.

## 1. INTRODUCTION

Fuzzy logic control has attracted increasing attention because it can provide an alternative approach to the control of plants that are complex, uncertain, ill-defined, and has available qualitative knowledge from domain experts for their controller design during the last four decades. Among several model-based fuzzy logic control approaches, the method based on the Takagi-Sugeno (T-S) fuzzy model has become quit popular in the past three decades. T-S fuzzy model can drastically reduce the number of rules in modeling higher nonlinear systems. Consequently, T-S fuzzy models are less prone to the curse of dimensionality than other fuzzy models. More importantly, T-S fuzzy models provide a basis for development of systematic approaches to stability analysis and controller design of fuzzy control systems [6].

Since the T-S fuzzy model were proposed by Takagi and Sugeno [16], there have been dramatic progress in stability analysis and control design of this model-based fuzzy systems [2, 15, 17]. When parametric uncertainties appear in a T-S fuzzy system, the robust stability problem was addressed in [12, 18], where the stability conditions were expressed in terms of LMIs. Sucient conditions for the solvability of the robust $H_\infty$ fuzzy control problem for uncertain T-S fuzzy systems were proposed in [2, 3, 9, 11] by using the algebraic Riccati inequality-based approach and the LMI-based approach, respectively. Time delays are frequently encountered in various engineering systems such as aircraft, long transmission lines in pneumatic systems, and chemical or process control systems. It has been shown that the existence of time delays is often one of the main causes of instability and poor performance of a control system [8]. T-S fuzzy systems with time delays have been studied over the last decades. In [4], the stability analysis and stabilization problems for such T-S fuzzy delay systems were considered, and state feedback fuzzy controllers and fuzzy observers were designed via the so-called parallel distributed compensation (PDC) scheme. The robust $H_\infty$ control problem for uncertain T-S fuzzy systems with time delays was investigated in [10]. The corresponding results for the discrete case can be found in [5, 23, 26].

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On the other hand, the study of stochastic systems has attracted considerable attention. Since stochastic modeling has come to play an important role in many branches of science and engineering applications, many fundamental results for deterministic systems have been extended to stochastic systems [20, 24]. When parameter uncertainties and time-delay appear in stochastic systems, the problems of robust stochastic stability analysis and controller design have been studied in [14, 21, 22], where some useful robust stochastic stability conditions were proposed. The combination of stochastic system and T-S fuzzy model can be seen in [13], where the stochastic stability and fuzzy controller design were investigated. The delaydependent stabilization problem for stochastic fuzzy systems with constant time delays was addressed in [25]. A sliding mode fuzzy controller was designed in [7] to stabilize a stochastic T-S fuzzy system with unknown nonlinearities and constant time delays. It should be pointed out that the studies in [7, 13, 25] are concerned with the continuous-time case. To the best of our knowledge, so far, no results on the delaydependent roust stabilization for discrete stochastic T-S fuzzy systems are available in the literature, which is still open and remains unsolved. This motivates the present study.

In this paper, we are concerned with the delay-dependent stabilization problem for uncertain discrete stochastic fuzzy systems with time-varying delays. The parameter uncertainties are assumed to be timevarying norm-bounded, and the system delay is supposed to be time-varying but bounded. We aim at designing a state-feedback fuzzy controller such that the resulting closed-loop system is robustly stochastically stable. A delay-dependent sucient condition for the solvability of the formulated problem is proposed. The desired state feedback controller is constructed by solving certain LMIs, which can be easily implemented by using standard numerical algorithms [1]. We also provide a numerical example to demonstrate the eectiveness and applicability of the proposed method.

**Notation:** Throughout this paper, for real symmetric matrices $X$ and $Y$ , $X \geq Y$ (respectively, $X > Y$) means that the matrix $X–Y$ is positive semi-definite (respectively, positive definite). I is an identity matrix with appropriate dimension. The superscript "$T$" represents the transpose of a matrix. The notation "*" is used as an ellipsis for terms that are induced by symmetry. $\mathbb{N}$ is the set of natural numbers. $l_2\,[0,\infty)$ refers to the space of square summable infinite vector sequences. The notation $\|\cdot\|$ stands for the usual $l_2[0,\infty)$ norm while $|\cdot|$ refers to the Euclidean vector norm. We use $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$, respectively, to denote the minimum and maximum eigenvalue of a symmetric matrix. $diag(A_1, A_2, \ldots, A_n)$ denotes the following diagonal matrix,

$$\begin{bmatrix} A_1 & 0 & \cdots & 0 \\ 0 & A_2 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & A_n \end{bmatrix}.$$

Matrices, if the dimensions are not explicitly stated, are assumed to have compatible dimensions for algebraic operations.

## 2. DEFINITIONS AND PROBLEM FORMULATION

Consider the following uncertain discrete stochastic T-S fuzzy system with time-varying delays:

**Plant Rule** $i$: **IF** $s_1(k)$ is $\mu_{i1}$ and $s_2(k)$ is $\mu_{i2}$ . . . and $sg(k)$ is $\mu_{ig}$ THEN

$$\begin{aligned} x(k+1) &= A_i(k)x(k) + A_{di}(k)x(k-\tau(k)) + B_{1i}(k)u(k) \\ &\quad +[E_i(k)x(k) + E_{di}(k)(k-\tau(k)) + B_{2i}(k)u(k)]\omega(k), \qquad (1) \\ x(k) &= \phi(k),\ k = -\tau_2, -\tau_2+1, \cdots, 0.\ i = 1, 2, \cdots, r, \qquad (2) \end{aligned}$$

where $\mu_{ij}$ is the fuzzy set and $r$ is the number of IF-THEN rules; $x(k) \in \mathbb{R}^n$ is the state; $u(k) \in \mathbb{R}^m$ is the control input; $s_1(k), s_2(k), \ldots, sg(k)$ are the premise variables which do not depend on the input variables $u(k)$. Throughout this paper, we make the following assumptions:

**Assumption 1:** $\omega(k) \in \mathbb{R}^q$ is a real scalar process on a probability space $(\Omega,\mathcal{F},\mathcal{P})$ relative to an increase family $(\mathcal{F}_k)_{k\in\mathbb{N}}$ of σ- algebras $\mathcal{F}_k \subset \mathcal{F}$ generated by $(\omega(k)_{k\in\mathbb{N}}$, $\mathbb{N}$ is the set of natural numbers,

$$\mathcal{E}\{\omega(k)\} = 0,\ \mathcal{E}\{\omega(k)^2\} = \sigma,$$

where the scalar $\sigma > 0$, and the stochastic process $\omega(0), \omega(1), \cdots$, are independent.

**Assumption 2:** $\tau(k) > 0$ is an integer representing the time-varying delay of the system, which satisfies

$$\tau_1 \le \tau(k) \le \tau_2,$$

where $\tau_1 > 0$ and $\tau_2 > 0$ are integers. $x(k) = \phi(k)$, $k = -\tau_2, -\tau_2 + 1, \cdots, 0$ are the initial conditions and independent of the process $\{\omega(k)\}$.

**Assumption 3:** $A_i(k)$, $A_{di}(k)$, $B_{1i}(k)$, $E_i(k)$, $E_{di}(k)$, $B_{2i}(k)$ are known real constant matrices of appropriate dimensions, $\Delta A_i(k)$, $\Delta A_{di}(k)$, $\Delta E_i(k)$, $\Delta E_{di}(k)$, $\Delta B_{1i}(k)$, $\Delta B_{1i}(k)$ are unknown matrices representing time-varying parameter uncertainties, $F_i(\cdot) : \mathbb{N} \mapsto \mathbb{R}^{l_1 \times l_2}$ is an unknown time-varying matrix function, where for $i = 1, 2, \ldots, r$,

$$A_i(k) = A_i + \Delta A_i(k),\ A_{di}(k) = A_{di} + \Delta A_{di}(k),\ E_{di}(k) = E_{di} + \Delta E_{di}(k),$$

$$E_i(k) = E_i + \Delta E_i(k),\ B_{1i}(k) = B_{1i} + \Delta B_{1i}(k),\ B_{2i}(k) = B_{2i} + B_{2i}(k),$$

$$[\Delta A_i(k)\ \Delta A_{di}(k)\ \Delta E_i(k)\ \Delta E_{di}(k)\ \Delta B_{1i}(k)\ \Delta B_{2i}(k)] = M_i F_i(k)[N_{1i}\ N_{2i}\ N_{3i}\ N_{4i}\ N_{5i}\ N_{6i}], \tag{3}$$

$$F_i(k)^T Fi(k) \le I,\ \forall k \in \mathbb{N}. \tag{4}$$

The uncertain matrices $\Delta A_i(k)$, $\Delta A_{di}(k)$, $\Delta E_i(k)$, $\Delta E_{di}(k)$, $\Delta B_{1i}(k)$, $\Delta B_{1i}(k)$ are said to be admissible if both (3) and (4) hold.

We denote by $l_{e2}([0,\infty); \mathbb{R}^r)$ the space of $r$-dimensional nonanticipatory square-summable stochastic processes $f(\cdot) = (f(k)_{k\in\mathbb{N}})$ on $\mathbb{N}$ with respect to $(\mathcal{F}_k)$ $k\in\mathbb{N}$ satisfying

$$\|f\|_{e_2}^2 = E\left\{\sum_{k=0}^{\infty} |f(k)|^2\right\} = \sum_{k=0}^{\infty} E\left\{|f(k)|^2\right\} < \infty.$$

By using a center-average defuzzier, product inference, and singleton fuzzifier, the final output dynamic fuzzy model can be inferred as follows:

$$\Sigma: \quad x(k+1) = \sum_{i=1}^{r} h_i(s(k))\{A_i(k)x(k) + A_{di}(k)x(k-\tau(k)) + B_{1i}(k)u(k)$$
$$+[E_i(k)x(k) + E_{di}(k)x(k-\tau(k)) + B_{2i}(k)u(k)]\omega(k)\} \tag{5}$$

where

$$h_i(s(k)) = \frac{\overline{\omega}_i(s(k))}{\sum_{j=1}^{r} \overline{\omega}_j(s(k))},$$

$$\overline{\omega}_i(s(k)) = \prod_{j=1}^{g} \mu_{ij}(s_j(k)),$$

$$s(k) = [s_1(k)\ s_2(k)\ \ldots\ s_g(k)],$$

and $\mu_{ij}(s_j(k))$ is the grade of membership of $s_j(k)$ in $\mu_{ij}$. Then, it can be seen that

$$\bar{\omega}_i(s(k)) \geq 0, i = 1, 2, \ldots, r,$$

$$\sum_{j=1}^{r} \bar{\omega}_j(s(k)) > 0, \quad \forall k \in \mathbb{N},$$

and

$$hi(s(k)) > 0,\ i = 1, 2, \ldots, r,$$

$$\sum_{i=1}^{r} h_i(s(k)) = 1, \quad \forall k \in \mathbb{N}.$$

Throughout this paper, we shall use the following definitions:

**Definition 1:** The uncertain discrete stochastic time-delay system ($\Sigma$) is said to be robustly stochastically stable with $u(k) = 0$ if there exists a scalar $c > 0$ such that for all admissible uncertainties

$$\mathcal{E}\left\{\sum_{k=0}^{\infty} |x(k)|^2\right\} \leq c \sup_{-\tau_2 \leq k \leq 0} \mathcal{E}\{|\phi(k)|^2\},$$

where $x(k)$ denotes the solution of equation (5) at time $k$ under initial conditions in (3).

In the present study, by the parallel distributed compensation(PDC) technique [17], we are interested in designing a fuzzy state feedback controller in the following form:

**Control Rule** $i$: **IF** $s_1(k)$ is $\mu_{i1}$ and $s_2(k)$ is $\mu_{i2}$ . . . and $s_g(k)$ is $\mu_{ig}$ THEN

$$u(k) = K_{ix}(k),\ i = 1, 2, \ldots, r, \tag{6}$$

where $K_i \in \mathbb{R}^{m\times n}$ is the controller gain to be determined. Then, the overall fuzzy state feedback controller is given by

$$u(k) = \sum_{i=1}^{r} h_i(s(k)) K_i x(k). \tag{7}$$

The objective of this paper is to design a state feedback fuzzy controller in the form of (7) such that the resulting closed-loop system is robustly stochastically stable.

Before concluding this section, we introduce the following lemma, which will be used in the derivation of our main results in the next sections.

**Lemma 1:** [19] Let $\mathcal{A}$ , $\mathcal{D}$, $\mathcal{S}$, $\mathcal{W}$ and $\mathcal{F}$ be real matrices with appropriate dimensions such that $\mathcal{W} > 0$ and $\mathcal{F}^T\mathcal{F} \leq I$. Then we have the following:

(1) For any scalar $\epsilon > 0$ and vectors $x$ and $y$ of appropriate dimensions,

$$2x^T\mathcal{D}\mathcal{F}\mathcal{S}y \leq \epsilon^{-1}x^T\mathcal{D}\mathcal{D}^T x + \epsilon y^T\mathcal{S}^T\mathcal{S}y.$$

(2) For any scalar $\epsilon > 0$ such that $\mathcal{W} - \epsilon\mathcal{D}\mathcal{D}^T > 0$,

$$(\mathcal{A} + \mathcal{D}\mathcal{F}\mathcal{S})^T\mathcal{W}^{-1}(\mathcal{A} + \mathcal{D}\mathcal{F}\mathcal{S}) \leq \mathcal{A}^T(\mathcal{W} - \epsilon\mathcal{D}\mathcal{D}^T)^{-1}\mathcal{A} + \epsilon^{-1}\mathcal{S}^T\mathcal{S}.$$

## 3. ROBUST STABILIZATION

In this section, we will give some sucient conditions for the solvability of the delay-dependent stabilization problem for the stochastic time-delay T-S fuzzy systems. The result on robust stochastic stability analysis for system ($\Sigma$) is provided in the following theorem:

**Theorem 1:** The uncertain discrete stochastic fuzzy time-delay system $\Sigma$ with $u(k) = 0$ is robustly stochastically

stable if there exist matrices $P > 0$, $Q > 0$, and scalars $\epsilon_{1i} > 0$ and $\epsilon_{2i} > 0$ such that the following LMIs hold for all $i = 1, 2, \cdots, r$,

$$\begin{bmatrix} \Omega_{11i} & \Omega_{12i} & A_i^T P & \sqrt{\sigma} E_i^T P & 0 & 0 \\ * & \Omega_{22i} & A_{di}^T P & \sqrt{\sigma} E_{di}^T P & 0 & 0 \\ * & * & -P & 0 & PM_i & 0 \\ * & * & * & -P & 0 & PM_i \\ * & * & * & * & -\epsilon_{1i} I & 0 \\ * & * & * & * & * & -\epsilon_{2i} I \end{bmatrix} < 0, \tag{8}$$

where

$$\bar{\tau} = 1 + \tau_2 - \tau_1, \tag{9}$$

$$\Omega_{11i} = -\bar{\tau} Q - P + \epsilon_{1i} N_{1i}^T N_{1i} + \sigma \epsilon_{2i} N_{3i}^T N_{3i}, \tag{10}$$

$$\Omega_{12i} = \epsilon_{1i} N_{1i}^T N_{2i} + \sigma \epsilon_{2i} N_{3i}^T N_{4i}, \tag{11}$$

$$\Omega_{22i} = \epsilon_{1i} N_{2i}^T N_{2i} + \sigma \epsilon_{2i} N_{4i}^T N_{4i} - Q. \tag{12}$$

**Proof:** Pre- and post-multiplying the LMI in (8) by diag($I$, $I$, $P^{-1}$, $P^{-1}$, $I$, $I$) respectively, result in

$$\begin{bmatrix} \Omega_{11ii} & \Omega_{12i} & A_i^T & \sqrt{\sigma} E_i^T & 0 & 0 \\ * & \Omega_{22i} & A_{di}^T & \sqrt{\sigma} E_{di}^T & 0 & 0 \\ * & * & -P^{-1} & 0 & M_i & 0 \\ * & * & * & -P^{-1} & 0 & M_i \\ * & * & * & * & -\epsilon_{1i} I & 0 \\ * & * & * & * & * & -\epsilon_{2i} I \end{bmatrix} < 0, \tag{13}$$

which, by the Schur complement formula, we have that there exists a scalar $\delta > 0$ such that

$$P^{-1} - \bar{\delta}_{1i}^{-1} M_i M_i^T > 0, \quad P^{-1} - \epsilon_{2i}^{-1} M_i M_i^T > 0, \tag{14}$$

$$\Omega + \bar{A}_i^T (P^{-1} - \epsilon_{1i}^{-1} M_i M_i^T)^{-1} \bar{A}_i + \sigma \bar{E}_i^T (P^{-1} - \epsilon_{2i}^{-1} M_i M_i^T)^{-1} \bar{E}_i \tag{15}$$

$$+ \epsilon_{1i} \bar{N}_{1i}^T \bar{N}_{1i} + \sigma \epsilon_{2i} \bar{N}_{2i}^T \bar{N}_{2i} < \begin{bmatrix} -\delta I & 0(16) \\ 0 & 0 \end{bmatrix}, \tag{16}$$

where

$$\Omega = \begin{bmatrix} \tau Q - P & 0 \\ 0 & -Q \end{bmatrix},$$

$$\bar{A}_i = [A_i A_{di}], \quad \bar{E}_i = [E_i E_{di}],$$

$$\bar{N}_{1i} = [N_{1i} N_{2i}], \bar{N}_{2i} = [N_{3i} N_{4i}].$$

For system ($\Sigma$), we choose the following Lyapunov functional candidate:

$$V_k(\mathcal{X}_k) = x(k)^T Px(k) + \bar{V}_k(\mathcal{X}_k) + \tilde{V}_k(\mathcal{X}_k) \tag{17}$$

where

$$\mathcal{X}_k = [x(k - \tau(k)), x(k - \tau(k) + 1), \ldots, x(k)], \tag{18}$$

$$\bar{V}_k(X_k) = \sum_{i=k-\tau(k)}^{k-1} x(i)^T Qx(i) \tag{19}$$

$$\tilde{V}_k(X_k) = \sum_{j=-\tau_2+2}^{-\tau_1+1} \sum_{i=k+j-1}^{k-1} x(i)^T Qx(i). \tag{20}$$

Then, we have

$$\epsilon\{\bar{V}_{k+1}(\mathcal{X}_{k+1}) \,|\, \mathcal{X}_k\} - \bar{V}_k(\mathcal{X}_k)$$

$$\begin{aligned}
&= \sum_{i=k+1-\tau(k+1)}^{k} x(i)^T Qx(i) - \sum_{i=k-\tau(k)}^{k-1} x(i)^T Qx(i) \\
&= \sum_{i=k+1-\tau(k+1)}^{k-\tau_1} x(i)^T Qx(i) + \sum_{i=k+1-\tau_1}^{k-1} x(i)^T Qx(i) + x(k)^T Qx(k) \\
&\quad - \sum_{i=k+1-\tau(k)}^{k-1} x(i)^T Qx(i) - x(k-\tau(k))^T Qx(k-\tau(k)) \\
&\le \sum_{i=k+1-\tau(k+1)}^{k-\tau_1} x(i)^T Qx(i) + x(k)^T Qx(k) - x(k-\tau(k))^T Qx(k-\tau(k))
\end{aligned} \tag{21}$$

$$\begin{aligned}
&\mathcal{E}\{\tilde{V}_{k+1}(\mathcal{X}_{k+1}) \,|\, \mathcal{X}_k\} - \tilde{V}_k(\mathcal{X}_k) \\
&= \sum_{j=-\tau_2+2}^{-\tau_1+1} \left[ \sum_{i=k+j}^{k} x(i)^T Qx(i) - \sum_{i=k+j-1}^{k-1} x(i)^T Qx(i) \right] \\
&= (\tau_2 - \tau_1) x(k)^T Qx(k) - \sum_{i=k+1-\tau_2}^{k-\tau_1} x(i)^T Qx(i)
\end{aligned} \tag{22}$$

From (21) and (22) it is easy to see

$$\mathcal{E}\{[\overline{V}_{k+1}(\mathcal{X}_{k+1})+\tilde{V}_{k+1}(\mathcal{X}_{k+1})]\,|\,\mathcal{X}_k\}-\overline{V}_k(\mathcal{X}_k)-\tilde{V}_k(\mathcal{X}_k)$$
$$\leq \overline{\tau}_1 x(k)^T Qx(k)-x(k-\tau(k))^T Qx(k-\tau(k))$$

and

$$\mathcal{E}\,\{V_{k+1}(\mathcal{X}_{k+1})|\mathcal{X}_k\}-V_k(\mathcal{X}_k)$$
$$\leq \mathcal{E}\{x(k+1)^T Px(k+1)\}+x(k)^T(\overline{\tau}Q-P)x(k)-x(k-\tau(k))^T Qx(k-\tau(k)) \tag{23}$$

Now, by Lemma 1 we can see

$$\mathcal{E}(x(k+1)^T Px(k+1)) = \frac{1}{2}\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(s(k))h_j(s(k))\xi(k)^T$$
$$\times\{(\overline{A}_i+M_iF_i(k)\overline{N}_{1i})^T P(\overline{A}_i+M_jF_j(k)\overline{N}_{1j})+(\overline{A}_j+M_jF_j(k)\overline{N}_{1j})^T P(\overline{A}_i+M_iF_i(k)\overline{N}_{1i})$$
$$+\sigma[(\overline{E}_i+M_iF_i(k)\overline{N}_{2i})^T P(\overline{E}_j+M_jF_j(k)\overline{N}_{2j})+(\overline{E}_j+M_jF_j(k)\overline{N}_{2j})^T P(\overline{E}_i+M_iF_i(k)\overline{N}_{2i})]\}\times\xi(k)$$

$$\leq \sum_{i=1}^{r} h_i(s(k))\xi(k)^T$$
$$\times\{(\overline{A}_i+M_iF_i(k)\overline{N}_{1i})^T P(\overline{A}_i+M_iF_i(k)\overline{N}_{1i})+\sigma(\overline{E}_i+M_iF_i(k)\overline{N}_{2i})^T P(\overline{E}_i+M_iF_i(k)\overline{N}_{2i})\}\times\xi(k)$$

$$\leq \sum_{i=1}^{r} h_i(s(k))\xi(k)^T$$
$$\times\{\overline{A}_i^T(P^{-1}-\varepsilon_{1i}^{-1}M_iM_i^T)^{-1}\overline{A}_i+\varepsilon_{1i}\overline{N}_{1i}^T\overline{N}_{1i}+\sigma[\overline{E}_i^T(P^{-1}-\varepsilon_{2i}^{-1}M_iM_i^T)^{-1}\overline{E}_i+\varepsilon_{2i}\overline{N}_{2i}^T\overline{N}_{2i}]\}\times\xi(k),$$

where

$$\xi(k)=[x(k)^T\;x(k-\tau(k))^T]^T. \tag{24}$$

Therefore, by (16), for all $\xi(k)\neq 0$, we obtain

$$\mathcal{E}\,\{V_{k+1}(\mathcal{X}_{k+1})|\mathcal{X}_k\}-V_k(\mathcal{X}_k)<-\delta\mathcal{E}|x(k)|^2. \tag{25}$$

Now, taking expectation of both sides of the inequality in (25) gives

$$\mathcal{E}\,\{V_{k+1}(\mathcal{X}_{k+1})|\mathcal{X}_k\}-\mathcal{E}\,\{V_k(\mathcal{X}_k)\}<-\delta\mathcal{E}\,\{|x(k)|^2\}. \tag{26}$$

Therefore, for any integer $N>1$, summing up both sides of (26) from 0 to $N$ results in

$$\mathcal{E}\{V_{N+1}(\mathcal{X}_{N+1})\}-\mathcal{E}\{V_0(\mathcal{X}_0)\}<-\delta\mathcal{E}\left\{\sum_{k=0}^{N}|x(k)|^2\right\}, \tag{27}$$

which implies

$$\mathcal{E}\left\{\sum_{k=0}^{N}|x(k)|^2\right\} < \frac{1}{\delta}\big[\mathcal{E}\{V_0(\mathcal{X}_0)\}-\mathcal{E}\{V_{N+1}(\mathcal{X}_{N+1})\big]$$

$$< \frac{1}{\delta}\mathcal{E}\{V_0(\mathcal{X}_0)$$

$$< c \sup_{-\tau_2 \le i \le 0} \mathcal{E}\{|\phi(i)|^2\},$$

where

$$c = \frac{1}{\delta}\{\max\{\lambda_{\max}(P), \lambda_{\max}(Q)\}\left(\frac{\tau_2^2 - \tau_1^2 + \tau_1 + \tau_2}{2} + 1\right), \tag{28}$$

which completes the proof.

Now, we are in a position to present the result on the robust stabilization problem for the uncertain discrete stochastic time-delay system $(\Sigma)$.

**Theorem 2:** Consider the uncertain discrete stochastic fuzzy time-delay system $(\Sigma)$. This system is robustly stochastically stabilizable if there exist matrices $X > 0$, $\hat{Q} > 0, Y_i$ and scalars $\epsilon_{1ij} > 0$ and $\epsilon_{2ij} > 0$, $1 \le i \le j \le r$ such that the following LMI holds:

$$\begin{bmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} & \Psi_{14} & \Psi_{15} & 2\overline{\tau}X \\ * & \Psi_{22} & 0 & 0 & 0 & 0 \\ * & * & \Psi_{33} & 0 & 0 & 0 \\ * & * & * & -\epsilon_{1ij}I & 0 & 0 \\ * & * & * & * & -\epsilon_{2ij}I & 0 \\ * & * & * & * & * & -\overline{\tau}\hat{Q} \end{bmatrix} < 0, \tag{29}$$

where $\overline{\tau}$ is defined in (9) and

$$\Psi_{11} = \begin{bmatrix} -4X & 0 \\ * & -4\hat{Q} \end{bmatrix}, \quad M_{ij} = [M_i M_j],$$

$$\Psi_{12} = \begin{bmatrix} XA_i^T + XA_j^T + Y_i^T B_{1i}^T + Y_j^T B_{1j}^T \\ \hat{Q}A_{di}^T + \hat{Q}A_{dj}^T \end{bmatrix},$$

$$\Psi_{13} = \sqrt{\sigma}\begin{bmatrix} XE_i^T + XE_j^T + Y_i^T B_{2i}^T + Y_j^T B_{2j}^T \\ \hat{Q}E_{di}^T + \hat{Q}E_{dj}^T \end{bmatrix},$$

$$\Psi_{14} = \begin{bmatrix} XN_{1i}^T + Y_j^T N_{5i}^T & XN_{1j}^T + Y_i^T N_{5j}^T \\ \hat{Q}N_{2i}^T & \hat{Q}N_{2j}^T \end{bmatrix},$$

$$\Psi_{15} = \begin{bmatrix} XN_{3i}^T + Y_j^T N_{6i}^T & XN_{3j}^T + Y_i^T N_{6j}^T \\ \hat{Q}N_{4i}^T & \hat{Q}N_{4j}^T \end{bmatrix},$$

$$\Psi_{22} = \epsilon_{1ij} M_{ij} M_{ij}^T - X \quad \Psi_{33} = \epsilon_{2ij} M_{ij} M_{ij}^T - X.$$

In this case, a desired stabilizing fuzzy controller can be chosen as in (7) with the state feedback gains as

$$K_i = Y_i X^{-1},\ i = 1, 2, \ldots, r. \tag{30}$$

**Proof:** Pre- and post-multiplying the LMI in (29) by diag($X^{-1}$, $\hat{Q}^{-1}$, $I, I, , I, I$) respectively, result in

$$\begin{bmatrix} \tilde{\Psi}_{11} & \tilde{\Psi}_{12} & \tilde{\Psi}_{13} & \tilde{\Psi}_{14} & \tilde{\Psi}_{15} \\ * & \tilde{\Psi}_{22} & 0 & 0 & 0 \\ * & * & \tilde{\Psi}_{33} & 0 & 0 \\ * & * & * & -\epsilon_{1ij} I & 0 \\ * & * & * & * & -\epsilon_{2ij} I \end{bmatrix} < 0, \tag{31}$$

where $X^{-1} = P, \hat{Q}^{-1} = Q$, $K_i$ is given in (30), $i = 1, 2, \ldots, r$ and

$$\tilde{\Psi}_{11} = 4\begin{bmatrix} \bar{\tau}Q - P & 0 \\ * & -Q \end{bmatrix},$$

$$\tilde{\Psi}_{12} = \begin{bmatrix} A_i^T + A_j^T + K_i^T B_{1i}^T + K_j^T B_{1j}^T \\ A_{di}^T + A_{dj}^T \end{bmatrix},$$

$$\tilde{\Psi}_{13} = \sqrt{\sigma}\begin{bmatrix} E_i^T + E_j^T + K_i^T B_{2i}^T + K_j^T B_{2j}^T \\ E_{di}^T + E_{dj}^T \end{bmatrix},$$

$$\tilde{\Psi}_{14} = \begin{bmatrix} N_{1i}^T + K_j^T N_{5i}^T & N_{1j}^T + K_i^T N_{5j}^T \\ N_{2i}^T & N_{2j}^T \end{bmatrix},$$

$$\tilde{\Psi}_{15} = \begin{bmatrix} N_{3i}^T + K_j^T N_{6i}^T & N_{3j}^T + K_i^T N_{6j}^T \\ N_{4i}^T & N_{4j}^T \end{bmatrix},$$

$$\tilde{\Psi}_{22} = \epsilon_{1ij} M_{ij} M_{ij}^T - P^{-1} \quad \tilde{\Psi}_{33} = \epsilon_{2ij} M_{ij} M_{ij}^T - P^{-1}.$$

Now, applying the state feedback controller in (7) with the controller gains given in (30) to system $\Sigma$, we obtain the following closed-loop system:

$$\Sigma^C : x(k+1) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(s(k))h_j(s(k))\{A_{ij}(k)x(k) + A_{di}(k)x(k-\tau(k))$$

$$+[E_{ij}(k)\, x(k) + E_{di}(k)\, x(k-\tau(k))]\omega(k)\}, \tag{32}$$

where

$$A_{ij} = A_i + B_{1i}Kj\ ,\ E_{ij} = E_i + B_{2i}K_j, \tag{33}$$

$$\Delta A_{ij}(k) = A_i(k) + B_{1i}(k)K_j\ ,\ E_{ij}(k) = Ei(k) + B_{2i}(k)Kj, \tag{34}$$

$$A_{ij}(k) = A_{ij} + A_{ij}(k),\ Eij(_k) = E_{ij} + E_{ij}(k). \tag{35}$$

For system ($\Sigma^c$), using the same Lyapunov functional as in (17) we obtain

$$\mathcal{E}(x(k+1)^T Px(k+1)) = \frac{1}{4}\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r}\sum_{l=1}^{r} h_i(s(k))h_j(s(k))h_k(s(k))h_l(s(k))\xi(k)^T$$

$$\times\{[\mathcal{A}_{ij} + \mathcal{A}_{ji} + \Delta\mathcal{A}_{ij}(k) + \Delta\mathcal{A}_{ji}(k)]^T P[\mathcal{A}_{kl} + \mathcal{A}_{lk} + \Delta\mathcal{A}_{kl}(k) + \Delta\mathcal{A}_{lk}(k)]$$

$$+\sigma[\mathcal{E}_{ij} + \mathcal{E}_{ji} + \Delta\mathcal{E}_{ij}(k) + \Delta\mathcal{E}_{ji}(k)]^T P[\mathcal{E}_{kl} + \mathcal{E}_{lk} + \Delta\mathcal{E}_{kl}(k) + \Delta\mathcal{E}_{lk}(k)]\} \times \xi(k)$$

$$= \frac{1}{8}\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r}\sum_{l=1}^{r} h_i(s(k))h_j(s(k))h_k(s(k))h_l(s(k))\xi(k)^T$$

$$\times\{[\mathcal{A}_{ij} + \mathcal{A}_{ji} + \Delta\mathcal{A}_{ij}(k) + \Delta\mathcal{A}_{ji}(k)]^T P[\mathcal{A}_{kl} + \mathcal{A}_{lk} + \Delta\mathcal{A}_{kl}(k) + \Delta\mathcal{A}_{lk}(k)]$$

$$+[\mathcal{A}_{kl} + \mathcal{A}_{lk} + \Delta\mathcal{A}_{kl}(k) + \Delta\mathcal{A}_{lk}(k)]^T P[\mathcal{A}_{ij} + \mathcal{A}_{ji} + \Delta\mathcal{A}_{ij}(k) + \Delta\mathcal{A}_{ji}(k)]$$

$$+\ \sigma\ [\mathcal{E}_{ij} + \mathcal{E}_{ji} + \Delta\mathcal{E}_{ij}(k) + \Delta\mathcal{E}_{ji}(k)]^T P[\mathcal{E}_{kl} + \mathcal{E}lk + \Delta\mathcal{E}_{kl}(k) + \Delta\mathcal{E}_{lk}(k)]$$

$$+[\mathcal{E}_{kl} + \mathcal{E}_{lk} + \Delta\mathcal{E}_{kl}(k) + \Delta\mathcal{E}_{lk}(k)]^T P[\mathcal{E}_{ij} + \mathcal{E}_{ji} + \Delta\mathcal{E}_{ij}(k) + \Delta\mathcal{E}_{ji}(k)]\} \times \xi(k)$$

$$\leq \frac{1}{4}\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(s(k))h_j(s(k))\xi(k)^T$$

$$\times\ \{[\mathcal{A}_{ij} + \mathcal{A}_{ji} + \Delta\mathcal{A}_{ij}(k) + \Delta\mathcal{A}_{ji}(k)]^T P[\mathcal{A}_{ij} + \mathcal{A}_{ji} + \Delta\mathcal{A}_{ij}(k) + \Delta\mathcal{A}_{ji}(k)]$$

$$+\ \sigma[\mathcal{E}_{ij} + \mathcal{E}_{ji} + \Delta\mathcal{E}_{ij}(k) + \Delta\mathcal{E}_{ji}(k)]^T P[\mathcal{E}_{ij} + \mathcal{E}_{ji} + \Delta\mathcal{E}_{ij}(k) + \Delta\mathcal{E}_{ji}(k)]\} \times \xi(k), \tag{36}$$

where $\xi(k)$ is defined in (24) and

$$\mathcal{A}_{ij} = [A_{ij}\quad A_{di}],\ \mathcal{E}_{ij} = [\mathcal{E}_{ij}\quad \mathcal{E}_{di}], \tag{37}$$

$$\Delta\mathcal{A}_{ij}(k) = [\Delta\mathcal{A}_{ij}(k)\quad \Delta\mathcal{A}_{di}(k)],\ \Delta\mathcal{E}_{ij}(k) = [\Delta\mathcal{E}_{ij}(k)\quad \Delta\mathcal{E}_{di}(k)]. \tag{38}$$

It follows from (3), (4) and (34) that

$$\Delta\mathcal{A}_{ij}(k) + \Delta\mathcal{A}_{ji}(k) = M_iF_i(k)[N_{1i} + N_{5i}K_j\ ] + M_jF_j(k)[N_{1j} + N_{5j}K_i]$$

$$= M_{ij}F_{ij}(k)N_{1ij}\ ,$$

$$\Delta\mathcal{E}_{ij}(k) + \Delta\mathcal{E}_{ji}(k) = M_iF_i(k)[N_{3i} + N_{6iKj}\ ] + M_jF_j(k)[N_{3j} + N_{6j}K_i]$$

$$= M_{ij}F_{ij}(k)N_{2ij}\ ,$$

where

$$F_{ij}(k) = \begin{bmatrix} F_i(k) & 0 \\ 0 & F_j(k) \end{bmatrix}, \tag{39}$$

$$N_{1ij} = \begin{bmatrix} N_{1i} + N_{5i}K_j & N_{2i} \\ N_{1j} + N_{5j}K_i & N_{2j} \end{bmatrix}, \tag{40}$$

$$N_{2ij} = \begin{bmatrix} N_{3i} + N_{6i}K_j & N_{4i} \\ N_{3j} + N_{6j}K_i & N_{4j} \end{bmatrix}, \tag{41}$$

(42)

By Lemma 1 we have

$$[\mathcal{A}_{ij} + \mathcal{A}_{ji} + \mathcal{A}_{ij}(k) + \mathcal{A}_{ji}(k)]^T P[\mathcal{A}_{ij} + \mathcal{A}_{ji} + \mathcal{A}_{ij}(k) + \mathcal{A}_{ji}(k)]$$
$$\leq [\mathcal{A}_{ij} + \mathcal{A}_{ji}]^T (P^{-1} - \epsilon_{1ij} M_{ij} M_{ij}^T)^{-1} [A_{ij} + A_{ji}] + \varepsilon_{1ij}^{-1} N_{1ij}^T N_{1ij}$$

$$\sigma[\mathcal{E}_{ij} + \mathcal{E}_{ji} + \Delta\mathcal{E}_{ij}(k) + \Delta\mathcal{E}_{ji}(k)]^T P[\mathcal{E}_{ij} + \mathcal{E}_{ji} + \Delta\mathcal{E}_{ij}(k) + \Delta\mathcal{E}_{ji}(k)]$$

$$\leq \sigma[\mathcal{E}_{ij} + \mathcal{E}_{ji}]^T (P^{-1} - \epsilon_{2ij} M_{ij} M_{ij}^T)^{-1} [\mathcal{E}_{ij} + \mathcal{E}_{ji}] + \epsilon_{2ij}^{-1} N_{2ij}^T N_{2ij}$$

By the same deduction as in Theorem 1 and together with (23) and (36) we can see

$$\mathcal{E}\{V_{k+1}(\mathcal{X}_{k+1}) \mid \mathcal{X}_k\} - V_k(\mathcal{X}_k)$$

$$\leq \frac{1}{4}\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(s(k))h_j(s(k))\xi(k)^T$$

$$\times\{[\mathcal{A}_{ij} + \mathcal{A}_{ji}]^T (P^{-1} - \epsilon_{1ij} M_{ij} M_{ij}^T)^{-1} [\mathcal{A}_{ij} + \mathcal{A}_{ji}] + \epsilon_{1ij}^{-1} N_{1ij}^T N_{1ij}$$

$$+\sigma[\mathcal{E}_{ij} + \mathcal{E}_{ji}]^T (P^{-1} - \epsilon_{2ij} M_{ij} M_{ij}^T)^{-1} [\mathcal{E}_{ij} + \mathcal{E}_{ji}] + \epsilon_{2ij}^{-1} N_{2ij}^T N_{2ij} + 4\Omega\}\xi(k), \tag{43}$$

where is define in (17).

On the other hand, from (31), (37), (40) and (41) we can see

$$\tilde{\Psi}_{11} = 4\Omega, \quad \tilde{\Psi}_{12} = [\mathcal{A}_{ij} + \mathcal{A}_{ji}]^T,$$
$$\tilde{\Psi}_{13} = [\mathcal{E}_{ij} + \mathcal{E}_{ji}]^T, \quad \tilde{\Psi}_{14} = N_{1ij}^T, \quad \tilde{\Psi}_{15} = N_{2ij}^T.$$

So, by Schur complement formula, together with (31) and (43) we obtain

$$\mathcal{E}\{V_{k+1}(\mathcal{X}_{k+1}) | \mathcal{X}_k\} - V_k(\mathcal{X}_k) < 0. \tag{44}$$

Thus, similar to the proof of Theorem 1, we obtain that system $\Sigma^c$ is robustly stochastically stable. This completes the proof.

## 4. NUMERICAL EXAMPLE

In this section, by applying the PDC technique, we provide a simulation example to illustrate the $H_\infty$ controller design approach developed in this paper. The uncertain T-S fuzzy neutral delay system considered in this example is with two rules:

**Plant Rule 1:** If $x_1(k)$ is $\mu_{11}$ then

$$
\begin{aligned}
x(k+1) &= A_1(k)x(k) + A_{d1}(k)(k-\tau(k)) + B_{11}(k)u(k) \\
&\quad + [E_1(k)x(k) + E_{d1}(k)(k-\tau(k)) + B_{21}(k)u(k)]\omega(k),
\end{aligned}
$$

**Plant Rule 2:** If $x_1(k)$ is $\mu_{21}$ then

$$
\begin{aligned}
x(k+1) &= A_2(k)x(k) + A_{d2}(k)(k-\tau(k)) + B_{12}(k)u(k) \\
&\quad +[E_2(k)x(k) + E_{d1}(k)x(k-\tau(k)) + B_{22}(k)u(k)]\omega(k),
\end{aligned}
$$

**Control Rule 1:** If $x_1(k)$ is $\mu_{11}$ then

$$u(k) = K_1x(k),$$

**Control Rule 2:** If $x_1(k)$ is $\mu_{21}$ then

$$u(k) = K_2x(k),$$

where

$$A_1 = \begin{bmatrix} -0.9 & 0.4 \\ 0.5 & 0.8 \end{bmatrix}, A_{d1} = \begin{bmatrix} 0.24 & -0.13 \\ 0 & 0.12 \end{bmatrix}, E_1 = \begin{bmatrix} -0.14 & 0.09 \\ 0.17 & 0.35 \end{bmatrix},$$

$$E_{d1} = \begin{bmatrix} -0.12 & -0.24 \\ -0.12 & -0.48 \end{bmatrix}, B_{11} = \begin{bmatrix} 2.5 & 0.8 \\ -0.6 & 1.5 \end{bmatrix}, B_{21} = \begin{bmatrix} 0.8 & 1 \\ -0.6 & 2.5 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 1.1 & 0.6 \\ 0.1 & -0.4 \end{bmatrix}, A_{d2} = \begin{bmatrix} -0.11 & 0.23 \\ 0 & -0.12 \end{bmatrix}, E_2 = \begin{bmatrix} 0.25 & -0.56 \\ 0.33 & 0.17 \end{bmatrix},$$

$$E_{d2} = \begin{bmatrix} 0.13 & -0.26 \\ 0.13 & 0.52 \end{bmatrix}, B_{12} = \begin{bmatrix} 1.8 & 1.3 \\ 0.6 & 1.1 \end{bmatrix}, B_{22} = \begin{bmatrix} 1.4 & 1.6 \\ -0.9 & 1.5 \end{bmatrix}.$$

The parameter uncertainties $\Delta Ai(k)$, $\Delta A_{di}(k)$, $\Delta E_i(_k)$, $\Delta E_{di}(k)$ $\Delta B_{1i}(k)$ and $\Delta B_{2i}(k)$, $i = 1, 2$, are assumed to satisfy (3) and (4) with

$$M_1 = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, M_2 = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix},$$

$$
\begin{aligned}
&N_{11} = [0.01 \quad 0], && N_{21} = [-0.01 \quad 0.01], && N_{31} = [0.03 \quad 0], \\
&N_{41} = [0.02 \quad 0], && N_{51} = [-0.02 \quad 0.01], && N_{61} = [0.04 \quad 0.02], \\
&N_{12} = [0 \quad 0.01], && N_{22} = [0.01 \quad 0.01], && N_{32} = [0.02 \quad 0.01], \\
&N_{42} = [0.02 \quad 0.01], && N_{52} = [-0.02 \quad 0], && N_{62} = [0.03 \quad 0].
\end{aligned}
$$

The membership functions of $\mu_{11}$ and $\mu_{21}$ are shown as follows:

$$h_1(x_1(k)) = \mu_{11} = \begin{cases} \frac{1}{2}, & x_1 < -1 \\ \frac{1}{2} + \frac{1}{2}x_1, & |x_1| \le 1, \\ 1, & x_1 > 1 \end{cases} \quad h_2(x_1(k)) = \mu_{21} = \begin{cases} \frac{1}{2}, & x_1 < -1 \\ \frac{1}{2} + \frac{1}{2}x_1, & |x_1| \le 1. \\ 0, & x_1 > 1 \end{cases}$$

The purpose of this example is to design a fuzzy state feedback controller such that the resulting closedloop system is robustly stochastically stable. The time delay level is specified to be $\tau_1 \le \tau(k) \le \tau_2$ with $\tau_1 = 2$ and $\tau_2 = 4$ and $\mathcal{E}\{\omega(k)_2\} = \sigma$ is satisfied with $\sigma = 1$. Then, by using the Matlab LMI Control Toolbox to solve the LMIs in (29), we obtain a set of feasible solutions as follows:

$$X = \begin{bmatrix} 18.1651 & -7.7165 \\ -7.7165 & 5.0125 \end{bmatrix}, \quad \hat{Q} = \begin{bmatrix} 70.3736 & -30.7473 \\ -30.7473 & 21.7422 \end{bmatrix},$$

$$Y_1 = \begin{bmatrix} 7.4424 & -4.2374 \\ 1.0579 & -0.7452 \end{bmatrix}, \quad Y_2 = \begin{bmatrix} -3.1239 & 1.2835 \\ -4.5360 & 1.9180 \end{bmatrix},$$

$$\epsilon_{111} = 0.0777, \quad \epsilon_{112} = 0.1874, \quad \epsilon_{122} = 0.0676,$$
$$\epsilon_{211} = 0.1769, \quad \epsilon_{212} = 0.1980, \quad \epsilon_{222} = 0.0218.$$

Then, by Theorem 2, the final controller are obtained as follows:

$$u(k) = \left( h_1(x_1(k)) \begin{bmatrix} 0.1462 & -0.6203 \\ -0.0142 & -0.1705 \end{bmatrix} + h_2(x_2(k)) \begin{bmatrix} -0.1826 & -0.0251 \\ -0.2519 & -0.0051 \end{bmatrix} \right) x(k).$$

## 5. CONCLUSIONS

This paper has provided sucient conditions for the solvability of the problem of robust stochastic stabilization for a class of uncertain stochastic T-S fuzzy systems with time-varying delays. These conditions are expressed in terms of LMIs, which can be easily tested by using commercially available software. It has been shown that a desired state feedback fuzzy controller can be constructed when the given LMIs are feasible. The eectiveness and applicability of the proposed design method have been demonstrated by a numerical example.

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