Journal of Mathematical Control Science and Applications (JMCSA)
Vol. 1 No. 2 (July-December, 2015), ISSN : 0974-0570

Non-fragile Control of Fuzzy Systems With Parametric Uncertainties and Distributed Delays

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Abstract: This paper investigates the problem of non-fragile controller design for Takagi-Sugeno fuzzy systems with norm-bounded time-varying parametric uncertainties and distributed delays. The controller to be designed is assumed to have time-varying norm-bounded uncertainties in the additive form. A sufficient condition in terms of linear matrix inequalities is derived, which guarantees the robust asymptotic stability of the closed-loop system for all admissible uncertainties in both the system and controller. An illustrative example is provided to show the effectiveness of the proposed method.

Keywords: T-S fuzzy model, distributed delays, uncertainties, non-fragile control

1. INTRODUCTION

Parameter uncertainties are frequently encountered in many practical control systems, which render it difficult to design controllers for uncertain systems. In the past years, various methods have been developed to deal with this issue, among which a successful approach is the so-called fuzzy control. However, stability analysis of a fuzzy system is difficult. To study this problem, the idea that a linear system is adopted as the consequent part of a fuzzy rule has evolved into the innovative Takagi-Sugeno (T-S) model [2], which becomes quite popular these days. In the last few years, a great number of results on the control of systems based on T-S fuzzy model have been reported in the literature; see, e.g., [3, 4, 5, 6, 7], and the references therein.

Time delay is frequently encountered in various engineering systems; it is often a primary source of instability and performance degradation [1, 10] of a control system. In recent years, a lot of researchers have been concerned with the study of time-delay fuzzy systems [8]. It is noted that these results are derived for systems with discrete delays. When the number of summands in a system equation is increased and the differences between neighboring arguments values are decreased, systems with distributed delays will arise [11]. Therefore, control systems with distributed delays and uncertainties have received much attention in the past years [12, 13, 14]. Note that all of these mentioned design approaches rely on the resulting controllers which will be implemented exactly. In practical applications, however, many limitations lead the controller to be implemented imprecision, e.g. analog/ digital converters and inherent imprecision in analog devices, which implies that even though a controller is robust to plant uncertainties, it may be very sensitive to the uncertainties in the controller itself. This requires that a designed controller must be non-fragile with respect to its gain uncertainties [15, 16].

Considering controller gain uncertainty, this paper investigates the synthesis problem of robust non-fragile controller for fuzzy distributed time delay systems with parametric uncertainties. A sufficient condition in terms of LMIs is given to guarantee robust stabilization for a class of T-S fuzzy systems with parametric uncertainties in both the system and controllers. An illustrative example is provided to show the effectiveness of the proposed method.

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Notation: Throughout this paper, the notation $X \geq Y$ (respectively, $X > Y$) for real symmetric matrices X and Y means that the matrix $X - Y$ is positive semi-definite (respectively, positive definite). I is an identity matrix with appropriate dimension. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

2. PROBLEM FORMULATIONS

Consider a continuous-time T-S fuzzy system with distributed delays in which the i th rule is formulated in the following form:

Rule i:

If

$$x_1(t) \text{ is } \Gamma_1^i \text{ and} \ldots \text{and } x_n(t) \text{ is } \Gamma_n^i,$$

Then

$$\dot{x}(t) = (A_i + \Delta A_i)x(t) + (A_{1i} + \Delta A_{1i})x(t-\tau)$$

$$+(A_{2i} + \Delta A_{2i})\int_{-\tau}^{t} x(s)ds + (B_i + \Delta B_i)u(t) \quad i = 1,2,\cdots,r, \tag{1}$$

$$x(t) = \varphi(t) \qquad t \in [-\tau, 0]. \tag{2}$$

where Γ_j^i is a fuzzy set; $x(t) \in R^n$ is the state vector; $u(t) \in R^m$ is the control input vector, $A_i \in R^{n\times n}$, $A_{1i} \in R^{n\times n}$, $A_{2i} \in R^{n\times n}$ and $B_i \in R^{n\times m}$ are known matrices. $\Delta A_i, \Delta A_{1i}, \Delta A_{2i}$ and ΔB_i are time-varying matrices with appropriate dimensions, which represent parametric uncertainties in the model, and i is the number of rules of this T-S fuzzy model; $\tau > 0$ is the constant time delay; $\varphi(t)$ is a real-valued continuous initial function on $[-\tau, 0]$.

The defuzzified output of the T-S fuzzy system (1) is represented as follows:

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(x(t))[(A_i + \Delta A_i)x(t) + (A_{1i} + \Delta A_{1i})x(t-\tau)$$

$$+ (A_{2i} + \Delta A_{2i})\int_{t-\tau}^{t} x(s)ds + (B_i + \Delta B_i)u(t)] \qquad i = 1,2,\cdots,r \tag{3}$$

where

$$\omega_i(x(t)) = \prod_{j=1}^{n} \Gamma_i^j(x_j(t)),$$

$$h_i(x(t)) = \omega_i(x(t)) \Big/ \sum_{i=1}^{r} \omega_i(x(t)).$$

in which $\Gamma_j^i(x_j(t))$ is the grade of membership of $x_j(t)$ in Γ_j^i. Some basic properties of $\omega_i(t)$ are

$$\omega_i(x(t)) \geq 0,$$

$$\sum_{i=1}^{r} \omega_i(x(t)) > 0, \qquad i = 1,2,\cdots,r.$$

Then, it can be seen that

$$h_i(x(t)) \geq 0,$$

$$\sum_{i=1}^{r} h_i(x(t)) = 1, \qquad i = 1,2,\cdots,r.$$

Suppose a fuzzy model of state-feedback controller for the T-S fuzzy model is formulated as follows:

Rule i :

If

$$x_1(t) \text{ is } \Gamma_1^i \text{ and} \ldots \text{and } x_n(t) \text{ is } \Gamma_n^i,$$

then

$$u(t) = (K_i + \Delta K_i)x(t), \qquad i = 1,2,\cdots,r, \tag{4}$$

where $K_i \in R^{m\times n}$ is a constant control gain to be determined. ΔK_i is a time-varying matrix with appropriate dimensions, which represents parametric uncertainties in the controller.

Assumption 1: The parametric uncertain matrices considered are norm-bounded in the form

$$[\Delta A_i \quad \Delta A_{1i} \quad \Delta A_{2i} \quad \Delta B_i] = D_i F_i(t)[E_{0i} \quad E_{1i} \quad E_{2i} \quad E_{3i}],$$
$$\Delta K_i = D_{1i} F_{ki}(t) E_{4i}.$$

where $D_i, D_{1i}, E_{0i}, E_{1i}, E_{3i}$ and E_{4i} are known real constant matrices of appropriate dimensions, and $F_i(t)$, $F_{ki}(t)$ are unknown matrix functions with Lebesgue-measurable elements and satisfy

$$F_i(t)^T F_i(t) \leq I,$$

$$F_{ki}(t)^T F_{ki}(t) \leq I.$$

Then, the problem to be addressed is the design of a fuzzy state feedback controller in (4) such that the resulting closed-loop system from (3) and (4) is robustly asymptotically stable for all admissible uncertainties in both the system and controller.

3. MAIN RESULTS

In this section, an LMI approach will be developed to solve the non-fragile controller design problem for uncertain fuzzy distributed delay systems formulated in the previous section. We first introduce the following lemmas which will be used in the proof of our main results.

Lemma 1: [17] Let A, D, S, W and F be real matrices of appropriate dimensions such that $W > 0$ and $F^T F \leq I$.

Then we have the following:

(1) For scalar $\varepsilon > 0$ and vectors x and y of appropriate dimensions,

$$2x^T DFSy \leq \varepsilon^{-1} x^T DD^T x + \varepsilon y^T S^T Sy.$$

(2) For any scalar $\varepsilon > 0$ such that $W - \varepsilon DD^T > 0$,

$$(A + DFS)^T W^{-1} (A + DFS) \leq A^T (W - \varepsilon DD^T)^{-1} A + \varepsilon^{-1} S^T S.$$

The following theorem provides a sufficient condition for the solvability of the non-fragile control problem for uncertain fuzzy distributed delay systems.

Theorem 1 The uncertain fuzzy distributed delay system in (3) is robustly stabilized by controller (4) if there exist matrices $\hat{P} > 0, \hat{Q}_1 > 0, \hat{Q}_2 > 0$, M_i, and scalars $\varepsilon_{1ij} > 0, \varepsilon_{2ij} > 0, \varepsilon_{3ij} > 0$ $(i, j = 1,2,\cdots,r)$, such that the following LMIs hold:

$$\begin{bmatrix} \Pi & * & * & * & * \\ \mathrm{E} & -\varepsilon_{1ii} I & * & * & * \\ D_B & 0 & \mathrm{E}_D & * & * \\ \mathrm{E}_4 & 0 & 0 & -\varepsilon_{2ii} I & * \\ \Xi & 0 & 0 & 0 & -\Phi \end{bmatrix} < 0 \qquad (1 \leq i \leq r), \tag{5}$$

$$\begin{bmatrix} \hat{\Pi} & * & * & * & * \\ \hat{\mathrm{E}} & -\varepsilon_{1ij} I & * & * & * \\ \hat{D}_B & 0 & \hat{\mathrm{E}}_D & * & * \\ \hat{\mathrm{E}}_4 & 0 & 0 & -\varepsilon_{2ij} I & * \\ \Xi & 0 & 0 & 0 & -0.5\Phi \end{bmatrix} < 0 \qquad (1 \leq i < j \leq r). \tag{6}$$

where * represents the transposed elements in the symmetric positions, and

$$\Pi = \begin{bmatrix} \hat{\Phi}_i + (\varepsilon_{1ii} + \varepsilon_{3ii}) D_i D_i^T & * & * \\ \hat{Q}_1 A_{1i}^T & -\hat{Q}_1 & * \\ \hat{Q}_2 A_{2i}^T & 0 & -\hat{Q}_2 \end{bmatrix}, \quad \hat{\Pi} = \begin{bmatrix} \hat{\Phi}_{ij} + (\varepsilon_{1ij} + \varepsilon_{3ij}) \hat{D}\hat{D}^T & * & * \\ \hat{Q}_1 (A_{1i} + A_{1j})^T & -2\hat{Q}_1 & 0 \\ \hat{Q}_2 (A_{2i} + A_{2j})^T & 0 & -2\hat{Q}_2 \end{bmatrix},$$

$$\hat{\Phi}_i = \hat{P} A_i^T + A_i \hat{P} + B_i M_i + M_i^T B_i^T,$$

$$\hat{\Phi}_{ij} = \hat{P}(A_i + A_j)^T + (A_i + A_j)\hat{P} + B_i M_j + B_j M_i + M_i^T B_j^T + M_j^T B_i^T.$$

$$D_B = \begin{bmatrix} \varepsilon_{2ii} D_{1i}^T B_i^T & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \hat{D}_B = \begin{bmatrix} \varepsilon_{2ij} D_{1j}^T B_i^T & 0 & 0 \\ \varepsilon_{2ij} D_{1i}^T B_j^T & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \hat{D} = \begin{bmatrix} D_i & D_j \end{bmatrix},$$

$$\mathrm{E} = \begin{bmatrix} E_{0i}\hat{P} + E_{3i}M_i & E_{1i}\hat{Q}_1 & E_{2i}\hat{Q}_2 \end{bmatrix}, \hat{\mathrm{E}} = \begin{bmatrix} E_{0i}\hat{P} + E_{3i}M_j & E_{1i}\hat{Q}_1 & E_{2i}\hat{Q}_2 \\ E_{0j}\hat{P} + E_{3j}M_i & E_{1j}\hat{Q}_1 & E_{2j}\hat{Q}_2 \end{bmatrix},$$

$$\mathrm{E}_D = \begin{bmatrix} -\varepsilon_{2ii} I & * \\ \varepsilon_{2ii} E_{3i} D_{1i} & -\varepsilon_{3ii} I \end{bmatrix}, \qquad \hat{\mathrm{E}}_D = \begin{bmatrix} -\varepsilon_{2ij} I & * & * & * \\ 0 & -\varepsilon_{2ij} I & * & * \\ \varepsilon_{2ij} E_{3i} D_{1j} & 0 & -\varepsilon_{3ij} I & * \\ 0 & \varepsilon_{2ij} E_{3j} D_{1i} & 0 & -\varepsilon_{3ij} I \end{bmatrix},$$

$$\mathrm{E}_4 = \begin{bmatrix} E_{4i}\hat{P} & 0 & 0 \end{bmatrix}, \quad \hat{\mathrm{E}}_4 = \begin{bmatrix} E_{4j}\hat{P} & 0 & 0 \\ E_{4i}\hat{P} & 0 & 0 \end{bmatrix}, \quad \Xi = \begin{bmatrix} \hat{P} & 0 & 0 \\ \tau\hat{P} & 0 & 0 \end{bmatrix}, \quad \Phi = \begin{bmatrix} \hat{Q}_1 & 0 \\ 0 & \hat{Q}_2 \end{bmatrix}.$$

In this case, the desired controller gains can be designed as follows:

$$K_i = M_i \hat{P}^{-1}. \tag{7}$$

Proof. The closed-loop system from (3) and (4) with the controller gain in (7) is obtained as

$$\begin{aligned} \dot{x}(t) = {} & \sum\nolimits_{i=1}^{r} h_i^2(x(t))\{[(A_i + \Delta A_i) + (B_i + \Delta B_i)(M_i P + \Delta K_i)]x(t) \\ & + (A_{1i} + \Delta A_{1i})x(t-\tau) + (A_{2i} + \Delta A_{2i})\alpha(t)\} + \sum\nolimits_{i<j}^{r} h_i(x(t))h_j(x(t))\{[(A_i + A_j \\ & + \Delta A_i + \Delta A_j) + (B_i + \Delta B_i)(M_j P + \Delta K_j) + (B_j + \Delta B_j)(M_i P + \Delta K_i)]x(t) \\ & + (A_{1i} + A_{1j} + \Delta A_{1i} + \Delta A_{1j})x(t-\tau) + (A_{2i} + A_{2j} + \Delta A_{2i} + \Delta A_{2j})\alpha(t)\}, \end{aligned} \tag{8}$$

where

$$P = \hat{P}^{-1}, \ \ \alpha(t) = \int_{t-\tau}^{t} x(\theta) d\theta.$$

Consider the Lyapunov functional candidate as

$$V(x(t)) = x^T(t)Px(t) + V_1(t) + V_2(t) + V_3(t),$$

where

$$V_1(t) = \int_{t-\tau}^{t} x(\lambda)^T Q_1 x(\lambda) d\lambda , \tag{9}$$

$$V_2(t) = \int_{t-\tau}^{t} [\int_{s}^{t} x(\theta)^T d\theta] Q_2 [\int_{s}^{t} x(\theta) d\theta] ds , \tag{10}$$

$$V_3(t) = \int_{0}^{\tau} \int_{t-\mu}^{t} (\lambda + \mu - t) x(\lambda)^T Q_2 x(\lambda) d\lambda d\mu . \tag{11}$$

$$Q_1 = \hat{Q}_1^{-1}, \ \ Q_2 = \hat{Q}_2^{-1}$$

Then, the time derivative of $V(x(t))$ along the trajectory of the system (7) is given by

$$\dot{V}(x(t)) = \dot{x}(t)^T P x(t) + x(t)^T P \dot{x}(t) + \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) , \tag{12}$$

where

$$\dot{V}_1(t) = x(t)^T Q_1 x(t) - x(t-\tau)^T Q_1 x(t-\tau) , \tag{13}$$

$$\dot{V}_2(t) = 2\int_{t-\tau}^{t} (\theta - t + \tau) x(t)^T Q_2 x(\theta) d\theta - \alpha(t)^T Q_2 \alpha(t) , \tag{14}$$

$$\dot{V}_3(t) = \frac{1}{2}\tau^2 x(t)^T Q_2 x(t) - \int_{t-\tau}^{t} (\lambda - t + \tau) x(\lambda)^T Q_2 x(\lambda) d\lambda . \tag{15}$$

According to Lemma 1, we have

$$\dot{V}_2(t) \le \int_{t-\tau}^{t} (\theta - t + \tau)[x(t)^T Q_2 x(t) + x(\theta)^T Q_2 x(\theta)] d\theta - \alpha(t)^T Q_2 \alpha(t)$$

$$= \frac{1}{2}\tau^2 x(t)^T Q_2 x(t) + \int_{t-\tau}^{t} (\theta - t + \tau) x(\theta)^T Q_2 x(\theta) d\theta - \alpha(t)^T Q_2 \alpha(t) .$$

This together with (12), (13) and (15) implies

$$\dot{V}(x(t)) \le \dot{x}(t)^T P x(t) + x(t)^T P \dot{x}(t) + x(t)^T \Lambda x(t) - x(t-\tau)^T Q_1 x(t-\tau) - \alpha(t)^T Q_2 \alpha(t)$$

$$= \sum_{i=1}^{r} h_i^2(x(t))\{ x(t)^T [(A_i + B_i M_i P)^T P + P(A_i + B_i M_i P) + \Lambda] x(t)$$

$$+ 2x(t)^T P A_{1i} x(t-\tau) + 2x(t)^T P A_{2i} \alpha(t) - x(t-\tau)^T Q_1 x(t-\tau) - \alpha(t)^T Q_2 \alpha(t)$$

$$+ 2x(t)^T P[(\Delta A_i + \Delta B_i M_i P) x(t) + \Delta A_{1i} x(t-\tau) + \Delta A_{2i} \alpha(t) + (B_i + \Delta B_i)\Delta K_i x(t)]\}$$

$$+ \sum_{i<j}^{r} h_i(x(t)) h_j(x(t)) \{ x(t)^T [(A_i + A_j + B_i M_j P + B_j M_i P)^T P + P(A_i + A_j + B_i M_j P$$

$$+ B_j M_i P) + 2\Lambda] x(t) + 2x(t)^T P(A_{1i} + A_{1j}) x(t-\tau) + 2x(t)^T P(A_{2i} + A_{2j}) \alpha(t)$$

$$- 2x(t-\tau)^T Q_1 x(t-\tau) - 2\alpha(t)^T Q_2 \alpha(t) + 2x(t)^T P[(\Delta A_i + \Delta A_j + \Delta B_i M_j P$$

$$+ \Delta B_j M_i P) x(t) + (\Delta A_{1i} + \Delta A_{1j}) x(t-\tau) + (\Delta A_{2i} + \Delta A_{2j}) \alpha(t)$$

$$+(B_i+\Delta B_i)\Delta K_j x(t)+(B_j+\Delta B_j)\Delta K_i x(t)]\}, \tag{16}$$

where

$$\Lambda = Q_1+\tau^2 Q_2 .$$

By Assumption 1 and Lemma 1, it is easy to see that

$$\begin{aligned}&2x(t)^T P[(\Delta A_i+\Delta B_i M_i P)x(t)+\Delta A_{1i}x(t-\tau)+\Delta A_{2i}\alpha(t)]\\ &\le \varepsilon_{1ii}x(t)^T PD_i D_i^T Px(t)+\varepsilon_{1ii}^{-1}\beta(t)^T \mathrm{E}_{1k}^T \mathrm{E}_{1k}\beta(t),\end{aligned} \tag{17}$$

$$\begin{aligned}&2x(t)^T P[(\Delta A_i+\Delta A_j+\Delta B_i M_j P+\Delta B_j M_i P)x(t)+(\Delta A_{1i}+\Delta A_{1j})x(t-\tau)+(\Delta A_{2i}+\Delta A_{2j})\alpha(t)]\\ &\le \varepsilon_{1ij}x(t)^T P\hat{D}\hat{D}^T Px(t)+\varepsilon_{1ij}^{-1}\beta(t)^T \mathrm{E}_{2k}^T \mathrm{E}_{2k}\beta(t),\end{aligned} \tag{18}$$

$$2x(t)^T P(B_i+\Delta B_i)\Delta K_i x(t) \tag{19}$$

$$\le \varepsilon_{2ii}x(t)^T P(B_i+\Delta B_i)D_{1i}D_{1i}^T(B_i+\Delta B_i)^T Px(t)+\varepsilon_{2ii}^{-1}x(t)^T E_{4i}^T E_{4i}x(t)$$

$$\le x(t)^T P[B_i D_{1i}(\varepsilon_{2ii}^{-1}I-\varepsilon_{3ii}^{-1}D_{1i}^T E_{3i}^T E_{3i}D_{1i})^{-1}D_{1i}^T B_i^T+\varepsilon_{3ii}D_i D_i^T]Px(t)+\varepsilon_{2ii}^{-1}x(t)^T E_{4i}^T E_{4i}x(t),$$

$$2x(t)^T P[(B_i+\Delta B_i)\Delta K_j+(B_j+\Delta B_j)\Delta K_i]x(t)$$

$$\le x(t)^T P(\hat{\Upsilon}_2+\varepsilon_{3ij}\hat{D}\hat{D}^T)Px(t)+\varepsilon_{2ij}^{-1}x(t)^T\begin{bmatrix}E_{4j}^T & E_{4i}^T\end{bmatrix}\begin{bmatrix}E_{4j}\\ E_{4i}\end{bmatrix}x(t), \tag{20}$$

where

$$\beta(t)=[x(t)^T \quad x(t-\tau)^T \quad \alpha(t)^T]^T,\ \hat{D}=\begin{bmatrix}D_i & D_j\end{bmatrix}$$

$$\mathrm{E}_{1k}=\begin{bmatrix}(E_{0i}+E_{3i}M_i P) & E_{1i} & E_{2i}\end{bmatrix},\ \mathrm{E}_{2k}=\begin{bmatrix}(E_{0i}+E_{3i}M_j P) & E_{1i} & E_{2i}\\ (E_{0j}+E_{3j}M_i P) & E_{1j} & E_{2j}\end{bmatrix},$$

$$\hat{\Upsilon}_2=\begin{bmatrix}B_i D_{1j} & B_j D_{1i}\end{bmatrix}\left(\varepsilon_{2ij}^{-1}I-\varepsilon_{3ij}^{-1}\begin{bmatrix}D_{1j}^T E_{3i}^T & 0\\ 0 & D_{1i}^T E_{3j}^T\end{bmatrix}\begin{bmatrix}E_{3i}D_{1j} & 0\\ 0 & E_{3j}D_{1i}\end{bmatrix}\right)^{-1}\begin{bmatrix}D_{1j}^T B_i^T\\ D_{1i}^T B_j^T\end{bmatrix}.$$

It then follows from (16) and (17)-(20) that

$$\begin{aligned}\dot{V}(x(t)) &\le \sum_{i=1}^r h_i^2(x(t))\beta(t)^T(\Omega_1+\varepsilon_{1ii}^{-1}\mathrm{E}_{1k}^T\mathrm{E}_{1k})\beta(t)\\ &\quad+\sum_{i<j}^r h_i(x(t))h_j(x(t))\beta(t)^T(\Omega_2+\varepsilon_{1ij}^{-1}\mathrm{E}_{2k}^T\mathrm{E}_{2k})\beta(t),\end{aligned} \tag{21}$$

where

$$\Omega_1=\begin{bmatrix}\Phi_i+\Lambda+(\varepsilon_{1ii}+\varepsilon_{3ii})PD_i D_i^T P+P\hat{\Upsilon}_1 P+\varepsilon_{2ii}^{-1}E_{4i}^T E_{4i} & * & *\\ A_{1i}^T P & -Q_1 & *\\ A_{2i}^T P & 0 & -Q_2\end{bmatrix},$$

$$\Omega_2 = \begin{bmatrix} \Phi_{ij} + 2\Lambda + (\varepsilon_{1ij} + \varepsilon_{3ij}) P\hat{D}\hat{D}^T P + P\hat{\Upsilon}_2 P + \varepsilon_{2ij}^{-1}\hat{E}_4^T\hat{E}_4 & * & * \\ (A_{1i} + A_{1j})^T P & -2Q_1 & * \\ (A_{2i} + A_{2j})^T P & 0 & -2Q_2 \end{bmatrix},$$

$$\Phi_i = (A_i + B_i M_i P)^T P + P(A_i + B_i M_i P),$$

$$\Phi_{ij} = (A_i + A_j + B_i M_j P + B_j M_i P)^T P + P(A_i + A_j + B_i M_j P + B_j M_i P),$$

$$\hat{E}_4 = \begin{bmatrix} E_{4j} \\ E_{4i} \end{bmatrix}, \quad \hat{\Upsilon}_1 = B_i D_{1i}(\varepsilon_{2ii}^{-1} I - \varepsilon_{3ii}^{-1} D_{1i}^T E_{3i}^T E_{3i} D_{1i})^{-1} D_{1i}^T B_i^T.$$

Applying Schur complement to LMI (5) results in

$$\begin{bmatrix} \Psi_i & * & * & * & * \\ \hat{Q}_1 A_{1i}^T & -\hat{Q}_1 & * & * & * \\ \hat{Q}_2 A_{2i}^T & 0 & -\hat{Q}_2 & * & * \\ E_{0i}\hat{P} + E_{3i} M_i & E_{1i}\hat{Q}_1 & E_{2i}\hat{Q}_2 & -\varepsilon_{1ii} I & * \\ D_{1i}^T B_i^T & 0 & 0 & 0 & \Theta \end{bmatrix} < 0, \tag{22}$$

where

$$\Theta = -\varepsilon_{2ii}^{-1} I + \varepsilon_{3ii}^{-1} D_{1i}^T E_{3i}^T E_{3i} D_{1i}.$$

$$\Psi_i = \hat{\Phi}_i + \hat{P}\Lambda\hat{P} + (\varepsilon_{1ii} + \varepsilon_{3ii}) D_i D_i^T + \varepsilon_{2ii}^{-1}\hat{P} E_{4i}^T E_{4i}\hat{P},$$

Then, by taking a congruence transformation and applying Schur complement again, we have

$$\sum\nolimits_{i=1}^{r} h_i^2(x(t))\beta(t)^T(\Omega_1 + \varepsilon_{1ii}^{-1}\mathrm{E}_{1k}^T\mathrm{E}_{1k})\beta(t) < 0. \tag{23}$$

Furthermore, we can obtain

$$\sum\nolimits_{i<j}^{r} h_i(x(t))h_j(x(t))\beta(t)^T(\Omega_2 + \varepsilon_{1ij}^{-1}\mathrm{E}_{2k}^T\mathrm{E}_{2k})\beta(t) < 0. \tag{24}$$

It then follows from (21)-(24) that

$$\dot{V}(x(t)) < 0.$$

Therefore, the closed-loop system is robustly asymptotically stable. This completes the proof.

4. ILLUSTRATIVE EXAMPLE

In this section, we provide an illustrative example to demonstrate the effectiveness of the proposed method.

Consider an uncertain distributed delay fuzzy system with two fuzzy rules as follows:

$$\dot{x}(t) = \sum_{i=1}^{2} h_i(x(t))[(A_i + \Delta A_i)x(t) + (A_{1i} + \Delta A_{1i})x(t-1)$$

$$+ (A_{2i} + \Delta A_{2i})\int_{t-1}^{t} x(s)ds + (B_i + \Delta B_i)u(t)],$$

where

$$A_1 = \begin{bmatrix} 0.5 & -0.3 \\ 0.1 & 1 \end{bmatrix},\quad A_{11} = \begin{bmatrix} 0.3 & 0.2 \\ -0.5 & 0.1 \end{bmatrix},\quad A_{21} = \begin{bmatrix} -0.2 & 0.8 \\ 0.6 & 0 \end{bmatrix},\quad B_1 = \begin{bmatrix} 1.5 & 0 \\ 0.3 & 1.6 \end{bmatrix},$$

$$E_{01} = [0.3 \quad 0.1],\quad E_{11} = [-0.1 \quad 0.3],\quad E_{21} = [0.3 \quad 0],\quad E_{31} = [-0.2 \quad 0.1],$$

$$E_{41} = [-0.2 \quad 0.3],\quad D_1 = [0 \quad 0.2]^T,\quad D_{11} = [0.1 \quad 0.1]^T$$

$$A_2 = \begin{bmatrix} 0.8 & 0 \\ -0.2 & 0.6 \end{bmatrix},\quad A_{12} = \begin{bmatrix} 0.6 & 0.2 \\ 0 & -0.3 \end{bmatrix},\quad A_{22} = \begin{bmatrix} 0.5 & -0.2 \\ -0.3 & 0.2 \end{bmatrix},\quad B_2 = \begin{bmatrix} 1 & -0.8 \\ 0.2 & 1.5 \end{bmatrix},$$

$$E_{02} = [0.2 \quad 0],\quad E_{12} = [0.1 \quad 0.1],\quad E_{22} = [0.2 \quad 0.1],\quad E_{32} = [-0.1 \quad 0],$$

$$E_{42} = [0.3 \quad 0.1],\quad D_2 = [-0.2 \quad 0.1]^T,\quad D_{12} = [0.3 \quad -0.1]^T,$$

$$h_1(x_1(t)) = \begin{cases} 1/3, & x_1 < -1 \\ \frac{2}{3} + \frac{1}{3}x_1, & |x_1| \le 1 \\ 1, & x_1 > 1 \end{cases} \text{£¬} \quad h_2(x_1(t)) = \begin{cases} 2/3, & x_1 < -1 \\ \frac{1}{3} - \frac{1}{3}x_1, & |x_1| \le 1 \\ 0, & x_1 > 1 \end{cases}.$$

In this example, we suppose $\tau = 1$ and $F_i(t) = \cos(t)I_{2\times 2}$, $(i = 1,2)$, $x(0) = [-0.8 \quad 1.2]^T$.

From Figure 1, it is easy to see that the open-loop system is not stable. Now, by Theorem 1, we use the Matlab LMI Control Toolbox to solve the LMIs in (5) and (6) and obtain a set of solutions as:

$$\hat{P} = \begin{bmatrix} 0.4521 & 0.0396 \\ 0.0396 & 0.3868 \end{bmatrix},$$

$$\hat{Q}_1 = \begin{bmatrix} 0.0564 & 0.0028 \\ 0.0028 & 0.0486 \end{bmatrix},$$

$$\hat{Q}_2 = \begin{bmatrix} 0.0555 & -0.0026 \\ -0.0026 & 0.0535 \end{bmatrix},$$

$$\varepsilon_{111} = 25.6153,\quad \varepsilon_{112} = 25.5891,\quad \varepsilon_{122} = 24.5596,$$

$$\varepsilon_{211} = 23.6612, \quad \varepsilon_{212} = 23.1411, \quad \varepsilon_{222} = 22.3465,$$

$$\varepsilon_{311} = 24.076, \quad \varepsilon_{312} = 24.6115, \quad \varepsilon_{322} = 23.9938.$$

Then, by Theorem 1, a set of desired fuzzy controller gains can be chosen as

$$K_1 = \begin{bmatrix} -4.7 & -0.0172 \\ 0.0553 & -3.9639 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} -6.3084 & -0.0517 \\ 0.2825 & -2.9633 \end{bmatrix}.$$

Simulation results of the designed controller and the state response of the closed-loop system are given in Figures 2 and 3, respectively.

5. CONCLUSIONS

In this paper, we have studied the problem of non-fragile controller design for fuzzy distributed delay systems with norm-bounded time-varying parametric uncertainties. It has been shown that a desired non-fragile controller can be constructed by solving certain LMIs. An example has been provided to illustrate the effectiveness of the proposed method.

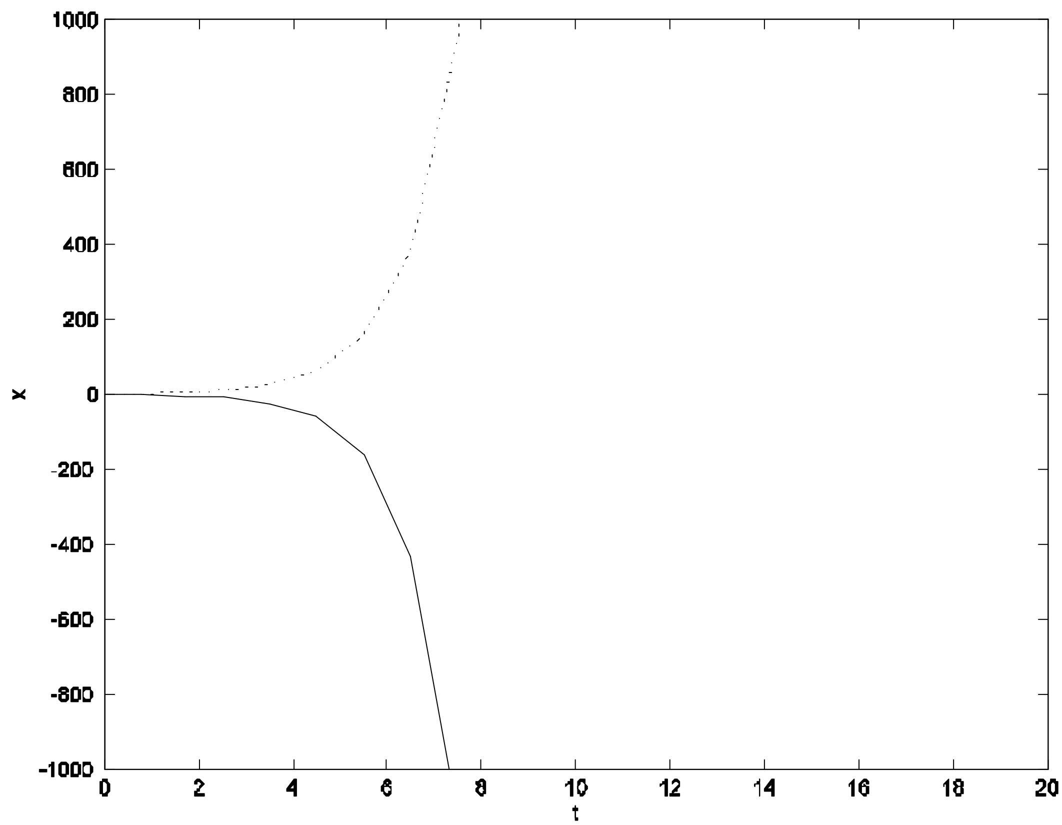

Figure 1: State Response $x_1(t)$ and $x_2(t)$ of the open-loop system

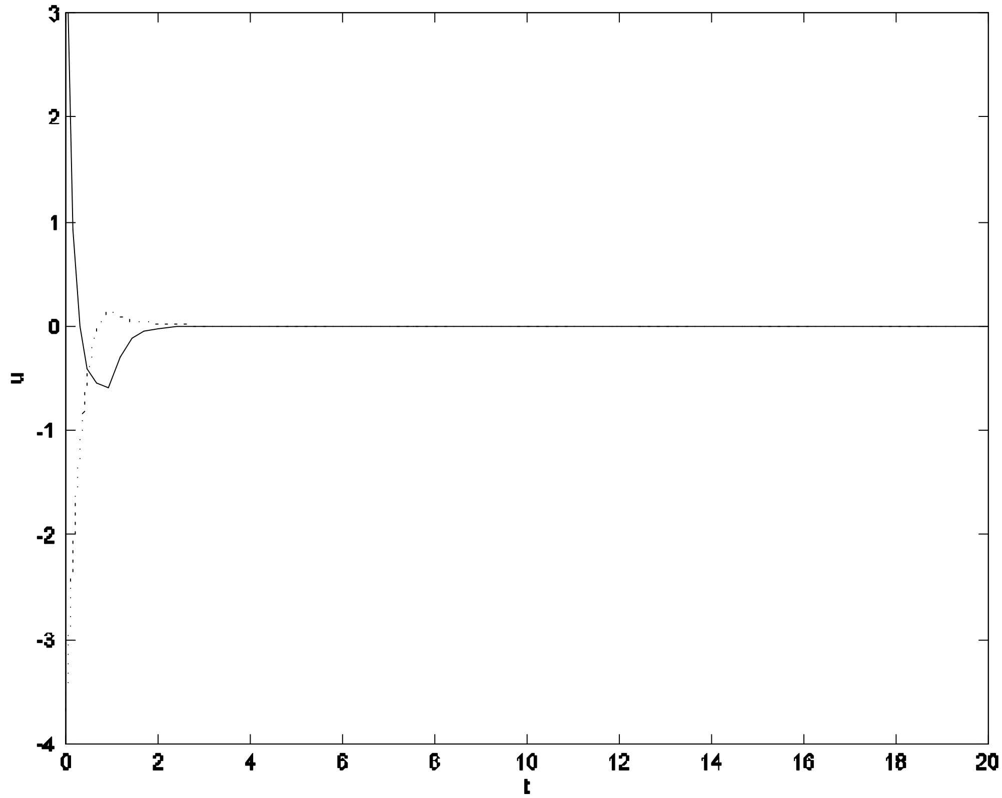

Figure 2: Control Vectors $u_1(t)$ and $u_2(t)$

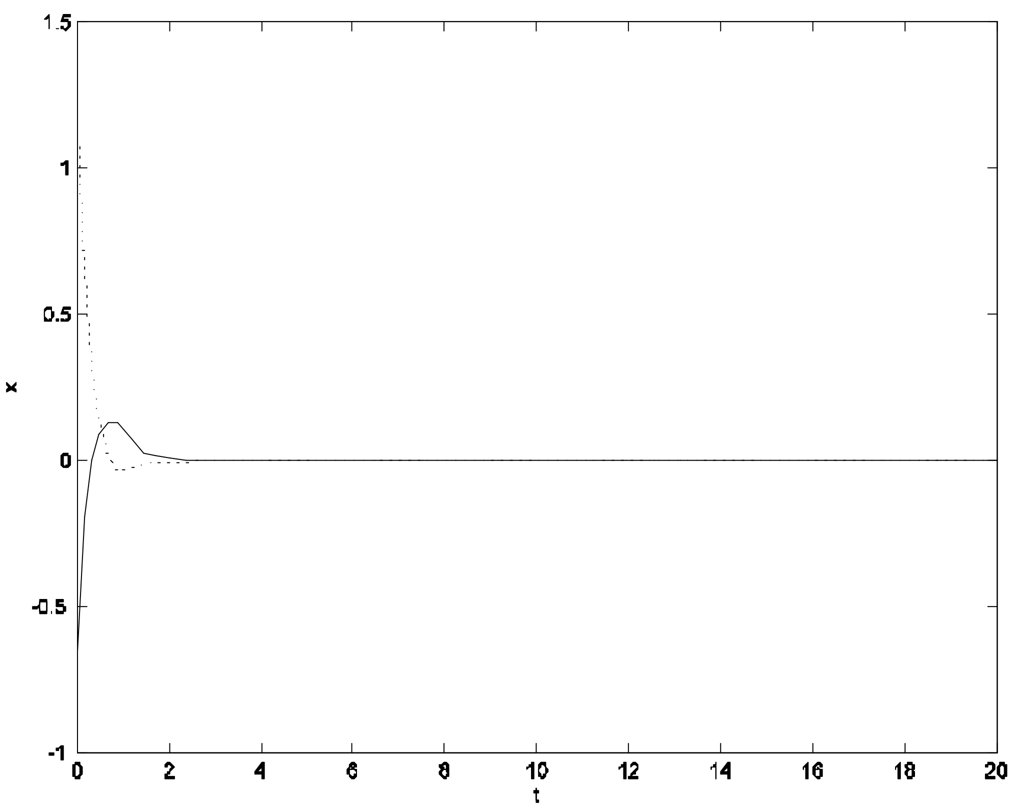

Figure 3: State Response $x_1(t)$ and $x_2(t)$ of the closed-loop system

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