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Robust Stability of Uncertain Lurie-type Neutral System with Delays

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Abstract: This paper introduces the robust stability of delay Lurie-type neutral control system. A new criterion is presented in term of linear matrix inequalities by using Lyapunov method. Since the criterion takes the size of the neutral and discrete delays into account, it is less conservative than the existing ones. Examples are given to that illustrate the advantage of our approach and reciprocal influences between the neutral and discrete delays.

Keyword: Lurie-type neutral system, Lyapunov method, Robust stability, Linear matrix inequality (LMI).

1. INTRODUCTION

The problem of the stability of Lurie-type control systems has been widely studied for several decades [1-3,7], which is due to theoretical interests as well as a powerful tool for practical system analysis and design. Since time delays are frequently encountered in various engineering systems and are often a source of instability, a considerable number of studies have also been done on the stability of delay control systems [3-10]. Recently, the stability analysis of neutral differential systems, which contain delays both in its state and in the derivatives of its states, has been widely investigated by many researcher [4,8-10]. Some stability criteria formulated in terms of linear matrix inequalities (LMIs), the linear matrix inequality can be easily solved.

The stability criteria are often classified into two categories according to their dependence on the size of the delays, namely, delay-dependent stability criteria and delay-independent stability criteria. In general, the delay-dependent stability criterion is less conservative than delay-independent stability one when the size of time-delay is small.

In this paper, using the Lyapunov functional technique, we present a novel delay-dependent stability criterion for delay Lurie differential systems. Based on a new method, we employ free weighting matrices to express the influences of, and the relationship between the terms $x(t-\tau)$ and $x(t)-\int_{t-\tau}^{t}\dot{x}(s)ds$. The new criterion is given in terms of LMIs, which make the free weighting matrices ease to select. Since this criterion is both neutral-delay-dependent and discrete-delay-dependent, it is less conservative than previous methods for mixed neutral- and discrete-delays. Two examples are given to show both the superiority of the present result to those in the literature and the influences between neutral and discrete delays.

Through the paper, R^n space denotes n-dimensional Euclidean space, $R^{n\times m}$ space is the set of all $n \times m$ real matrices, I denotes identity matrix of appropriate order, and $*$ represents the elements below the main diagonal of a symmetric block matrix. The notation $W > 0$ ($\geq, <, \leq 0$) denotes a symmetric positive definite (positive semidefinite, negative, negative semidefinite) matrix W.

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2. THE DESCRIPTION OF SYSTEMS AND PRELIMINARIES

Consider the Lurie neutral functional differential system described by the following equation:

$$\begin{cases} \dot{x}(t) - C\dot{x}(t-\tau_2) = (A+\Delta A(t))x(t) + (B+\Delta B(t))x(t-\tau_1) + (D+\Delta D(t))f(\sigma(t)) \\ \sigma(t) = c^T x(t) \end{cases} \tag{1}$$

where $x(t) \in R^n$ space is the state vector, $A \in R^{n\times n}$ space, $B \in R^{n\times n}$ space, $C \in R^{n\times n}$ space and $D \in R^{n\times 1}$ space, $c \in R^{n\times 1}$ space are known real constant matrices, τ_1, $\tau_2 > 0$ are unknown constant state delays. The initial condition of system (1) satisfy

$$x(t) = \phi(t), \quad t \in [-max\{\tau_1, \tau_2\}, 0]$$

and the nonlinear $f(\sigma)$ satisfy

$$f(\sigma) \in K[0,k] = \{f(\sigma) \mid \phi(0) = 0; 0 \le \sigma f(\sigma) \le k\sigma^2; \sigma \ne 0\} \tag{2}$$

for $0 < k < +\infty$. The uncertainties are assumed to be of the following form

$$[\Delta A(t) \quad \Delta B(t) \quad \Delta D(t)] = LF(t)[E_a \quad E_b \quad E_d] \tag{3}$$

where L, E_a, E_b, E_d are constant matrices with appropriate dimensions, and $F(t)$ is unknown real and possibly time-vary matrix with Lebesgue measurable elements bounded by

$$\| F_{(t)} \| \le 1, \quad \forall t \tag{4}$$

Before proceeding further, we will state well known Lemma.

Lemma 2.1:[13] The linear matrix inequality

$$\begin{bmatrix} \Omega_1 & \Omega_2^T \\ \Omega_2 & \Omega_3 \end{bmatrix} > 0$$

is equivalent to

$$\Omega_1 - \Omega_2^T \Omega_3^{-1} \Omega_2 > 0, \Omega_1 > 0$$

where $\Omega_1 = \Omega_1^T, \Omega_3 = \Omega_3^T$ and Ω_2 depend on x

Lemma 2.2: [14] For matrices $P, L, N, F \in R^{n\times n}$ with $P > 0$, $\| F \| \le 1$ and $\alpha > 0$, one has the following:

(1) $(LFN)^T P + P(LFN \le \alpha PLL^T P + \alpha^{-1} N^T N$

(2) If $P - \alpha MM^T > 0$, then

$$(A+LFN)^T P^{-1}(A+LFN) \le A^T(P-\alpha MM^T)^{-1}A + \alpha^{-1}N^T N.$$

3. MAIN RESULT

System (1) can be written as

$$\begin{cases} \dot{x}(t) - C\dot{x}(t-\tau_2) = Ax(t) + Bx(t-\tau_1) + Df(\sigma(t)) + Lu \\ y = E_a x(t) + E_b x(t-\tau_1) + E_d f(\sigma(t)) \\ u = F(t)y, \\ \sigma(t) = c^T x(t) \end{cases} \tag{5}$$

In order to simplify the treatment of the problem, We define an $\wp : D([-\tau_2, 0], R^n) \to R^n$ as $\wp(x_t) = x(t) - Cx(t - \tau_2)$

The stability of system $\wp$ is defined as follow:

Definition 3.1 (Hale [11])The operator $\wp$ is said to be stable if the zero solution of the homogeneous difference equation $\wp(x_t) = 0, t \geq 0, x_0 = \psi \in \{ f \in C([-\tau_2, 0] : \wp f = 0\}$ is uniformly asympotically stable. Now, we have the following theorem

Theorem 3.2 Given scalars $\tau_1 > 0$ and $\tau_2 > 0$, the system (1) is absolute stable if the operator $\wp(x_t)$ is stable and there exist the positive-definite matrices $P = P^T, Q_1 = Q_1^T, Q_2 = Q_2^T, R = R^T, X_{66} = X_{66}^T, Y_{66} = Y_{66}^T$, positive scalars $\alpha, \beta, \alpha_1, \alpha_2$ and any matrices X_{ij}, Y_{ij} of appropriate dimensions satisfying the following matrix inequalities:

$$\begin{bmatrix} S & SL \\ L^T S & \alpha_1 I \end{bmatrix} > 0 \tag{6}$$

$$\begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} & \phi_{14} & \phi_{15}+\alpha kc & PL & A^T S & 0 & \alpha_1 E_a^T & \alpha_2 E_a^T \\ * & \phi_{22} & \phi_{23} & \phi_{24} & \phi_{25} & 0 & B^T S & 0 & \alpha_1 E_b^T & \alpha_2 E_b^T \\ * & * & \phi_{33} & \phi_{34} & \phi_{35} & -C^T PL & 0 & 0 & 0 & 0 \\ * & * & * & \phi_{44} & \phi_{45} & 0 & C^T S & 0 & 0 & 0 \\ * & * & * & * & \phi_{55} - 2\alpha & \beta c^T L & D^T S & 0 & \alpha_1 E_d^T & \alpha_2 E_d^T \\ * & * & * & * & * & -\alpha_2 I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -S & SL & 0 & 0 \\ * & * & * & * & * & * & * & -\alpha_1 I & 0 & 0 \\ * & * & * & * & * & * & * & * & -\alpha_1 I & 0 \\ * & * & * & * & * & * & * & * & * & -\alpha_2 I \end{bmatrix} < 0 \tag{7}$$

$$\Psi_1 = \begin{bmatrix} X_{11} & X_{12} & X_{13} & X_{14} & X_{15} & X_{16} \\ * & X_{22} & X_{23} & X_{24} & X_{25} & X_{26} \\ * & * & X_{33} & X_{34} & X_{35} & X_{36} \\ * & * & * & X_{44} & X_{45} & X_{46} \\ * & * & * & * & X_{55} & X_{56} \\ * & * & * & * & * & X_{66} \end{bmatrix} \geq 0 \tag{8}$$

$$\Psi_2 = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} & Y_{15} & Y_{16} \\ * & Y_{22} & Y_{23} & Y_{24} & Y_{25} & Y_{26} \\ * & * & Y_{33} & Y_{34} & Y_{35} & Y_{36} \\ * & * & * & Y_{44} & Y_{45} & Y_{46} \\ * & * & * & * & Y_{55} & Y_{56} \\ * & * & * & * & * & Y_{66} \end{bmatrix} \geq 0 \tag{9}$$

where

$$\begin{aligned}
\phi_{11} &= PA + A^T P + Q_1 + Q_2 + X_{16} + X_{16}^T + Y_{16} + Y_{16}^T + \tau_1 X_{11} + \tau_2 Y_{11} \\
\phi_{12} &= PB - X_{16} + X_{26}^T + Y_{26}^T + \tau_1 X_{12} + \tau_2 Y_{12} \\
\phi_{13} &= -A^T PC + X_{36}^T - Y_{16} + Y_{36}^T + \tau_1 X_{13} + \tau_2 Y_{13} \\
\phi_{14} &= X_{46}^T + Y_{46}^T + \tau_1 X_{14} + \tau_2 Y_{14} \\
\phi_{15} &= PD + \beta A^T c + X_{56}^T + Y_{56}^T + \tau_1 X_{15} + \tau_2 Y_{15} \\
\phi_{22} &= -Q_1 - X_{26} - X_{26}^T + \tau_1 X_{22} + \tau_2 Y_{22} \\
\phi_{23} &= -B^T PC - X_{36}^T - Y_{26} + \tau_1 X_{23} + \tau_2 Y_{23} \\
\phi_{24} &= -X_{46}^T + \tau_1 X_{24} + \tau_2 Y_{24} \\
\phi_{25} &= \beta B^T c - X_{56}^T + \tau_1 X_{25} + \tau_2 Y_{25} \\
\phi_{33} &= -Q_2 - Y_{36}^T - Y_{36} + \tau_1 X_{33} + \tau_2 Y_{33} \\
\phi_{34} &= -Y_{46}^T + \tau_1 X_{34} + \tau_2 Y_{34} \\
\phi_{35} &= -C^T PD - Y_{56}^T + \tau_1 X_{35} + \tau_2 Y_{35} \\
\phi_{44} &= -R + \tau_1 X_{44} + \tau_2 Y_{44} \\
\phi_{45} &= \beta C^T c + \tau_1 X_{45} + \tau_2 Y_{45} \\
\phi_{55} &= 2\beta D^T c + \tau_1 X_{55} + \tau_2 Y_{55} \\
S &= R + \tau_1 X_{66} + \tau_2 Y_{66}
\end{aligned} \tag{10}$$

Proof For $P > 0, Q_1 > 0, Q_2 > 0, R > 0, X_{66} > 0, Y_{66} > 0$ and $\beta > 0$, We choose the Lyapunov functional to be

$$V = V_1 + V_2 + V_3 + V_4 + V_5 + V_6 + V_7$$

where

$$\begin{aligned}
V_1 = \wp^T(x_t) P \wp(x_t) + \int_{t-\tau_1}^{t} x^T(s) Q_1 x(s) ds + \int_{t-\tau_2}^{t} x^T(s) Q_2 x(s) ds + \int_{t-\tau_2}^{t} \dot{x}^T(s) R \dot{x}(s) ds \\
+ \int_{-\tau_1}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) X_{66} \dot{x}(s) ds d\theta + \int_{-\tau_2}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) Y_{66} \dot{x}(s) ds d\theta + 2\beta \int_0^{\sigma} f(\alpha) d\alpha.
\end{aligned}$$

Taking the time derivative of V along the solution of (5) gives that

$$\dot{V} = 2[x(t) - Cx(t-\tau_2)]^T P\left[Ax(t) + Bx(t-\tau_1) + Df(\sigma) + Lu\right] + x^T(t)Q_1x(t)$$
$$-x^T(t-\tau_1)Q_1x(t-\tau_1) + x^T(t)Q_2x(t) - x^T(t-\tau_2)Q_2x(t-\tau_2) + \dot{x}^T(t)R\dot{x}(t)$$
$$-\dot{x}^T(t-\tau_2)R\dot{x}(t-\tau_2) + \tau_1\dot{x}^T(t)X_{66}\dot{x}(t) - \int_{t-\tau_1}^{t}\dot{x}^T(s)X_{66}\dot{x}(s)ds + \tau_2\dot{x}^T(t)Y_{66}\dot{x}(t)$$
$$-\int_{t-\tau_2}^{t}\dot{x}^T(s)Y_{66}\dot{x}(s)ds + 2\beta[Ax(t) + Bx(t-\tau_1) + C\dot{x}(t-\tau_2) + Df(\sigma) + Lu]^T cf(\sigma). \tag{11}$$

Applying Lemma 2.2(1), the following inequality hold:

$$2[x^T(t)P - x^T(t-\tau_2)C^TP + \beta f^T(\sigma)c^T]Lu = 2[x^T(t)P - x^T(t-\tau_2)C^TP + f^T(\sigma)\beta c^T]LF(t)y$$
$$\leq \alpha_2^{-1}[x^T(t)PL - x^T(t-\tau_2)C^TPL + f^T(\sigma)\beta c^TL]^T[x^T(t)PL - x^T(t-\tau_2)C^TPL + f^T(\sigma)\beta c^TL] + \alpha_2 y^Ty. \tag{12}$$

Note that $S = R + \tau_1 X_{66} + \tau_2 Y_{66}$, and from LMI (6) and Lemma 2.2(2), we have

$$\begin{aligned}\dot{x}^T(t)S\dot{x}(t) \leq{}& [Ax(t) + Bx(t-\tau_1) + C\dot{x}(t-\tau_2) + Df(\sigma)]^T(S^{-1} - \alpha_1^{-1}LL^T)^{-1}\\ &[Ax(t) + Bx(t-\tau_1) + C\dot{x}(t-\tau_2) + Df(\sigma)] + \alpha_1 y^Ty.\end{aligned} \tag{13}$$

Using the Leibniz-Newton formula, we can write

$$x(t-\tau_1) = x(t) - \int_{t-\tau_1}^{t}\dot{x}(s)ds$$
$$x(t-\tau_2) = x(t) - \int_{t-\tau_2}^{t}\dot{x}(s)ds. \tag{14}$$

According to (14), for any matrices $X_{i6}, Y_{i6}, (i = 1,\ldots,5)$, the following equation hold:

$$\begin{aligned}&2[x^T(t)X_{16} + x^T(t-\tau_1)X_{26} + x^T(t-\tau_2)X_{36} + \dot{x}^T(t-\tau_2)X_{46}\\ &\quad + f^T(\sigma)X_{56}][x(t) - x(t-\tau_1) - \int_{t-\tau_1}^{t}\dot{x}(s)ds] = 0\end{aligned} \tag{15}$$

$$\begin{aligned}&2[x^T(t)Y_{16} + x^T(t-\tau_1)Y_{26} + x^T(t-\tau_2)Y_{36} + \dot{x}^T(t-\tau_2)Y_{46}\\ &\quad + f^T(\sigma)Y_{56}][x(t) - x(t-\tau_2) - \int_{t-\tau_2}^{t}\dot{x}(s)ds] = 0.\end{aligned} \tag{16}$$

On the other hand, for any appropriately matrices X_{ij}, Y_{ij}, let $E_{ij} = \tau_1(X_{ij} - X_{ij}) - \tau_2(Y_{ij} - Y_{ij}), (i = 1,\ldots,5; i \leq j \leq 5)$, so $E_{ij} = 0$ the following equation also hold;

$$Z_1^T\begin{bmatrix} E_{11} & E_{12} & E_{13} & E_{14} & E_{15}\\ E_{12}^T & E_{22} & E_{23} & E_{24} & E_{25}\\ E_{13}^T & E_{23}^T & E_{33} & E_{34} & E_{35}\\ E_{14}^T & E_{24}^T & E_{34}^T & E_{44} & E_{45}\\ E_{15}^T & E_{25}^T & E_{35}^T & E_{45}^T & E_{55}\end{bmatrix}Z_1 = 0 \tag{17}$$

where

$$Z_1^T = \left[x^T(t)\ x^T(t-\tau_1)\ x^T(t-\tau_2)\ \dot{x}^T(t-\tau_2)\ f^T(\sigma) \right]^T$$

Then we add the terms on the left side of Eqs. (15)-(17) to $\dot{V}(x_t)$; and consider the fact that, for any $r > 0$ and any function $f(u), \int_{t-r}^{t} f(u)ds = rf(u)$. Then, $\dot{V}(x_t)$ can be expressed as follows:

$$\dot{V}(x_t) \le Z_1^T(t)\Omega_0 Z_1(t) - \int_{t-\tau_1}^{t} Z_2^T(t,s)\Psi_1 Z_2(t,s)ds - \int_{t-\tau_2}^{t} Z_2^T(t,s)\Psi_2 Z_2(t,s)ds + \alpha_1 y^T y + \alpha_2 y^T y$$

$$+ Z_1^T M^T (S^{-1} - \alpha_1^{-1} LL^T)^{-1} M Z_1 + \alpha_2^{-1} Z_1^T N^T N Z_1 \tag{18}$$

where

$$M = \begin{bmatrix} A & B & 0 & C & D \end{bmatrix},\ N = \begin{bmatrix} L^T P & 0 & -L^T PC & 0 & \beta L^T c \end{bmatrix},\ Z_2(t,s) = [Z_1^T(t)\dot{x}^T(s)]^T,$$

$$\Omega_0 = \begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} & \phi_{14} & \phi_{15} \\ * & \phi_{22} & \phi_{23} & \phi_{24} & \phi_{25} \\ * & * & \phi_{33} & \phi_{34} & \phi_{35} \\ * & * & * & \phi_{44} & \phi_{45} \\ * & * & * & * & \phi_{55} \end{bmatrix}, \tag{19}$$

and $\Psi_1, \Psi_2, \phi_{ij}, (i = 1,\ldots,5; i \le j \le 5)$ and S are defined in Eqs. (8) (9) (10). In addition, from condition (2) we can see $f(\sigma(t))$ and $\sigma(t)$ have the same positive definite or oppositive definite, so condition (2) are equivalent to

$$f(\sigma(t))(f(\sigma(t)) - kc^T x(t)) \le 0. \tag{20}$$

Using (20) and applying S-procedure shows that if there exists $\alpha_1 > 0, \alpha_2 > 0, \Psi_1 \ge 0, \Psi_2 \ge 0$ and

$$\Sigma = \Omega_0 + M^T (S^{-1} - \alpha_1^{-1} LL^T)^{-1} M + \alpha_2^{-1} N^T N + (\alpha_1 + \alpha_2) N_1^T N_1 < 0,$$

such that

$$Z_1^T(t)\Sigma Z_1(t) - \int_{t-\tau_1}^{t} Z_2^T(t,s)\Psi_1 Z_2(t,s)ds - \int_{t-\tau_2}^{t} Z_2^T(t,s)\Psi_2 Z_2(t,s)ds - 2\alpha f(\sigma(t))(f(\sigma(t)) - kc^T x(t)) < 0 \tag{21}$$

here $N_1 = \begin{bmatrix} E_a & E_b & 0 & 0 & E_d \end{bmatrix}$, then $\dot{V}(x_t) < 0$ for any $Z_1(t) \ne 0$ under the condition (2). Since the terms $\int_{t-\tau_1}^{t} Z_2^T(t,s)\Psi_1 Z_2(t,s)ds$ and $\int_{t-\tau_2}^{t} Z_2^T(t,s)\Psi_2 Z_2(t,s)ds$ in (21) is positive-definite, note that by Lemma 3.1 (Schur complement), (21) is equivalent to

$$\begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} & \phi_{14} & \phi_{15}+\alpha kc & PL & A^T & 0 & E_a^T & E_a^T \\ * & \phi_{22} & \phi_{23} & \phi_{24} & \phi_{25} & 0 & B^T & 0 & E_b^T & E_b^T \\ * & * & \phi_{33} & \phi_{34} & \phi_{35} & -C^T PL & 0 & 0 & 0 & 0 \\ * & * & * & \phi_{44} & \phi_{45} & 0 & C^T & 0 & 0 & 0 \\ * & * & * & * & \phi_{55}-2\alpha & \beta c^T L & D^T & 0 & E_d^T & E_d^T \\ * & * & * & * & * & -\alpha_2 I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -S^{-1} & L & 0 & 0 \\ * & * & * & * & * & * & * & -\alpha_1 I & 0 & 0 \\ * & * & * & * & * & * & * & * & -\alpha_1^{-1} I & 0 \\ * & * & * & * & * & * & * & * & * & -\alpha_2^{-1} I \end{bmatrix} < 0. \tag{22}$$

Pre- and post-multiplying by diag $(I,I,I,I,I,I,S,I,\alpha_1 I,\alpha_2 I)$ result in (7). So system (1) is absolute stable if the LMI (6)-(9) are feasible. This completes our proof.

Remark 3.3 Since for any $r > 0$ and any function $f(u), \int_{t-r}^{t} f(u)ds = rf(u)$, so according to (15)(16)(17), we have two following equations:

$$Z_1^T(-\tau_1)\begin{bmatrix} X_{11} & X_{12} & X_{13} & X_{14} & X_{15} \\ X_{12}^T & X_{22} & X_{23} & X_{24} & X_{25} \\ X_{13}^T & X_{23}^T & X_{33} & X_{34} & X_{35} \\ X_{14}^T & X_{24}^T & X_{34}^T & X_{44} & X_{45} \\ X_{15}^T & X_{25}^T & X_{35}^T & X_{45}^T & X_{55} \end{bmatrix} Z_1 + 2[x^T(t)X_{16} + x^T(t-\tau_1)X_{26} + x^T(t-\tau_2)X_{36}$$

$$+\dot{x}^T(t-\tau_2)X_{46} + f^T(\sigma)X_{56}]\times(-\int_{t-\tau_1}^{t} \dot{x}(s)ds) - \int_{t-\tau_1}^{t} \dot{x}^T(s)X_{66}\dot{x}(s)ds = -\int_{t-\tau_1}^{t} Z_2^T(t,s)\Psi_1 Z_2(t,s)ds$$

and

$$Z_1^T(-\tau_2)\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} & Y_{15} \\ Y_{12}^T & Y_{22} & Y_{23} & Y_{24} & Y_{25} \\ Y_{13}^T & Y_{23}^T & Y_{33} & Y_{34} & Y_{35} \\ Y_{14}^T & Y_{24}^T & Y_{34}^T & Y_{44} & Y_{45} \\ Y_{15}^T & Y_{25}^T & Y_{35}^T & Y_{45}^T & Y_{55} \end{bmatrix} Z_1 + 2[x^T(t)Y_{16} + x^T(t-\tau_1)Y_{26} + x^T(t-\tau_2)Y_{36}$$

$$+\dot{x}^T(t-\tau_2)Y_{46} + f^T(\sigma)Y_{56}]\times(-\int_{t-\tau_2}^{t} \dot{x}(s)ds) - \int_{t-\tau_2}^{t} \dot{x}^T(s)Y_{66}\dot{x}(s)ds = -\int_{t-\tau_2}^{t} Z_2^T(t,s)\Psi_2 Z_2(t,s)ds.$$

Remark 3.4: In the Theorem 1, the relationships between the term $x(t-\tau_1)$ and $x(t)-\int_{t-\tau_1}^{t}\dot{x}(s)ds,$ and $x(t-\tau_2)$ and $x(t)-\int_{t-\tau_2}^{t}\dot{x}(s)ds,$ have been considered through the free weighting matrices, X_{i6} and $Y_{i6}, (i=1\cdots 5)$ and the optimal weighting matrices can be selected by solving the LMIs (7) and (8). In contrast, previous methods employ fixed weighting matrices, which are not usually optimal ones.

Remark 3.5: In previous methods, it is very hard to handle the case where the neutral and discrete delays are different, the result are almost discrete delays dependent and neutral delay independent. But our method overcomes the difficulty that appears in previous studies, the criterion in Theorem 1 includes information on the sizes of τ_1 and τ_2, which makes it both discrete delays dependent and neutral delay dependent stability criterion. So this method yields a less conservative criterion than previous criteria.

Remark 3.6: By Theorem 3.2, we can easy obtain a criterion for the system described by (1) with the special case of $D = 0$, if we set $R = Q_2 = Y_{66} = 0$, then the Lyapunov functional reduces to

$$\tilde{V} = x^T(t)Px(t)+\int_{t-\tau_1}^{t} x^T(s)Q_1x(s)ds+\int_{-\tau_1}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)X_{66}\dot{x}(s)dsd\theta+2\beta\int_0^{\sigma} f(\alpha)d\alpha$$

Similar to the proof of Theorem 3.2, we can draw the conclusion of Theorem 7 in the paper of [3]

Remark 3.7: When the system doesn't contain time-varying structureed uncertainties, it can be described by

$$\begin{cases}\dot{x}(t)-C\dot{x}(t-\tau_2)=Ax(t)+Bx(t-\tau_1)+Df(\sigma)\\ \sigma=c^Tx(t)\end{cases} \tag{23}$$

We construct a Lyapunov functional which is same as that in Theorem 3.2. Similar to the proof of Theorem 3.2, the stability of system (23) can be stated as:

Given scalars $\tau_1>0$ and $\tau_2>0$, the system (23) is absolute stable if the operator $\wp(x_t)$ is stable and there exist the symmetric positive-definite matrices $P,Q_1,Q_2,R,X_{66},Y_{66}$, positive scalars $\alpha,\beta,$ and any matrices $X_{ij},Y_{ij}(i,j=1,\cdots,i<j\le 5)$ of appropriate dimensions satisfying (8)(9) and the following matrix inequalities:

$$\Omega=\begin{bmatrix}\phi_{11} & \phi_{12} & \phi_{13} & \phi_{14} & \phi_{15}+\alpha kc & A^TS\\ \phi_{12}^T & \phi_{22} & \phi_{23} & \phi_{24} & \phi_{25} & B^TS\\ \phi_{13}^T & \phi_{23}^T & \phi_{33} & \phi_{34} & \phi_{35} & 0\\ \phi_{14}^T & \phi_{24}^T & \phi_{34}^T & \phi_{44} & \phi_{45} & C^TS\\ \phi_{15}^T & \phi_{25}^T & \phi_{35}^T & \phi_{45}^T & \phi_{55}-2\alpha I & D^TS\\ SA & SB & 0 & SC & SD & -S\end{bmatrix}<0 \tag{24}$$

Here ϕ_{ij} and S is the same as Theorem 3.2.

Remark 3.8: The above result can be easy extended to the stability of uncertain Lurie-type neutral system with multiple neutral and discrete delays.

4. EXAMPLES

Example 4.1 Consider the Lurie-type neutral system (23) where

$$A = \begin{bmatrix} -2 & -1 \\ -1 & 0 \end{bmatrix}, B = \begin{bmatrix} -0.5 & -0.1 \\ 0.1 & -0.5 \end{bmatrix}, C = \begin{bmatrix} -0.2 & 0 \\ 0.2 & -0.1 \end{bmatrix}, D = \begin{bmatrix} -0.2 \\ -0.3 \end{bmatrix}, c = \begin{bmatrix} 0.6 \\ 0.8 \end{bmatrix}$$

Applying Remark 3.7 and using the Matlab LMI Toolbox [13], it is found that for $\tau_1 = \tau_2$ and $k = 1$ the maximum upper bounds on the allowable size to be $\tau_1 = \tau_2 = 1.987$, Table 1 list the upper bounds on τ_1 that guarantee the stability of the system for values of τ_2 from 0.1 to 1.0. It can be seen the upper bound on τ_1 decrease as τ_2 when τ_2 is small.

Table 1
The upper bounds on τ_1 with the difficult values of τ_2

τ_2	0.1	0.2	0.3	0.4	0.5
τ_1	2.057	2.050	2.042	2.035	2.029
τ_2	0.6	0.7	0.8	0.9	1.0
τ_1	2.023	2.018	2.013	2.009	2.006

Example 4.2: Consider the system described by (1) with

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, B = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}, C = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, D = \begin{bmatrix} -0.2 \\ -0.3 \end{bmatrix}, c = \begin{bmatrix} 0.6 \\ 0.8 \end{bmatrix}$$

$$E_a = E_b = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, E_d = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, L = \begin{bmatrix} \gamma & 0 \\ 0 & \gamma \end{bmatrix}$$

Applying Theorem 3.2, it can be seen that for $\tau_1 = \tau_2$ and $k = 1$, $\gamma = 0.1$ the maximum upper bounds on the allowable size to be $h_{max} = \tau_1 = \tau_2 = 2.33$.

For $k = 1$, we now consider the effect of uncertainty bound γ on the maximum time-delay $h = \tau_1 = \tau_2$ for stability of system (1). Table 2 illustrates the numerical for different γ. As γ increases, h decreases.

Table 2
Allowable time delay h with different γ

γ	0.05	0.1	0.15	0.2	0.25
h_{max}	2.87	2.33	1.91	1.57	1.26
γ	0.3	0.35	0.4	0.45	0.5
h_{max}	0.97	0.76	0.60	0.49	0.39

For $k = 1$, $\gamma = 0.1$, Table 3 list the maximum bounds on τ_1 that guarantee the stability of the system for different τ_2 from 0.1 to 1.2. It also can be seen that the upper bound on τ_1 decrease as τ_2 when τ_2 is small.

It is clear that our results are significant better for a constant k when $\tau_1 \neq \tau_2$. If the maximum upper bound on the delay τ_2 is small, then the maximum upper bound on τ_1 is larger than that when $\tau_1 = \tau_2$.

Table 3
The upper bounds on τ_1 with the difficult values of τ_2

τ_2	0.1	0.2	0.3	0.4	0.5	0.6
τ_1	2.87	2.83	2.79	2.75	2.70	2.66
τ_2	0.7	0.8	0.9	1.0	1.1	1.2
τ_1	2.61	2.57	2.53	2.49	2.46	2.42

5. CONCLUSION

This paper presents a new delay-dependent stability criterion for uncertainty Lurie-type neutral system with delays. The criterion takes into account the relationships between the term $x(t-\tau_1)$ and $x(t)-\int_{t-\tau_1}^{t}\dot{x}(s)ds,$ and $x(t-\tau_2)$ and $x(t)-\int_{t-\tau_2}^{t}\dot{x}(s)ds,$ the free weighting matrices used to express the relationships between, and reciprocal influences of neutral and discrete delays. The criterion has been expressed in the form of LMIs and is both neutral and discrete delay dependent. Two examples illustrate that this method provides over previous methods.

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