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COMMON FIXED POINT THEOREMS FOR MULTIVALUED MAPS IN PARTIAL METRIC SPACES

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ABSTRACT. Recently, Suzuki in 2008 obtained a generalization of Banach contraction principle. Subsequently, a number of new fixed/common fixed point theorems for mappings in metric spaces/partial metric spaces have been established by many authors. In this paper, we obtain a common fixed point theorem for multivalued maps in partial metric spaces which generalizes some well known results and also extends many results in the settings of partial metric.

1. Introduction

In 1994, Mathews[12] introduced the notion of partial metric spaces(PMS) as a part of denotational semantics of data for networks and proved the Banach contraction principle in partial metric context for the applications in program verification.

Definition 1.1. [12] A partial metric on a non empty set X is a function $p : X \times X \to \mathbb{R}^+$ such that for all $x, y, z \in X$:

(a) $x = y \Leftrightarrow p(x, x) = p(x, y) = p(y, y)$.
(b) $p(x, x) \leq p(x, y)$.
(c) $p(x, y) = p(y, x)$.
(d) $p(x, y) \leq p(x, z) + p(z, y) - p(z, z)$.

Then the pair (X, p) is said to be partial metric space.

If $p(x, y) = 0$, then (a) and (b) imply that $x = y$. But converse is not true in general. An obvious example of the partial metric space is $(\mathbb{R}^+, p)$, where partial metric p is defined as $p(x, y) = max\,\{x, y\}$.

If p is a partial metric on X, then the mapping $p^s : X \times X \to \mathbb{R}^+$ defined by $p^s(x, y) = 2p(x, y) - p(x, x) - p(y, y)$ is a metric on X.

Let (X, p) be a partial metric space. Then a sequence $\{x_n\}$ in X is called:

(i) Convergent to a point $x \in X$ iff $\lim_{n\to+\infty} p(x_n, x) = p(x, x)$;
(ii) Cauchy sequence whenever $\lim_{m,n\to+\infty} p(x_m, x_n)$ exists and finite.

A partial metric space (X, p) is said to be complete if every Cauchy sequence $\{x_n\}$ in X is convergent with respect to τ_p. Furthermore,

$$\lim_{m,n\to+\infty} p(x_m, x_n) = \lim_{n\to+\infty} p(x, x_n) = p(x, x).$$

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Lemma 1.2. [12] *Let (X,p) be a partial metric space, then*

(i) *A sequence $\{x_n\}$ in X is a Cauchy sequence in (X,p) iff it is Cauchy sequence in metric space (x,p^s).*
(ii) *A partial metric space (X,p) is complete if and only if the metric space (X,p^s) is complete.*

After the introduction of above concept by Mathews[12], various generalizations of contraction principle are obtained by researchers in partial metric spaces(see for instance, [2], [3], [6] and references therein). Most of the contractive conditions used in proving existence of fixed points in partial metric spaces (PMS) are extensions of well known contractive conditions used to investigate the existence of fixed points for maps in metric spaces (see, [16]). Nadler [13] was the first who extended the Banach contraction concept for multivalued mappings and proved the remarkable result for multivalued contractions in metric spaces. Afterwards, there appears many generalizations of Nadler's result (see for instance, [5], [8], [9], [10], [18] and references therein). Recently, Ayadi et al.[4] introduced the partial Hausdorff metric, showing that the Nadler's fixed point theorem can be generalized to the partial metric spaces also.

Let (X,p) be a partial metric space and $CB^p(X)(resp.\, CL^p(X))$ be the collection of non-empty closed and bounded (resp. closed) subsets of Xrespectively. The Hausdorff (resp. Generalized Hausdorff) metric H_p is defined by

$$H_p(A,B) = max\left\{\sup_{x\in A} p(x,B), \sup_{y\in B} p(y,A)\right\}$$

for every $A,B \in CB^p(X)(resp.\, CL^p(X))$, where $p(x,A) = \inf_{y\in A} p(x,y)$.
For a non-empty subset A of a partial metric space (X,p), $a \in \overline{A}$ if and only if $p(a,A) = p(a,a)$, where $\overline{A}$ denotes the closure of A with respect to partial metric p. Note that A is closed in (X,p) if and only if $A = \overline{A}$.

Throughout this paper, for $x,y \in X$, we follow the following notations, where f,g,S and T are mappings to be defined specifically in a particular context:

$$M(Sx,Ty) = max\left\{p(x,y), \frac{p(x,Sx)+p(y,Ty)}{2}, \frac{p(x,Ty)+p(y,Sx)}{2}\right\}$$

$$M_1(Sx,Ty) = max\left\{p(fx,gy), \frac{p(fx,Sx)+p(gy,Ty)}{2}, \frac{p(fx,Ty)+p(gy,Sx)}{2}\right\}$$

Suzuki[19], in 2008, introduced a new type of mapping and proved a good generalization of Banach contraction principle. Then, many results appeared in the literature on Suzuki type contraction conditions for the existence of fixed points for singlevalued as well as multivalued mappings in metric spaces (see [5], [7], [11], [17] and references therein).

Further, these results have been extended in the setting of partial metric spaces by many authors (see [1],[2], [14], [15] and references therein).

Recently, Rao et al.[15] introduced the new condition (W.C.C.) and obtained the Suzuki type fixed point theorems for a generalized multivalued mappings on partial Hausdorff metric spaces.

Definition 1.3. [15] Let (X,p) be a partial metric space with $f,g: X \to X$ and $S: X \to CB^p(X)$. Then the triplet $(f,g;S)$ is said to satisfy condition (W.C.C.) if $p(fx,gy) \leq p(y,Sx)$ for all $x,y \in X$.

Theorem 1.4. [15] *Let (X,p) be a complete partial metric space and $S,T: X \to CB^p(X)$ and $f,g: X \to X$. Assume that there exists $0 \leq r < 1$ such that for every $x,y \in X$,*

$$\phi(r)\, min\{p(fx,Sx), p(gy,Ty)\} \leq p(fx,gy) \Rightarrow H_p(Sx,Ty) \leq r\, M_1(Sx,Ty) \quad (1.1)$$

where, $\cup_{x\in X} Sx \subset g(X)$ and $\cup_{x\in X} Tx \subset f(X)$ and $\phi: [0,1) \to (0,1]$ defined as

$$\phi(r) = \begin{cases} 1 & \text{if } 0 \leq r < \frac{1}{2} \\ (1-r) & \text{if } \frac{1}{2} \leq r < 1. \end{cases} \quad (1.2)$$

If the triplet $(f,g;S)$ or the triplet $(f,g;T)$ satisfy the condition(W.C.C.), then f, g, S and T have a unique common fixed point in X.

Here, we remark that a common fixed point for the mappings exists even if the conditions of the above theorem are not satisfied. To prove our claim, we give following counter example where the conditions (1.1) and (W.C.C) of the above theorem are not satisfied but there is a common fixed point for the maps.

Example 1.5. Let $X = \{0,1,2\}$ be endowed with the partial metric $p: X \times X \to \mathbb{R}^+$ defined by

$$\begin{aligned} p(0,0) &= p(1,1) = 0, \qquad p(2,2) = \frac{1}{3}, \\ p(0,1) &= p(1,0) = \frac{1}{4}, \\ p(0,2) &= p(2,0) = \frac{2}{5}, \\ p(1,2) &= p(2,1) = \frac{13}{20}. \end{aligned}$$

Define the mappings $f,g: X \to X$ as identity maps and $S,T: X \to CB^p(X)$ by

$$Sx = \begin{cases} \{0\} & \text{if } x \in \{0,1\} \\ \{0,1\} & \text{if } x = 2 \end{cases} \quad \text{and } Tx = \begin{cases} \{0\} & \text{if } x \in \{0,1\} \\ \{1\} & \text{if } x = 2. \end{cases}$$

In this example, if we take $x=1, y=1$, then $p(fx,gy) = 0$, $H_p(Sx,Ty) = 0$, $min\{p(fx,Tx), p(gy,Ty)\} = \frac{1}{4}$ and $M(Sx,Ty) = \frac{1}{4}$. It is clear that

$$\phi(r) min\{p(fx,Tx), p(gy,Ty)\} > p(fx,gy) \text{ but } H_p(Sx,Ty) \leq r\, M(Sx,Ty).$$

And also, if we take $x=1, y=0$, then $p(fx,gy) = \frac{1}{4}$, $p(y,Sx) = 0$ and $p(y,Tx) = 0$. It means that condition (W.C.C.) is also not satisfied but 0 is the common fixed point of S and T.

Now, we give our main result which is more general for the existence of common fixed points of mappings in partial metric spaces. We use the following lemma essentially due to Nadler [13] in case of metric spaces and also true for the partial metric context as in [4].

Lemma 1.6. *Let $A, B \in CL^p(X)$ and $a \in A$, then for any $\epsilon > 0$, there exists a point $b \in B$ such that $p(a,b) \leq H_p(A,B) + \epsilon$.*

2. Main Results

Theorem 2.1. *Let X be a complete partial metric space and let S and T be maps from X to $CL^p(X)$. If there exists $r \in [0,1)$ such that for all $x, y \in X$,*

$$min\{p(x,Sx), p(y,Ty)\} \leq (1+r)p(x,y) \text{ implies } H_p(Sx,Ty) \leq rM(Sx,Ty). \tag{2.1}$$

Then there exists an element $z \in X$ such that $z \in Sz \cap Tz$.

Proof. Here, we take $M(Sx,Ty) > 0$. Otherwise, if $M(Sx,Ty) = 0$ then $x = y$ is a common fixed point of S and T. Let $\beta = r + \epsilon$ where $\epsilon > 0$. Let $x_0 \in X$ and $x_1 \in Tx_0$, by lemma 1.6, there exists $x_2 \in Sx_1$ such that

$$p(x_2, x_1) \leq H_p(Sx_1, Tx_0) + M(Sx_1, Tx_0).$$

Similarly there exists $x_3 \in Tx_2$ such that

$$p(x_3, x_2) \leq H_p(Tx_2, Sx_1) + \epsilon M(Tx_2, Sx_1).$$

Continuing in this manner, we find a sequence $\{x_n\}$ in X such that

$$x_{2n+1} \in Tx_{2n} \text{ and } x_{2n+2} \in Sx_{2n+1}$$

and

$$p(x_{2n+1}, x_{2n}) \leq H_p(Tx_{2n}, Sx_{2n-1}) + \epsilon M(Tx_{2n}, Sx_{2n-1})$$

$$p(x_{2n+2}, x_{2n+1}) \leq H_p(Sx_{2n+1}, Tx_{2n}) + M(Sx_{2n+1}, Tx_{2n}).$$

Now, we show that for any $n \in \mathbb{N}$,

$$p(x_{2n+1}, x_{2n}) \leq \beta\, p(x_{2n-1}, x_{2n}). \tag{2.2}$$

Suppose $p(x_{2n-1}, Sx_{2n-1}) \geq p(x_{2n}, Tx_{2n})$, then

$$min\{p(x_{2n-1}, Sx_{2n-1}), p(x_{2n}, Tx_{2n})\} \leq p(x_{2n-1}, Sx_{2n-1}) \leq (1+r)p(x_{2n-1}, x_{2n}).$$

By (2.1), we get

$$\begin{aligned}
p(x_{2n+1}, x_{2n}) &\leq H_p(Tx_{2n}, Sx_{2n-1}) \\
&\leq rM(Sx_{2n-1}, Tx_{2n}) \\
&\leq rM(Sx_{2n-1}, Tx_{2n}) + \epsilon\, M(Sx_{2n-1}, Tx_{2n}) \\
&= \beta\, M(Sx_{2n-1}, Tx_{2n}) \\
&= \beta\, max\Big\{p(x_{2n-1}, x_{2n}), \frac{p(x_{2n-1}, Sx_{2n-1}) + p(x_{2n}, Tx_{2n})}{2}, \\
&\quad \frac{p(x_{2n-1}, Tx_{2n}) + p(x_{2n}, Sx_{2n-1})}{2}\Big\} \\
&\leq \beta\, max\Big\{p(x_{2n-1}, x_{2n}), \frac{p(x_{2n-1}, x_{2n}) + p(x_{2n}, x_{2n+1})}{2}, \\
&\quad \frac{p(x_{2n-1}, x_{2n+1}) + p(x_{2n}, x_{2n})}{2}\Big\} \\
&\leq \beta\, max\{p(x_{2n-1}, x_{2n}), p(x_{2n}, x_{2n+1})\} \\
&\leq \beta\, p(x_{2n-1}, x_{2n})
\end{aligned}$$

which proves (2.2).
If $p(x_{2n}, Tx_{2n}) \geq p(x_{2n-1}, Sx_{2n-1})$ then

$$\begin{aligned}
min\{p(x_{2n-1}, Sx_{2n-1}), p(x_{2n}, Tx_{2n})\} &= p(x_{2n-1}, Sx_{2n-1}) \\
&\leq p(x_{2n-1}, x_{2n}) \leq (1 + r)p(x_{2n-1}, x_{2n}).
\end{aligned}$$

Now, from (2.1), we have

$$\begin{aligned}
p(x_{2n+1}, x_{2n}) &\leq H_p(Tx_{2n}, Sx_{2n-1}) \\
&\leq M(Sx_{2n-1}, Tx_{2n}) + \epsilon M(Sx_{2n-1}, Tx_{2n}) \\
&= \beta\, M(Sx_{2n-1}, Tx_{2n}) \\
&= \beta\, max\Big\{p(x_{2n-1}, x_{2n}), \frac{p(x_{2n-1}, Sx_{2n-1}) + p(x_{2n}, Tx_{2n})}{2}, \\
&\quad \frac{p(x_{2n-1}, Tx_{2n}) + p(x_{2n}, Tx_{2n-1})}{2}\Big\} \\
&\leq \beta\, max\Big\{p(x_{2n-1}, x_{2n}), \frac{p(x_{2n-1}, x_{2n}) + p(x_{2n}, x_{2n+1})}{2}, \\
&\quad \frac{p(x_{2n-1}, x_{2n+1}) + p(x_{2n}, x_{2n})}{2}\Big\} \\
&\leq \beta\, max\{p(x_{2n-1}, x_{2n}), p(x_{2n}, x_{2n+1})\} \\
&\leq \beta\, p(x_{2n-1}, x_{2n}).
\end{aligned}$$

This yields (2.2). In an analogous manner, we can show that

$$p(x_{2n+2}, x_{2n+1}) \leq \beta p(x_{2n+1}, x_{2n}). \tag{2.3}$$

Now, we conclude from (2.2) and (2.3) that for any $n \in \mathbb{N}$,

$$\begin{aligned}
& p(x_n, x_{n+1}) \leq \beta\, p(x_n, x_{n-1}) \\
\Rightarrow \quad & p(x_n, x_{n+1}) \leq \beta^n\, p(x_1, x_0).
\end{aligned}$$

Thus, for all $m, n \in \mathbb{N}$ with $m > n$, we get

$$\begin{aligned} p(x_m, x_n) &\leq p(x_n, x_{n+1}) + p(x_{n+1}, x_{n+2}) + \ldots + p(x_{m-1}, x_m) \\ &\leq \beta^n p(x_1, x_0) + \beta^{n+1} p(x_1, x_0) + \ldots + \beta^{m+n-1} p(x_1, x_0) \\ &\leq \beta^n \left(\frac{1-\beta^m}{1-\beta}\right) p(x_1, x_0) \leq \left(\frac{\beta^n}{1-\beta}\right) p(x_1, x_0). \end{aligned}$$

It implies $\lim_{m,n\to+\infty} p(x_m, x_n) = 0$ and so $\{x_n\}$ is a Cauchy sequence. Since X is complete, $\{x_n\}$ converges to some point $z \in X$, i.e., $\lim_{n\to+\infty} p(x_n, z) = p(z, z)$. Furthermore,

$$\lim_{m,n\to+\infty} p(x_m, x_n) = \lim_{n\to+\infty} p(x_n, z) = p(z, z) = 0.$$

Since $x_n \to z$, there exists $n_0 \in \mathbb{N}$ such that

$$p(x_n, z) \leq \frac{1}{3} p(z, y) \qquad \text{for } y \neq z \text{ and all } n \geq n_0.$$

As in ([14], 913),

$$\begin{aligned} (1+r)^{-1} p(x_{2n-1}, Sx_{2n-1}) &\leq p(x_{2n-1}, Sx_{2n-1}) \\ &\leq p(x_{2n-1}, x_{2n}) \leq p(x_{2n-1}, z) + p(z, x_{2n}) \\ &\leq \frac{2}{3} p(y, z) = p(y, z) - \frac{1}{3} p(y, z) \\ &\leq p(y, z) - p(x_{2n-1}, z) \\ &\leq p(x_{2n-1}, y). \end{aligned}$$

Therefore

$$p(x_{2n-1}, Sx_{2n+1}) \leq (1+r) p(x_{2n-1}, y). \tag{2.4}$$

Now, either $p(x_{2n-1}, Sx_{2n-1}) \leq p(y, Ty)$ or $p(y, Ty) \leq p(x_{2n-1}, Sx_{2n-1})$.
In either case by (2.4) and (2.1), we have

$$\begin{aligned} p(x_{2n}, Ty) &\leq H_p(Sx_{2n-1}, Ty) \leq rM(Sx_{2n-1}, Ty) \\ &\leq r\,max \left\{ p(x_{2n-1}, y), \frac{p(x_{2n-1}, Sx_{2n-1}) + p(y, Ty)}{2}, \frac{p(x_{2n-1}, Ty) + p(y, Sx_{2n-1})}{2} \right\}. \end{aligned}$$

Making $n \to \infty$, we get

$$\begin{aligned} p(z, Ty) &\leq r\,max \left\{ p(z, y), \frac{p(z, z) + p(y, Ty)}{2}, \frac{p(z, Ty) + p(y, z)}{2} \right\} \\ &\leq r\,max \left\{ p(y, z), \frac{p(z, Ty) + p(y, z)}{2} \right\} \end{aligned}$$

$$\Rightarrow \qquad p(z, Ty) \leq r\,max \left\{ p(y, z), \frac{p(z, Ty) + p(y, z)}{2} \right\}. \tag{2.5}$$

It is clear from (2.5) that

$$p(z, Ty) \leq r\, p(z, y). \tag{2.6}$$

Now, we show that

$$H_p(Sz, Tz) \leq r\,max \left\{ p(z, y), \frac{p(z, Sz) + p(y, Ty)}{2}, \frac{p(z, Ty) + p(y, Sz)}{2} \right\}. \tag{2.7}$$

Suppose that $y \neq z$, then for every $n \in \mathbb{N}$ there exists $z_n \in Ty$ such that

$$p(z, z_n) \leq p(z, Ty) + \frac{1}{n} p(y, z).$$

So by (2.6), we obtain

$$\begin{aligned} p(z, Ty) &\leq p(y, z_n) \leq p(y, z) + p(z, z_n) \\ &\leq p(y, z) + p(z, Ty) + \frac{1}{n} p(y, z) \\ &\leq p(y, z) + r\, p(y, z) + \frac{1}{n} p(y, z) \\ &= (1 + r + \frac{1}{n}) p(y, z). \end{aligned}$$

Hence

$$p(z, Ty) \leq (1 + r)\, p(y, z). \tag{2.8}$$

Now either $p(z, Sz) \leq p(y, Ty)$ or $p(y, Ty) \leq p(z, Sz)$.
So in either case, by (2.8) and the assumption we have

$$H_p(Sz, Ty) \leq r\, M(Sz, Ty)$$

which is (2.7). Now taking $y = x_{2n}$ in (2.7), we get

$$\begin{aligned} P(Sz, x_{2n+1}) &\leq H_p(Sz, Tx_{2n}) \\ &\leq r\, max \left\{ p(z, x_{2n}), \tfrac{p(z,Sz)+p(x_{2n},Tx_{2n})}{2}, \tfrac{p(z,Tx_{2n})+p(x_{2n},Sz)}{2} \right\} \\ &\leq r\, max \left\{ p(z, x_{2n}), \tfrac{p(z,Sz)+p(x_{2n},x_{2n+1})}{2}, \tfrac{p(z,x_{2n+1})+p(x_{2n},Sz)}{2} \right\}. \end{aligned}$$

Taking the limit as $n \to \infty$ we have

$$\begin{aligned} & p(Sz, z) \leq \frac{r}{2} p(Sz, z) \\ \Rightarrow \quad & p(Sz, z) = 0 = p(z, z) \\ \Rightarrow \quad & z \in \overline{Sz} = Sz. \end{aligned}$$

With similar arguments, we can show that $z \in Tz$. Hence $z \in Sz \cap Tz$. □

Now we show that the Example 1.5 satisfies the conditions (2.1) of the Theorem 2.1 with $r = \frac{10}{21}$ for all $x, y \in X$. Note that Sx and Tx are closed for all $x \in X$ under the given partial metric p.

(i) If $x = y = 0$ then $H_p(Sx, Ty) = 0$, $min\{p(x, Sx), p(y, Ty)\} = 0$ and $M(Sx, Ty) = 0$.

(ii) If $x = 0, y = 1$ then $H_p(Sx, Ty) = 0$, $min\{p(x, Sx), p(y, Ty)\} = 0$ and $M(Sx, Ty) = \frac{1}{4}$.

(iii) If $x = 0, y = 2$ then $H_p(Sx, Ty) = \frac{1}{4}$, $min\{p(x, Sx), p(y, Ty)\} = 0$ and $M(Sx, Ty) = \frac{13}{20}$.

(iv) If $x = 1, y = 0$ then $H_p(Sx, Ty) = 0$, $min\{p(x, Sx), p(y, Ty)\} = 0$ and $M(Sx, Ty) = \frac{1}{4}$.

(v) If $x = 1, y = 1$ then $H_p(Sx, Ty) = 0$, $min\{p(x, Sx), p(y, Ty)\} = \frac{1}{4}$ and $M(Sx, Ty) = \frac{1}{4}$.

(vi) If $x = 1, y = 2$ then $H_p(Sx, Ty) = \frac{1}{4}$, $min\{p(x, Sx), p(y, Ty)\} = \frac{1}{4}$ and $M(Sx, Ty) = \frac{13}{20}$.
(vii) If $x = 2, y = 0$ then $H_p(Sx, Ty) = 0$, $min\{p(x, Sx), p(y, Ty)\} = 0$ and $M(Sx, Ty) = \frac{2}{5}$.
(viii) If $x = 2, y = 1$ then $H_p(Sx, Ty) = 0$, $min\{p(x, Sx), p(y, Ty)\} = \frac{1}{4}$ and $M(Sx, Ty) = \frac{13}{20}$.
(ix) If $x = 2, y = 2$ then $H_p(Sx, Ty) = \frac{1}{4}$, $min\{p(x, Sx), p(y, Ty)\} = \frac{2}{5}$ and $M(Sx, Ty) = \frac{21}{40}$.

Thus for all $x, y \in X$ with $r = \frac{10}{21}$, we get

$$min\{p(x, Tx), p(y, Ty)\} \leq (1 + r)p(x, y) \text{ implies } H_p(Sx, Ty) \leq rM(Sx, Ty).$$

Evidently, $0 \in S0 \cap T0$.

Here, we remark that our result, i.e. Theorem 2.1 is also generalization of the result of R. Kamal et al. ([11], Theorem 2.2) in partial metric context.

Now if we take S and T as single valued mappings of X, we get following result which is generalization of ([14], Theorem 2) and extension of ([11], Corollory 2.3).

Theorem 2.2. *Let X be a complete partial metric space and $S, T : X \to X$. Assume there exists $r \in [0, 1)$ such that for every $x, y \in X$,*

$$min\,\{p(x, Sx), p(y, Ty)\} \leq (1 + r)p(x, y) \text{ implies } d(Sx, Ty) \leq r\,M(Sx, Ty).$$

Then S and T have a unique common fixed point.

Proof. It can be proved easily by taking S and T as single valued maps in Theorem 2.1. Uniqueness of the common fixed point is obvious. □

Taking $S = T$ in Theorem 2.1, we get following Corollaries which are generalizations of results of [18] in the settings of partial metric.

Corollary 2.3. *Let X be a complete partial metric space and $T : X \to CL(X)$. Assume there exists $r \in [0, 1)$ such that for every $x, y \in X$,*

$$p(x, Tx) \leq (1 + r)p(x, y) \text{ implies } H_p(Tx, Ty) \leq r\,M(Tx, Ty).$$

Then there exists $z \in X$ such that $z \in Tz$.

Corollary 2.4. *Let X be a complete partial metric space and $T : X \to X$. Assume there exists $r \in [0, 1)$ such that for every $x, y \in X$,*

$$p(x, Tx) \leq (1 + r)p(x, y) \text{ implies } p(Tx, Ty) \leq r\,M(Tx, Ty).$$

Then T has a unique fixed point.

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