

Influence of The Filtering Method in The Kinematic Data Consistency of Biomechanical Systems: A Benchmark Example

F. J. Alonso¹, J. Cuadrado² and P. Pintado³

¹Departamento de Ingeniería Mecánica, Energética y de los Materiales, Universidad de Extremadura
Avda. de Elvas s/n, 06071 Badajoz, Spain

²Departamento de Ingeniería Industrial II, Universidad de La Coruña, Mendizábal s/n, 15403 Ferrol, Spain

³Departamento de Mecánica Aplicada, Universidad de Castilla La Mancha, Camilo José Cela s/n
13071 Ciudad Real, Spain

The inverse dynamic analysis of biomechanical systems is corrupted by numerous sources of error that reduce its usefulness. The most important errors are the raw displacement differentiation and the kinematic inconsistency induced by skin motion. The first type of error is mainly due to the amplification of high-frequency low-amplitude noise introduced by the motion capture system when the raw displacement signals are differentiated. The second source of error, the skin motion artifact, produces violations of the kinematic constraint equations of the multibody system. This work studies the influence of the filtering method in the kinematic consistency of biomechanical systems. The objective is to compare the obtained results using several classical and advanced filtering and smoothing schemes: Butterworth filter, GCVSPL (splines), singular spectrum Analysis and the Hodrick-Prescott filter and filter parameters. A benchmark example that includes computer generated data of a four-bar mechanism was processed using the filtering-consistency method to study the influence of several parameters. The results show that the most important error is the raw data problem. The kinematic consistency must be imposed on the smoothed data; in fact, the consistency condition does not eliminate the high-frequency low-amplitude noise present in the displacement signals. Another important conclusion is that the kinematic consistency can detect the presence of subfiltering or overfiltering.

Keywords: Biomechanics, Filtering, Smoothing, Signal processing, Skin Motion, Raw Data Problem, Kinematic Consistency.

1. INTRODUCTION: THE RAW DATA PROBLEM

The systems for motion capture currently used in biomechanical analysis introduce measurement errors that appear in the form of noise in the recorded displacement signals. In particular, the noise is amplified unacceptably when the displacements are differentiated in order to obtain velocities and accelerations (Vaughan, 1982). Since this may lead to large errors in the inverse dynamics analysis of biomechanical systems (Hatze, 2002), it is necessary to filter the displacement signal prior to differentiation. The filtering of displacement signals to obtain noiseless velocities and accelerations has been widely studied in the literature. Traditional filtering techniques include digital Butterworth filters, splines, and filters based on spectral analysis (Pezzack et al., 1977; Hatze, 1981; Dowling, 1985; D'Amico & Ferrigno, 1990, 1992; Van den Bogert, 1996; Giakas & Baltzopoulos, 1997a, 1997b, 1998; Vaughan, 1982; Walker, 1998; Yu et al., 1999). These traditional filtering methods are well suited for stationary noise and signal patterns, but are poorly suited to smoothing nonstationary signals. This drawback is particularly problematic in biomechanical analysis since many physical activities involve impacts (Woltring, 1995; Giakas et al., 2000; Georgiakis et al., 2002).

Instead, advanced filtering techniques such as discrete wavelet transforms (Adham & Shibab, 1999), the Wigner function (Giakas et al., 2000), singular spectrum analysis (Alonso et al., 2005a) and the Hodrick-Prescott filter (Alonso et al., 2005b) have been applied. While these methods yield better results than conventional techniques, their application is more complex and a number of additional parameters must be chosen. For instance, one has to introduce a mother wavelet function for discrete wavelet transforms, the parameters of the filtering function for the Wigner function, an appropriate window length and grouping strategy for singular spectrum analysis, and the proper smoothing parameter for the Hodrick-Prescott filter. The aim of this work is to compare the results obtained by using several classical and advanced filtering and smoothing schemes: Butterworth filter, GCVSPL (splines), singular spectrum analysis (SSA) and the Hodrick-Prescott filter; and to study the importance of the filtering method in the kinematic data consistency results: namely, the robustness of the results with respect to variations in the filtering parameters.

2. FILTERING AND SMOOTHING METHODS

The following sections briefly describe the major steps of the filtering and smoothing methods used in this work.

Let $\mathbf{x} = (x_0, x_1, \dots, x_{N-1})$ be the acquired raw displacement signal measured at equidistant time with sampling frequency f_s and $\mathbf{y} = (y_0, y_1, \dots, y_{N-1})$ the filtered signal.

2.1. Butterworth Filter

The Butterworth filter is a recursive filter, the equation that describes the filter is:

$$y_n = a_0 x_n + a_1 x_{n-1} + a_2 x_{n-2} + b_1 y_{n-1} + b_2 y_{n-2} \quad (1)$$

where x_n is the raw acquired signal, y_n is the filtered signal and a_0, a_1, a_2, b_1 and b_2 are the constants of the filter. The recursive character of the filter introduces a phase lag into the signal. To remove the phase lag it is necessary to apply the filter in both forward and backward direction. The constants are dependent on the sample rate and the desired cutoff frequency. Details of application of this filter to biomechanical signals are given in Pezzack *et al.* (1977), Challis (1999), Yu (1999) and Erer (2007).

2.2. Generalized Cross-validated Quintic Splines

The first and second derivatives of a given displacement signal can be calculated by fitting a quintic spline $y(t)$ to the displacement signal $\mathbf{x} = (x_0, x_1, \dots, x_{N-1})$ and differentiating the resulting equations (Woltring, 1986). The generalized cross validation (GCV) criterion as described by Woltring (1986) assumes uncorrelated, additive noise to be present in the data and selects automatically the optimal amount of filtering. The spline function $y(t)$ is chosen which minimizes

$$\min_y \left[\sum_{i=1}^N (y_i - x_i)^2 + p \int \left[y^{(n+1)}(t) \right]^2 dt \right] \quad (2)$$

where x_i are the raw data samples, and y^k denotes the k th derivative of $y(t)$. The weight factor p is the smoothing parameter, which is related to how closely the smoothed data tracks the raw data: for $p = 0$ no smoothing (spline passes through each point) is applied, for $p = \infty$ spline is 1-piece polynomial fit to entire data set. The GCVSPL has the advantage of producing continuous estimates which are needed in some applications. Details of application of this smoothing method are given in Woltring (1986).

2.3. Singular Spectrum Analysis (SSA)

SSA is a novel non-parametric technique used in the analysis of time series and based on principles of multivariate statistics. A concise description of the method will be given in this section, whereas Golyandina *et al.* (2001) have presented a complete derivation. Let $\mathbf{x} = (x_0, x_1, \dots, x_{N-1})$ be the length N time series representing the noisy displacement signal and let L represent a window length. The first step is to construct a Hankel matrix $\mathbf{X} = [\mathbf{X}_1, \mathbf{X}_2, \dots, \mathbf{X}_K]$ with $\mathbf{X}_j = (x_{j-1}, x_j, \dots, x_{j+L-2})^T$,

where $K = N - L + 1$, from the original signal by sliding a window with length L . The second step is to perform a singular value decomposition (SVD) of the Hankel matrix $\mathbf{X} = \mathbf{U}\mathbf{D}\mathbf{W}^T$. The final step is to keep only the r first singular values of \mathbf{X} that account for a selected (large) percentage of the total variance and to reconstruct the signal with the r value.

2.4. Hodrick-Prescott Filter

In Econometrics, the HP filter is one of the standard tools used to decompose a macroeconomic time series $\mathbf{x}^T = (x_0, x_1, \dots, x_{N-1})$ into a nonstationary trend component $\mathbf{y}^T = (y_0, y_1, \dots, y_{N-1})$ that represents the filtered signal, and a stationary residual component (noise signal) $\mathbf{c}^T = (c_0, c_1, \dots, c_{N-1}) = \mathbf{x}^T - \mathbf{y}^T$ (Baxter and King, 1999; Hodrick and Prescott, 1980; King and Rebelo, 1993; Reeves, 2000). The filter equations are derived from the following assumptions:

1. $\Delta^2 \mathbf{y} \sim \mathbf{N}(\mathbf{0}, \sigma_y^2 \mathbf{I})$
2. $\mathbf{c} \sim \mathbf{N}(\mathbf{0}, \sigma_c^2 \mathbf{I})$
3. $\Delta^2 \mathbf{y}$ is independent of \mathbf{c}
4. σ_c^2 and σ_y^2 are known

Under these assumptions, the maximum-likelihood estimate of the filtered signal \mathbf{y} is

$$\min_y \left(\mathbf{c}^T \mathbf{c} + \lambda (\Delta^2 \mathbf{y})^T \Delta^2 \mathbf{y} \right) \quad (3)$$

where λ is the ratio between the variances, $\lambda = \sigma_c^2 / \sigma_y^2$.

In the minimization problem (3), the effect of λ , usually termed the smoothing parameter, is to penalize the sum of the squared accelerations in the second term. Details of application of this filter to biomechanical signals are given in Alonso *et al.* (2005b).

3. THE SKIN MOTION ARTIFACT: KINEMATIC DATA CONSISTENCY

The skin motion artifact is due to the skin displacement and deformation that causes marker movement with respect to the underlying bone. This motion produces spurious reaction forces and driver moments when the inverse dynamic analysis (IDA) is performed, which result from violations of the kinematic constraint equations as seen in Fig. 1. This is regarded as one of the most critical sources of error in the IDA (Kuo, 1998; Gunther *et al.*, 2003; Leardini *et al.*, 2005). The techniques designed to minimize the contribution of this artifact can be divided into those which model the skin surface and those which include kinematic constraint equations of the biomechanical model.

In this work, we correct the filtered displacement signals in order to satisfy the kinematic multibody

equations of the biomechanical model to ensure that the length of each body segment remains constant during the simulation. The process is shown in Fig. 1. It is important to note that the raw data smoothing procedure does not ensure the kinematic data consistency in the biomechanical multibody model because the kinematic constraint equations are not necessarily satisfied after smoothing. This is due to the nature of the skin motion, which is a large-amplitude modulated oscillatory motion, and cannot be considered a part of the noise present in the displacement signal (Fig. 1). Moreover, the kinematic data consistency of the multibody model does not eliminate the noise present in the signal.

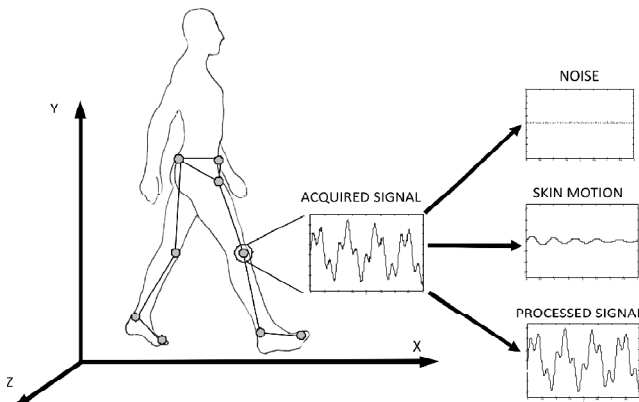


Figure 1: Biomechanical model with markers. The grey circles represent the natural coordinates of the multibody model. The figure illustrates the presence of measurement noise and skin motion artifact in the acquired signal

Silva and Ambrósio (2002) were the first to apply a systematic procedure using a multibody formalism to ensure kinematic data consistency. First, the initially inconsistent positions are used to calculate the constant lengths of the rigid bodies of the biomechanical model. Second, the intersegmental angles are calculated from the filtered displacement data, and the position analysis is performed using these angles. The kinematic analysis produces a new set of marker positions, which are consistent with the kinematic structure of the biomechanical model at each instant (Silva *et al.*, 2002, 2005).

To implement this procedure, the kinematic constraint equations of the biomechanical model are obtained

$$\Phi(\mathbf{q}) = 0 \quad (4)$$

where \mathbf{q} is the vector of generalized coordinates (marker coordinates) and $\Phi(\mathbf{q}) = 0$ is the vector of kinematic constraint equations of the multibody system. Newton–Raphson iterative method is applied to solve the nonlinear system (4) and obtain kinematically consistent data. The Newton–Raphson procedure starts by making an estimate

of the desired solution vector. Using the non-consistent (filtered raw-data) positions \mathbf{q}^* as an initial guess to the Newton–Raphson procedure (Silva *et al.*, 2002), consistent positions \mathbf{q} are obtained. The vector \mathbf{q}^* represents a correction of the inconsistent positions \mathbf{q}^* . Using the obtained corrected positions \mathbf{q} as a new initial guess to the Newton–Raphson procedure, and iterating the procedure one obtains a convergent solution after three or four iterations. This is due to the quadratic convergence rate of the method in the neighbourhood of the solution. The expression that describes the iterative procedure is:

$$\Phi_{\mathbf{q}}(\mathbf{q}_i) \Delta \mathbf{q}_i = -\Phi(\mathbf{q}_i) \quad (5)$$

where $\Phi_{\mathbf{q}}(\mathbf{q}_i)$ is the Jacobian matrix of the constraints at the inconsistent position \mathbf{q}_i , and $\Delta \mathbf{q}_i = \mathbf{q} - \mathbf{q}_i$ is the generalized coordinates correction at iteration i . Solving the above linear system iteratively one obtains a vector \mathbf{q} that satisfies the kinematic constraint equations with sufficient precision. The procedure is halted when the norm of the constraint equations matrix is smaller than a certain tolerance value, namely

$$\|\Phi(\mathbf{q}_i)\| < \varepsilon = 10^{-6} \quad (6)$$

Solving Eq. (5) for each instant of the simulation and applying the termination criterion (6), one obtains kinematically consistent data with sufficient precision. It is assumed that the Newton–Raphson method converges at each time of the simulation. This supposition is reasonable because the initial estimate of the desired solution is close to the exact solution due to the negligible amplitude of the skin motion with respect to the rigid body motion.

Consistent velocities $\dot{\mathbf{q}}$ and accelerations $\ddot{\mathbf{q}}$ are obtained by solving velocity and acceleration equations of the multibody system:

$$\begin{aligned} \Phi_{\mathbf{q}}(\mathbf{q})\dot{\mathbf{q}} &= 0 \\ \Phi_{\mathbf{q}}(\mathbf{q})\ddot{\mathbf{q}} &= -\dot{\Phi}_{\mathbf{q}}\dot{\mathbf{q}} \end{aligned} \quad (7)$$

No differences were observed when comparing the kinematically consistent positions, velocities and accelerations obtained using Eqs. (5-7) with a data set where consistent positions obtained using Eq. (5) are directly differentiated using finite differences to calculate velocities and accelerations (Silva, 2002).

This procedure produces reasonable good results (Silva *et al.*, 2002, 2005; Alonso *et al.*, 2007a). Nevertheless, the biomechanical model is driven by the angular histories calculated from the inconsistent input data, which are not the true angular histories. To overcome the calculation of the intersegmental angles from inconsistent data, we propose the simultaneous correction of the natural coordinates and intersegmental angles by performing a projection of the position solution

to the constraint manifold, in order to obtain a new set of positions \mathbf{q} that satisfies $\Phi = \mathbf{0}$ (Alonso *et al.*, 2007b). This scheme projects the set of natural coordinates measured with the motion capture system and calculates the angular histories from the consistent positions. The projection can be obtained by the solution of the following constrained minimization problem (Bayo and Ledesma, 1996):

$$\min_{\mathbf{q}} V = \frac{1}{2}(\mathbf{q} - \mathbf{q}^*)^T \mathbf{W}(\mathbf{q} - \mathbf{q}^*) \quad s.t. \quad \Phi = \mathbf{0} \quad (8)$$

where \mathbf{W} is a weighting matrix. Different weighting factors can be assigned to each natural coordinate to reflect the average degree of skin movement artifact associated to each coordinate. An identity weighting matrix has been used in this work. Using an augmented Lagrangian method to minimize the above function (Bayo and Ledesma, 1996), the following iterative scheme to calculate the consistent data positions \mathbf{q} is obtained:

$$(\mathbf{W} + \Phi_{\mathbf{q}}^T \alpha \Phi_{\mathbf{q}}) \Delta \mathbf{q}_{i+1} = -\mathbf{W} \Delta \mathbf{q}_i - \Phi_{\mathbf{q}}^T \alpha \Phi \quad (9)$$

where $\Delta \mathbf{q}_{i+1}$ and $\Delta \mathbf{q}_i$ are the position data corrections and the subscripts indicate the iteration number. Equation (9) can be solved iteratively until $\|\Delta \mathbf{q}_i\| < \varepsilon$, where ε is a user specified tolerance. The value of the penalty factor α only affects the convergence rate. Bayo *et al.* (1996) recommend penalty factors ranging from 10^5 to 10^7 to obtain a good convergence rate. The execution times to solve equations (5) and (8) were similar. In order to obtain consistent velocities $\dot{\mathbf{q}}$, we perform again an orthogonal projection of the velocities $\dot{\mathbf{q}}^*$ (calculated using the filtered data) to the velocity constraint manifold. This can be achieved by the solution of the linear equation:

$$(\mathbf{W} + \Phi_{\dot{\mathbf{q}}}^T \alpha \Phi_{\dot{\mathbf{q}}}) \dot{\mathbf{q}} = \mathbf{W} \dot{\mathbf{q}}^* \quad (10)$$

To obtain consistent accelerations, the projection of the accelerations $\ddot{\mathbf{q}}^*$ (calculated using the filtered data) onto the constraint manifold can be obtained through the solution of the following equation:

$$(\mathbf{W} + \Phi_{\ddot{\mathbf{q}}}^T \alpha \Phi_{\ddot{\mathbf{q}}}) \ddot{\mathbf{q}} = \mathbf{W} \ddot{\mathbf{q}}^* - \Phi_{\ddot{\mathbf{q}}}^T \alpha \dot{\Phi}_{\dot{\mathbf{q}}} \dot{\mathbf{q}} \quad (11)$$

4. RESULTS

To test the performance of the proposed procedure, several benchmark raw displacement signals (observed signals) were processed using the smoothing and filtering procedures presented in section 2, and the orthogonal projection scheme described in section 3. The computer-generated data correspond to the simulation of a four-bar crank-rocker mechanism during two crank revolutions (Fig. 2a). The input angular velocity and the lengths of

the links were fixed to $\dot{\theta} = 2\pi$ rad/s, $L_1 = 2m$, $L_2 = 8m$ and $L_3 = 5m$. The time step adopted was $h = 0.01s$ (sampling frequency $f_s = 100$ Hz) for a total time of simulation of 2 s. The original data $\mathbf{q} = (x_1, y_1, x_2, y_2)^T$ were corrupted to simulate both the skin motion artifact and noise introduced by the motion capture system using three different patterns: Gaussian noise, sinusoidal stationary noise and non-stationary noise. The non-stationary noise is simulated by adding lumped point masses connected to the system by viscoelastic unions. Figures 2a-2f show the original data and the corrupted data.

To study the influence of the filtering method in the obtained results, Butterworth (BW), GCVSPL, singular spectrum analysis (SSA), and the Hodrick-Prescott (HP) filter were applied to raw displacement signals.

The operation of the filter mainly depends on the filter parameter selection. Several attempts have been made to design a fully automatic optimal filter, but this objective has not been reached yet. Three filter parameter selection cases were studied: subfiltering (there exists some degree of amplification of high-frequency noise when the raw displacement signals are differentiated), normal filtering (proper filtering parameters are chosen), and overfiltering (the noise is filtered, but the filtering procedure produces loss of information of the displacement signal). A total number of $3 \times 4 \times 3 = 36$ studies were made considering the noise pattern, the filter method and the parameter selection.

There are no general rules for selecting the values of the filtering parameters. Their values depend on the type of signal to be analysed, and on the type of noise to be removed prior to differentiation. The choice is, as always, determined, on the one hand, by the desire to faithfully reproduce fast transients and, on the other hand, by the need to reduce noise in the accelerations. The literature presents a complete description of parameter selection criteria for each type of filter. To illustrate parameter selection, Table 1 presents the parameter selection for the Gaussian stationary noise. Figure 3 presents the Hodrick-Prescott smoothed kinematic signals y_2^* , \dot{y}_2^* , \ddot{y}_2^* along with the reference kinematic signals y_2 , \dot{y}_2 , \ddot{y}_2 using the parameters of Table 1.

Table 1
The Filters Parameter Set (Gaussian Stationary Noise)

Filter	Subfiltering	Normal	Overfiltering
BW	$f_c = 0.5 f_s$ Hz	$f_c = 0.05 f_s$ Hz	$f_c = 0.01 f_s$ Hz
GCVSPL	$f_c = 0.5 f_s$ Hz	$f_c = 0.05 f_s$ Hz	$f_c = 0.01 f_s$ Hz
SSA	$L = 100$ $r = 20$	$L = 100$ $r = 2$	$L = 100$ $r = 1$
HP	$\lambda = 10$	$\lambda = 1e3$	$\lambda = 1e5$

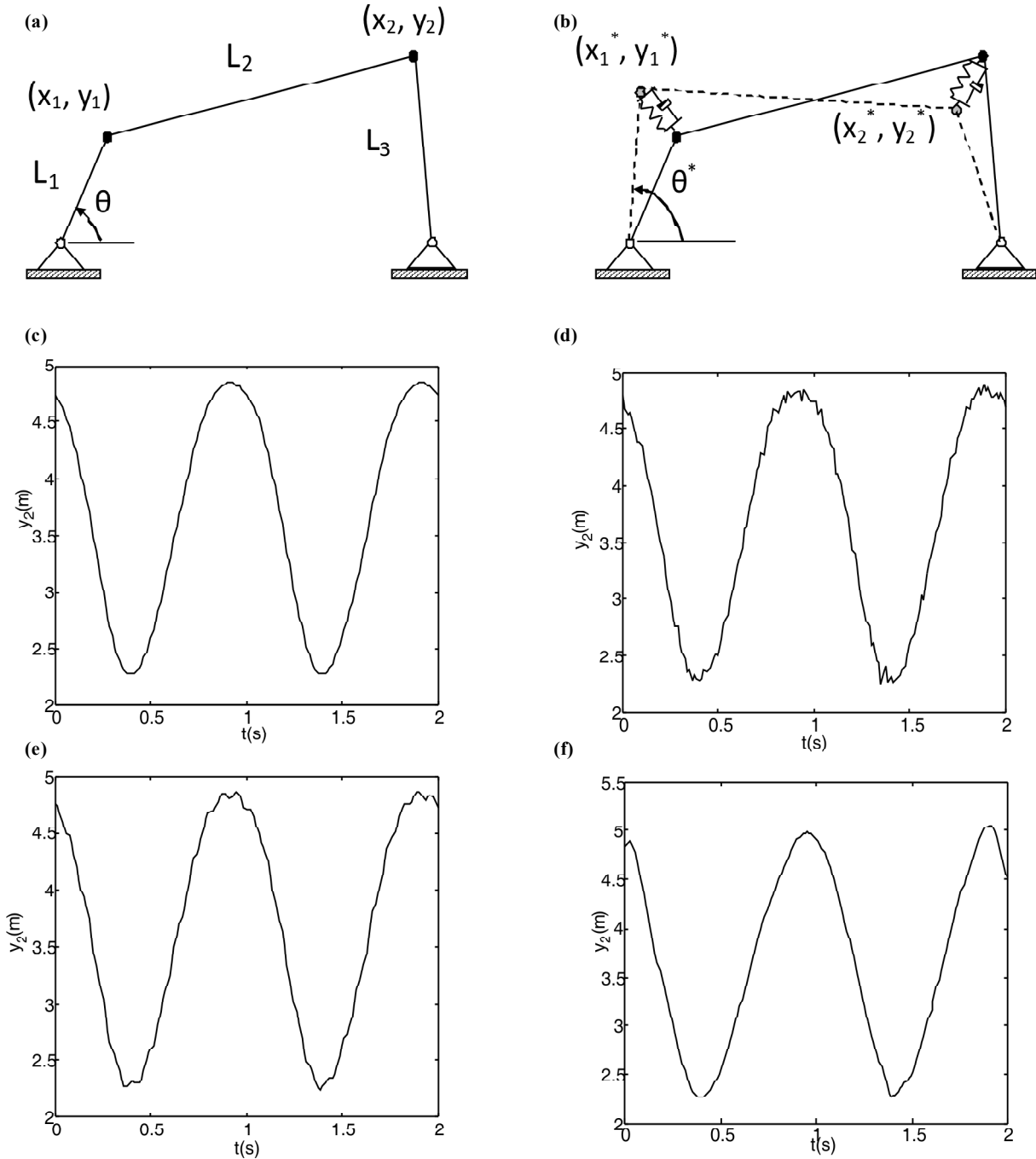


Figure 2: The four-bar benchmark example: (a) original coordinates; (b) model for the simulation of non-stationary noise; (c) the original y_2 ; (d) the corrupted y_2^* using Gaussian noise; (e) the corrupted y_2^* using sinusoidal stationary noise; (f) the corrupted y_2^* using non-stationary noise

In order to compare the results obtained by different procedures, the normalized root mean square (rms) of the error signal of acceleration is evaluated. The error signal is the difference between the reference acceleration signal and the signal obtained by the smoothing-consistency method.

$$\eta = \sqrt{\frac{\sum_{j=1}^5 \sum_{n=1}^N [\ddot{q}_{jn} - \ddot{q}_{jn}^*]^2}{\sum_{j=1}^5 \sum_{n=1}^N \ddot{q}_{jn}^2}} \quad (13)$$

where \ddot{q}_{jn} is the reference acceleration of coordinate j at instant n and \ddot{q}_{jn}^* is the acceleration of coordinate j at instant n obtained by the smoothing-consistency method.

Figures 4-6 show the residuals obtained for the Gaussian noise (amplitude $p = L_1/100$), the sinusoidal stationary noise, and the non-stationary noise, using the filtering methods presented in section 2 and, then, ensuring the kinematic consistency through the

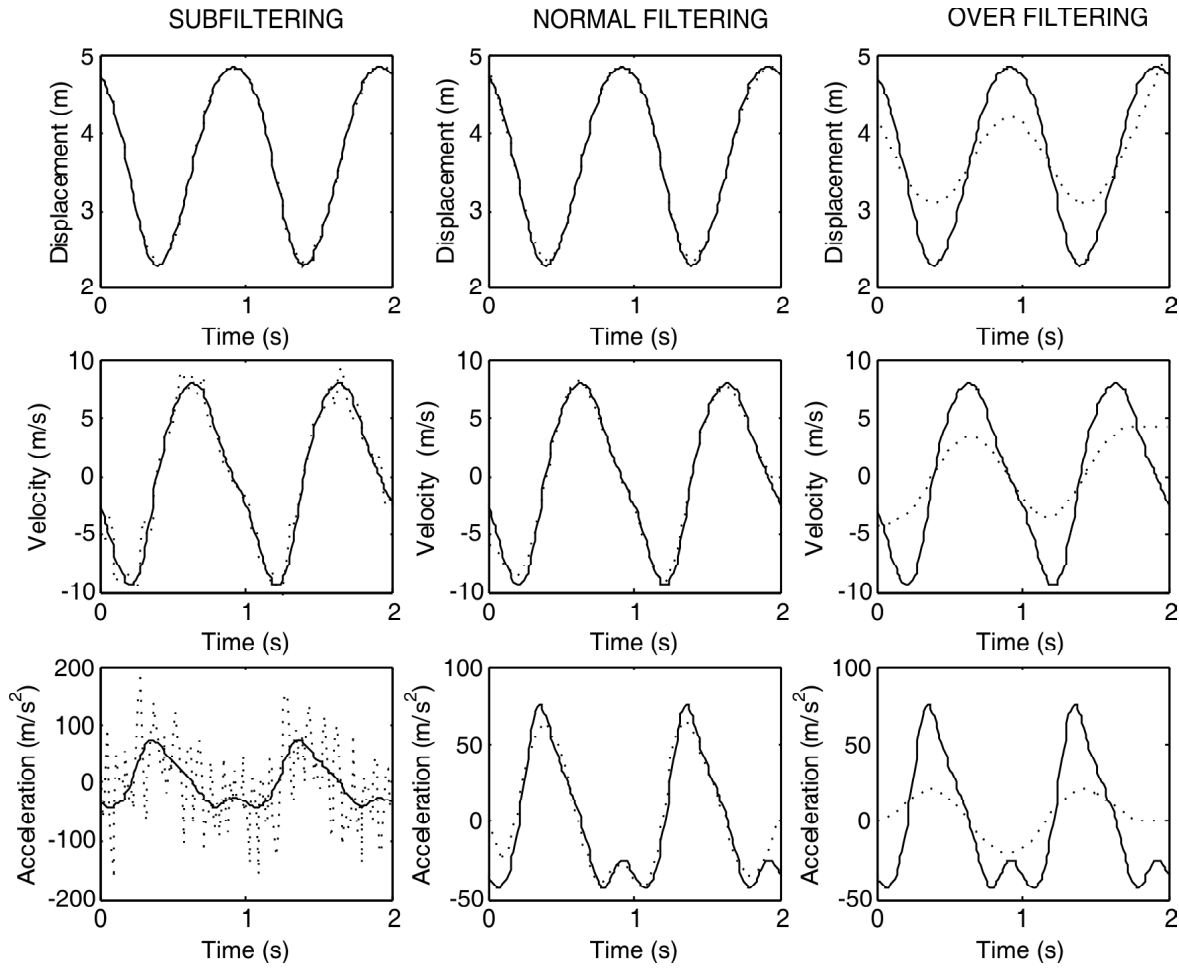


Figure 3: HP-smoothed kinematic signals (dashed line) and reference kinematic signals (solid line). Coordinate y_2^* and its derivatives (Gaussian stationary noise)

orthogonal projection methods for $\mathbf{q} = (x_1, y_1, x_2, y_2)^T$ and its higher derivatives.

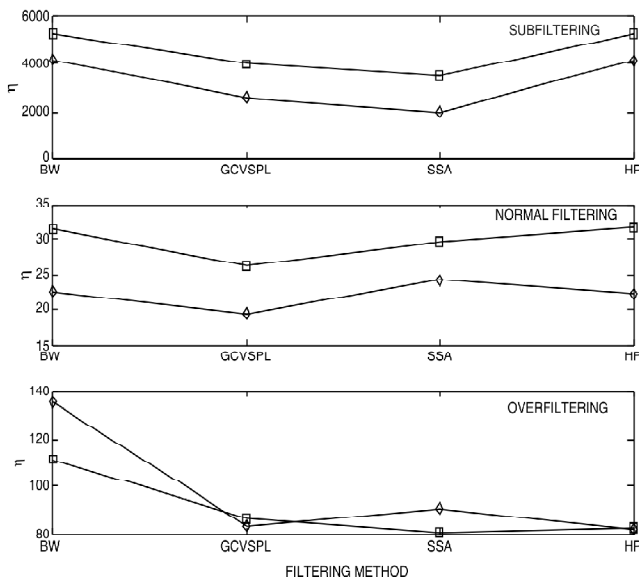


Figure 4: Acceleration residuals for Gaussian noise: (□) no kinematic consistency applied, (◇) kinematic consistency applied

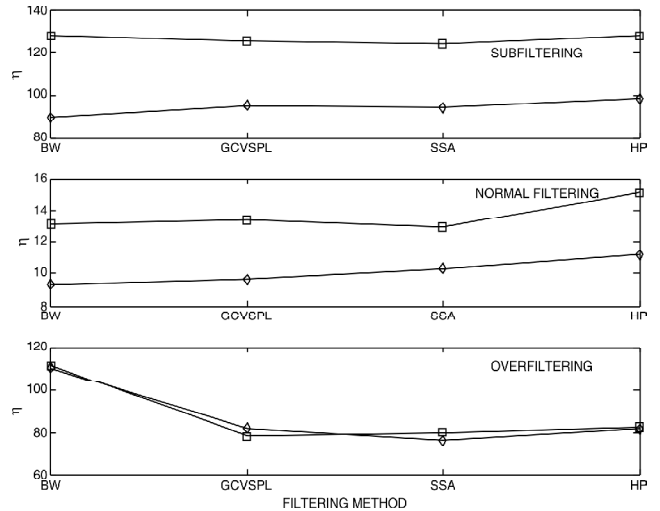


Figure 5: Acceleration residuals for sinusoidal stationary noise: (□) no kinematic consistency applied, (◇) kinematic consistency applied

Results show that the amount of filtering plays a key role in the kinematic data consistency. Namely, overfiltering causes the projection method to slightly improve the obtained acceleration residuals. In this case,

the displacement projection method does not converge. Overfiltering causes large changes in displacement signals that affects the lengths of the links of the four-bar mechanism (see Fig. 3). Signals in order to satisfy the kinematic multibody equations of the biomechanical model to ensure that the length of each body segment remains constant during the simulation.

The proper selection of filtering parameters to calculate the higher derivatives prior to projection dramatically improves the results, as shown in Figs. 4-6. Moreover, the projection method significantly improves the obtained acceleration residuals when optimal filtering is performed. This fact illustrates the importance of the raw displacement smoothing method in the kinematic consistency problem.

Another important conclusion is that the kinematic consistency can't be imposed directly on the raw data, because the consistency condition does not eliminate the high-frequency low-amplitude noise present in the displacement signals, as can be seen in the subfiltering case results (Figs. 4-6).

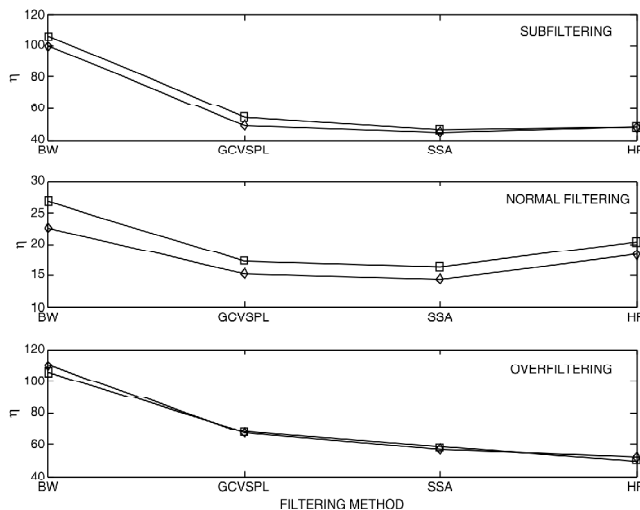


Figure 6: Acceleration residuals for non-stationary noise: (□) no kinematic consistency applied, (◇) kinematic consistency applied

5. DISCUSSION AND CONCLUSIONS

The results of this work show the influence of the filtering method in the kinematic data consistency, in the context of the analysis of biomechanical systems. Several noise patterns (including stationary and non-stationary), filtering methods (including classical and advanced) and amount of filtering (from subfiltering to overfiltering) have been tested. The results draw several relevant conclusions for biomechanical analysis that remark the importance of the selection of proper filtering parameters prior to projection:

- The consistency condition does not eliminate the high-frequency low-amplitude noise present in

the displacement signals, so the consistency condition cannot substitute the filtering of displacement signals.

- Overfiltering produces a noiseless displacement signal with loss of information that can affect the convergence of the orthogonal projection.
- The kinematic consistency can serve to check the presence of subfiltering or overfiltering since, in these cases, the projection method only achieves a slight improvement of the residuals.

Future works will need to focus on the possibilities of producing an automatic algorithm, and on the embedding of the algorithm in commercial biomechanical analysis packages. Namely, techniques will be devised to automatically choose the filtering parameters for smoothing and differentiation of the given signal. Further research is required in order to automate a procedure that detects the presence of subfiltering or overfiltering in the projection step and selects the optimal filtering parameters that produce a noiseless kinematic consistent data with minimum loss of information.

REFERENCES

- [1] Adham, R. I., Shihab, S. A., Discrete wavelet transform: a tool in smoothing kinematic data, *Journal of Biomechanics* **32**: 317–321, 1999.
- [2] Alonso, F. J., Del Castillo, J. M., Pintado, P., Application of singular spectrum analysis to the smoothing of raw kinematic signals, *Journal of Biomechanics* **38**: 1085–1092, 2005.
- [3] Alonso, F. J., Del Castillo, J. M., Pintado, P., Filtering of kinematic signals using the Hodrick-Prescott filter, *Journal of Applied Biomechanics* **21**: 271–285, 2005.
- [4] Alonso, F. J., Del Castillo, J. M., Pintado, P., Motion data processing and wobbling mass modelling in the inverse dynamics of skeletal models, *Mechanism and Machine Theory* **42**: 1153–1169, 2007.
- [5] Alonso, F. J., Cuadrado, J., Del Castillo, J. M., Pintado, P., Orthogonal projection methods for the kinematic data consistency of biomechanical systems, *Multibody Dynamics*, Milano, Italy, 2007.
- [6] Baxter, M., King, R. G., Measuring business cycles: Approximate band-pass filters for economic time series, *The Review of Economics and Statistics* **81**: 575–593, 1999.
- [7] Bayo, E., Ledesma, R., Augmented Lagrangian and mass-orthogonal projection methods for constrained multibody dynamics, *Nonlinear Dynamics* **9**: 113–130, 1999.
- [8] Challis, J. H., A procedure for automatic determination of filter cutoff frequency for the processing of biomechanical data, *Journal of Applied Biomechanics* **15**: 303–317, 1999.
- [9] D'Amico, M., Ferrigno, G., Technique for the evaluation of derivatives from noisy biomechanical displacement data using a model-based bandwidth selection procedure, *Medical and Biological Engineering and Computing* **28**: 407–415, 1990.
- [10] D'Amico, M., Ferrigno, G., Comparison between the more recent techniques for smoothing and derivative assessment in biomechanics, *Medical and Biological Engineering and Computing* **30**: 193–204, 1992.

- [11] Dowling, J., A modelling strategy for the smoothing of biomechanical data, In: B. Johnsson (Ed.), *Biomechanics*, Vol. XB. Human Kinetics, Champaign, IL, 1163–1167, 1985.
- [12] Erer, K. S., Adaptive usage of the Butterworth digital filter, *Journal of Biomechanics* **40**: 2934–2943, 1985.
- [13] Georgiakis, A., Stergioulas, L. K., Giakas, G., Wigner filtering with smooth roll-off boundary for differentiation of noisy nonstationary signals, *Signal Processing* **82**: 1411–1415, 2002.
- [14] Giakas, G., Baltzopoulos, V., A comparison of automatic filtering techniques applied to biomechanical walking data, *Journal of Biomechanics* **30**: 847–850, 1997.
- [15] Giakas, G., Baltzopoulos, V., Optimal digital filtering requires a different cut-off frequency strategy for the determination of the higher derivatives, *Journal of Biomechanics* **30**: 851–855, 1997.
- [16] Giakas, G., Baltzopoulos, V., Improved extrapolation techniques in recursive digital filtering: a comparison of least squares and prediction, *Journal of Biomechanics* **31**: 87–91, 1998.
- [17] Giakas, G., Stergioulas, L.K., Vourdas, A., Time–frequency analysis and filtering of kinematic signals with impacts using the Wigner function: accurate estimation of the second derivative, *Journal of Biomechanics* **33**: 567–574, 2000.
- [18] Golyandina, N., Nekrutkin, V., Zhigljavsky, A., *Analysis of time series structure – SSA and related techniques*, Chapman and Hall/CRC, Boca Raton, Florida, 2001.
- [19] Gunther, M., Sholukha, V. A., Kebler, D., Vank, W., Blickhan, R., Dealing with skin motion and wobbling masses in inverse dynamics, *Journal of Mechanics in Medicine and Biology* **3**: 439–444, 2003.
- [20] Hatze, H., The use of optimally regularized Fourier series for estimating higher-order derivatives of noisy biomechanical data, *Journal of Biomechanics* **14**: 13–18, 1981.
- [21] Hatze, H., The fundamental problem of myoskeletal inverse dynamics and its implications, *Journal of Biomechanics* **35**: 109–115, 2002.
- [22] Hodrick, R. J., Prescott, E. C., Post-war U. S. business cycles: An empirical investigation. Discussion paper 451, Carnegie-Mellon University, 1980.
- [23] King, R. G., Rebelo, S. T., Low frequency filtering and real business cycles. *Journal of Economic Dynamics and Control* **17**, 207–231, 1993.
- [24] Kuo, A. D., A least squares estimation approach to improving the precision of inverse dynamics computations, *ASME Journal of Biomechanical Engineering* **120**: 148–159, 1998.
- [25] Leardini, A., Chiari, L., Della Croce, U., Cappozzo, A., Human movement analysis using stereophotogrammetry, Part 2: Soft tissue artifact assesment and compensation, *Gait and Posture* **21**: 212–225, 2005.
- [26] Pezzack, J. C., Norman, R. W., Winter, D. A., An assessment of derivative determining techniques used for motion analysis, *Journal of Biomechanics* **10**: 377–382, 1977.
- [27] Reeves, J.J., The Hodrick-Prescott filter, a generalization, and a new procedure for extracting an empirical cycle from a series, *Studies in Nonlinear Dynamics and Econometrics* **4**: 447–454, 2000.
- [28] Silva, M.P.T., Ambròsio, J.A.C., Kinematic data consistency in the inverse dynamic analysis of biomechanical systems, *Multibody System Dynamics* **8**: 219–239, 2002.
- [29] Silva, M.P.T., Ambròsio, J.A.C., Sensitivity of the results produced by the inverse dynamic analysis of a human stride to perturbed input data, *Gait and Posture* **19**: 35–49, 2004.
- [30] Van den Bogert, A.J., de Koning, J.J., On optimal filtering for inverse dynamics analysis, Proceedings of the Ninth Canadian Society of Biomechanics Congress, Burnaby, BC, 214–215, 1996.
- [31] Vaughan, C. L., Smoothing and differentiation of displacement-time data: an application of splines and digital filtering, *International Journal of Bio-Medical Computing* **13**: 375–386, 1982.
- [32] Walker, J.A., Estimating velocities and accelerations of animal locomotion: a simulation experiment comparing numerical differentiation algorithms, *Journal of Experimental Biology* **201**: 981–995, 1998.
- [33] Woltring, H. J., A FORTRAN package for generalized, cross-validatory spline smoothing and differentiation, *Advances in Engineering Software*. **8**: 104–113, 1986.
- [34] Woltring, H. J., Smoothing and differentiation techniques applied to 3-D data. In: Allard, P., Stokes, I.A.F., Blanche, J.P. (Eds.), *Three Dimensional Analysis of Human Movement*. Human Kinetics Champaign, IL, 79–99, 1980.
- [35] Yu, B., Gabriel, D., Noble, L., An K.N., Estimate of the optimum cutoff frequency for the Butterworth low-pass digital filter, *Journal of Applied Biomechanics* **15**: 318–329, 1999.